

The Seven Liberal Arts,

A STUDY IN MEDIAEVAL CULTURE.

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To my wife,
Helen C. Abelson.

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INTRODUCTION.

Modern historical criticism lays great stress on two conceptions which it properly terms canons for the scientific study of history. Influenced by the theory of evolution, it assumes that there is a continuity in all history; that the present is deeply rooted in the past; that while "the old order changeth, yielding place to the new," the change is ever gradual and slow; that in the history of mankind cataclysms are no more to be found than in the development of the present shape of the earth. In the second place it maintains that the only fair way to judge the past is not from the standpoint of the present but in the light of the contemporary spirit of the particular age with which we are engaged. Nowadays, he is considered a poor historian indeed, who does not give proper weight to the *Zeitgeist* of by-gone days.

In no branch of historical investigation have these two canons wrought such remarkable changes as in the study of the Middle Ages. The new attitude towards this fascinating period of history has in some fields completely destroyed the value of much that passed for historical truth fifty years ago. To such an extent have our ideas about the middle ages been modified that the attitude of the modern scholar toward nearly all the traditional views of the period is one of skeptical reserve. "Things," he urges, "could not have been as we have been led to believe they were."

It is from this point of view that the writer has undertaken to investigate one phase of the culture of the middle ages. The aim of this monograph is to present an intensive study of the culture of the period by applying to the material presented the test of the two mentioned canons. The problem, in short, has been to discover how the inevitable spirit of change and adjustment interacted with the spirit of tradition ages in the single sphere of mediæval life—the culture of the average educated

man of the day. Such a task could obviously be accomplished in but one way—by a broad study of the varied fortunes of the instruments of culture of the period, namely, the agencies that trained the average educated man.

The investigation thus narrowed itself to a definition of the limits of the curriculum of the seven liberal arts; a determination of the scope of each of the subjects taught; the discovery of any intrinsic progress in the quantity and quality of the instruction afforded by the mediæval schools; and finally, the comprehension of the relation of this liberal education to the amount of knowledge possessed by the mediæval world during the period and to the ~~entire~~ scheme of the mediæval *Weltanschauung*.

Naturally the material for such an inquiry, as is the case in all of what the Germans conveniently term "*Kulturstudien*," was not to be found simply by working some neglected vein of historical ore. The problem was rather to seek among the scattered materials in the whole field of mediæval history for data that would throw light on the questions involved. The conclusions here presented are based primarily on a detailed examination of the typical text books used throughout the periods in the schools that offered a higher education in the study of the seven liberal arts. Proper evaluation of this material was made possible by recourse to the results of recent historical investigations in the field of Latin literature during the middle ages, the researches in the history of the mathematical sciences, and to some extent in the history of philosophy. The great mass of periodical literature, French, and especially German, devoted to mediæval historical research likewise yielded many results both positive and negative. Intensive studies in the general cultural conditions of limited territories in Western Europe have also been utilized.

I wish to express my thanks to Professors D. E. Smith, Paul Monroe and E. L. Thorndike of Teachers' College, Columbia University, for their assistance in the preparation of this monograph. Professor Smith read the chapters on the Quadrivium and gave me a number of helpful suggestions. Prof. Monroe has carefully gone over and criticised the entire volume. Professor Thorndike kindly assisted in seeing the monograph through the press. To my friend and colleague, Mr. Robert I.

Raiman, I am indebted for many valuable suggestions on matters of diction and style. But above all I am under special obligations to Professor James Harvey Robinson of Columbia University, at whose suggestion the writer first interested himself in this phase of mediæval culture. Such merit as the present work may possess is chiefly due to his stimulating instruction, helpful criticism, guidance and friendly counsel.

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The Seven Liberal Arts.

A STUDY IN MEDIAEVAL CULTURE.

CHAPTER I.

The Development of the Curriculum of the Seven Liberal Arts.

The question of the historical development of the curriculum comprising the seven liberal arts has received very little attention from scholars.¹ The available sources at hand are meagre, and the opinions of the authorities conflicting, on many interesting questions. On the chief point, however, all are agreed; namely: that historically the cycle of the seven liberal arts in the middle ages was an outgrowth of the ancient Greek and Roman systems of education; that from the Greek and Roman course of study modified by the introduction of Christian ideals of education the curriculum of the middle ages was gradually evolved. In the present chapter this development and adaptation will be briefly traced.

In earliest times Greek education consisted of the study of music and gymnastics.² With the development of Greek life and thought the subject of music was broadened to include

¹ The only three specific studies on the topic that the writer has been able to find come from the pens of English and American scholars: Thomas Davidson, "The Seven Liberal Arts," in *The Educational Review*, Vol. II., pp. 467-473, and also as an appendix to the same author's *Aristotle etc.*; H. Parker, "The Seven Liberal Arts," *The English Historical Review*, Vol. V., pp. 417-461; A. F. West, chapter on "Seven Liberal Arts" in his *Alcuin and the Rise of the Christian Schools*, pp. 4-27.

² P. Monroe, *Source Book of the History of Education*, p. 17 et seq.; p. 223.

poetry and letters. Plato's ideal scheme for the higher education of the "guardian class" contemplated a complete course in what we should call elementary, secondary and higher education. The elementary,—gymnastics, music, letters (i. e., grammar), was to last till the twentieth year; secondary education, lasting from the twentieth to the thirtieth year, was to embrace science:—arithmetic, geometry, astronomy, and musical harmony. The higher education, from the thirtieth to the thirty-fifth year, was to consist of the study of philosophy, as a final preparation for the practical life of the ideal citizen. In the first two divisions of the course we have a possible foreshadowing of the future separation of the curriculum into the Trivium and the Quadrivium.¹

Aristotle's curriculum, which was to serve as a foundation for higher studies, consisted of (1) reading and writing, (2) gymnastics, (3) music, and sometimes drawing. He too considered gymnastics and music important branches.² Aristotle's ideas on education as they have come down to us are very fragmentary. There is no evidence to support the theory that he placed grammar, rhetoric, dialectic, arithmetic, geometry and astronomy in the preparatory curriculum of Greek education, as has been asserted. From the text of Book VIII of the *Politics*, the only source on Aristotle's educational ideas, it certainly does not appear that these subjects of Plato's advanced curriculum of higher studies also formed what has been called "Aristotle's program of secondary education."³ In the absence of any definite statement by Aristotle as to his views on the subject it would be fair to assume that they agreed in the main with those of Plato, with the exception perhaps, that his plan for the scientific training of a man would call for more natural science than mathematics.⁴ But even these "scientific studies," whether mathematical or biological, were never considered part of the secondary curriculum to be

¹ Plato, *Republic*, Book II, par. 374 et seq., Book III, VI, VII, Jowett's translation. Cf. Bosanquet, *The Education of the Young in the Republic of Plato*, pp. 12-16.

² Aristotle, *Politics*, Jowett's translation, Chap. VIII, 3, pp. 245 et seq., Edition of 1885.

³ By Davidson, *Aristotle etc.* pp. 198, 199.

⁴ Cf. Burnet, *Aristotle on Education*, p. 135.

studied before the Greek boy had reached the "ephebean" age.¹

Set over against these "scientific studies" were the "practical studies" of the Sophists. Their ascendancy made *rhetoric* and not *natural science* the essential subject of the higher education of the Greeks. One of the effects of this change was that some study of mathematics had to be introduced into the preparatory curriculum since the value of mathematics to the Greeks was considered very great. Thus we find that in the third century the preparatory work of the Greek young man consisted of a study of the following subjects: (1) gymnastics, (2) grammar (rudiments of the language), (3) music, (4) drawing, (5) arithmetic, (6) geometry,—the last two being distinctly mentioned as the more advanced studies. Here again we have a foreshadowing of the distinction between the Trivium and the Quadrivium, the division of the later periods.² We find no further record of changes in the Greek curriculum of the liberal studies till the first century of the Christian era.

In the works of Philo Judaeus (fl. c. 30 A. D.), there are frequent references to the encyclical liberal arts. He includes in his list all those that were current three hundred years before his time, except drawing and gymnastics. He adds, however, rhetoric and dialectic. All these studies are spoken of as preparatory subjects and are clearly differentiated from the higher study—philosophy. Philo does not mention astronomy in his catalogue of preparatory subjects.³

¹ Cf. Laurie, *Survey of Pre-Christian Education*, pp. 287 et seq.

It should be noted here that while the works of Plato and Aristotle on education are theoretical plans, they represent essentially the practice of Greek education in their day. For preparatory education, the subject under discussion, Plato and Aristotle appear to have adopted in toto the prevailing Greek ideas of the time. Vide Monroe, *op. cit.* pp. 132, 133.

² Cf. Davidson, *Aristotle etc.*, p. 240, in which the Greek authorities are quoted. The quotation omits gymnastics, an obvious error, as at no time was that subject omitted from the Greek preparatory curriculum.

³ It should not be supposed that the instruction in these various fields, when relegated to the position of preparatory subjects, was as thorough as when they were considered as advanced studies. The higher school too often dominates the one of lower rank in the educational scale, and, as today the college largely determines the curriculum of the secondary school, so the rhetorical school in the Hellenistic period of Greek history, ranking as it did with the modern college, determined the character of the preparatory instruction. Hence the instruction in these different liberal studies must have been little

In the first half of the third century we find a writer alluding to (1) grammar, (2) rhetoric, (3) geometry, (4) arithmetic, (5) astronomy, and (6) music, in such a way as to leave no doubt that he speaks of them as liberal arts.¹ Dialectic, according to him, was then still considered a higher study.

In Greece, then, it would appear that until after the period of Alexander the Great, there were only three or four preparatory studies in the curriculum: letters, music, gymnastics, and drawing; that with the growth of knowledge, more subjects, like dialectics and mathematics, were added to the curriculum; and that in this way gradually there came to exist in free Hellas a cycle of studies which were considered the minimum of liberal training for the average free man. Although the number of these was not definitely fixed at this time, at least six of the seven subjects which later constituted the mediæval curriculum were studied in the first century of the present era.²

Passing on to the Romans, we find that only after the second Punic War did Rome begin to adopt Greek ideas of education. After the first period of innovation during which the education of the young Roman was conducted by Greek teachers in the Greek language, strong efforts were made to establish a national literature and a national education. This period marks the beginning of text books written in Latin. M. T. Varro (116–27 B.C.), contemporary of Cicero and Cæsar, attempted in common with them to build up a system of education based on Greek ideas, but in which Roman and not Greek literature should be the basis. To further his objects he wrote treatises on all the subjects taught in Greece in his day, but omitted drawing and included astronomy. His work, now lost, was entitled *Disciplinarum Libri Novem*, and contained treatises on grammar, rhetoric, dialectic, geometry, arithmetic, astrology, music, medicine and architecture.³ The books were undoubtedly written for the

more than elementary at this period, and surely was not better than the instruction afforded in the average school of the early middle ages. For full references to Philo vide Davidson, *op. cit.*, pp. 242–243.

¹ Sextus Empiricus, cited by Davidson, *op. cit.*, p. 243.

² L. Grasberger, *Erziehung und Unterricht im Klassischen Alterthum*, Bd. II, p. 237 et seq.

³ Ritschl, "De M. Varronis Disciplinarum Libris," in *Opuscula* III, pp.

secondary schools, which were at this time already clearly differentiated from the higher rhetorical schools. The practical secondary curriculum of the day was grammar in its narrow sense, and also literature, arithmetic, geometry and music—a repetition of the Greek curriculum of the third century before Christ.¹

Seneca (B.C. 2–A.D. 65), an authority for his age on matters pertaining to our subject, did not limit the number of secondary liberal studies to seven; in fact, his ideas as to what constituted the circle of the liberal studies seem to have been very indefinite. Now it is grammar, music, geometry, arithmetic and astronomy; now he speaks of medicine as a liberal art; again he plainly considers rhetoric and dialectic as higher studies and hence a part of philosophy.²

Nor does Quintilian (A.D. 35–95) seem to have had any conception of seven liberal arts. To be sure he speaks of the preparatory training given by the grammaticus as affording the Greek *ἐγκύκλια παιδεῖα*. But the cycle of preparatory studies which his model student of oratory was to master consisted only of grammar, a little music, geometry and astronomy; and even these, with the exception of grammar, were to be studied in a superficial manner. This was all the instruction given till the sixteenth year of the boy's age.³

So far, then, we are even further removed from the seven liberal arts than we were at the end of our survey of the Greek period. Apparently the influence of Varro had been of no avail to make permanent the curriculum which he had closely modelled after the Greeks. His failure to establish in Rome a modified Greek curriculum was due to the fact that soon after his time rhetoric and dialectic came to be considered advanced

352-372. West, *Alcuin etc.*, p. 7, accepts Ritschl's conclusions that these were the subjects of the nine treatises, and cites additional proof in support of the former's conclusion. All authorities agree that Varro did write treatises on Grammar, Rhetoric, Dialectic, Music, Geometry, and Architecture. Boissier, *Etude sur la Vie et les Ouvrages de M. T. Varron*, p. 333 et seq., thinks Ritschl has not proved his point in regard to the other three subjects, which conclusion Davidson (*op. cit.*) accepts.

¹ Cf. Laurie, *op. cit.* pp. 337-363.

² Seneca, Ep. 88, 89, *Opera*, Leipzig edition, 1882, Vol. IV. pp. 38 et seq.

³ Quintilian, *Instit.*, Book 1., Chaps. VIII–X, Watson's translation, Vol. 1. pp. 70-80.

studies. The Roman secondary course as compared with the Greek, which the Romans were plainly attempting to follow, was thus much briefer. In the first place the period of secondary education in Rome was shorter, lasting till the sixteenth year of a boy's age, not till the twenty-first as in Greece. Then again, the Romans' character made them reject from the curriculum anything not of a practical nature. Therefore the study of mathematics, in which the contemplative Greek delighted, was reduced to a minimum in the Roman course of study. It is only as practical studies that Quintilian advised the study of geometry, arithmetic and astronomy.

The Roman schools, partially Hellenized, maintained their efficient and practical character for some generations after Quintilian, but a decline set in after the third century. As the spirit of superficiality crept in, everything was sacrificed to the immediate practical needs of the time. Excepting in Gaul and Spain, where the racial characteristics of the Gallic mind encouraged the study of rhetoric and eloquence as the highest form of cultivation, the curriculum of the average Roman school contracted, and the grammaticus encroached upon the work of the rhetor, a result which Quintilian had foreseen.¹ In this way, while there was a decline in the field of higher education, the work in the secondary school was enriched by the addition of the elements of rhetoric and dialectics. The curriculum of the Roman school of this period consisted, then, of a superficial study of the elements of grammar, rhetoric, dialectic, arithmetic, geometry, music and astronomy. This undoubtedly was the accepted standard in the pagan schools at the beginning of the fourth century.

The evidence on this point becomes very convincing when we consider the latest researches as to the date of the work of Martianus Capella *De Nuptiis Philologiæ et Mercurii*.² Capella, judging from the very little that is known of his life, could hardly have been a formulator of a curriculum; hence his work which describes the content of the seven liberal arts—grammar, dialectic, rhetoric, geometry, arithmetic, astronomy, music, in the order given—would seem to have represented the accepted

¹ Quintilian, *Instit.*, Bk. II, Ch. 1, Watson's trans. I, pp. 96-99.

² Parker establishes beyond doubt the date of this epoch-making book as before 330; vide Parker, *English Historical Review* V., pp. 444-446.

standards of his time. Wishing to write a poetic account of the liberal arts, he most naturally turned to the work of Varro—the *Disciplinarum Libri Novem*—as a model. To a provincial of Africa like Capella, Varro, “vir Romanorum eruditissimus,” was certainly a sufficient authority to follow in matters of both style and content.¹ Again, the work of Capella like that of Varro, is divided into nine books, although since the first two books form the allegorical introduction, the subjects treated are only seven. The assumption that he followed Varro absolutely in treating the content is thus supported by the similarity in form of presentation. Indeed, according to his own statement, Capella omitted medicine and architecture, the two other subjects included in Varro’s treatise, only in order to keep within the mythological setting of his allegorical framework.²

The fourth century then, may be taken to represent the period when the curriculum of the Pagan schools in the Empire assumed the fixed character of a course in the seven liberal arts. This century of social and political transformation marked the final stages in the evolution of a course of study, which through the medium of the public school—a characteristic institution of the later empire—proved perhaps the most effective means of preserving the classical culture of the ancient world.

But during the three centuries in which this pagan curriculum was thus crystallizing, it was viewed with rooted suspicion by the Christian leaders. Throughout the second, third and fourth centuries men of the type of Origen, Tertullian, and Jerome felt that these very schools were the most formidable opponents of the new religion. And, when one considers how much was at stake in the contest, the vehement denunciations of the pagan education by these men become intelligible enough.

¹ His book is a medley of prose and verse, a form of literature which gained currency through Varro. Vide A. Ebert, *Allgemeine Geschichte der Literatur des Mittelalters im Abendlande*, Bd. I., p. 459 et seq.

² Vide Capella’s *De Nuptiis Philologiae et Mercurii*, Eyssenhardt’s edition 1866, p. 332, where the author expressly states that medicine and architecture are omitted because such purely material subjects have no place in a work treating of super-mundane interests. The whole subject of the relation of Capella’s work to that of Varro is exhaustively treated by Parker, *loc. cit.* passim.

The close of the fourth century, however, saw a decisive change; the triumph of Christian ideals was complete; and the decadence of the old education in nearly every part of the Empire was as marked as the decline of paganism. Now that these schools no longer menaced the supremacy of the rising church, the merits of the old curriculum could be considered dispassionately by Christian leaders. It did not require much discernment to see that while the rhetorical pagan schools of Gaul as they existed in the fourth and fifth centuries in their decadent form were worse than useless from the Christian point of view, the subject matter of their instruction,—literature, rhetoric and dialectic—was of real value to the Christians as an aid to the study of theology. Such in fact was the prevailing opinion among the devotees of the new religion even in the earlier period of strife with paganism. Even in the respective generations of Origen, Tertullian and Jerome the Christians had frequented these schools to a limited extent, and while a good Christian could not conscientiously become a teacher of rhetoric, it was not considered improper for him to study in the pagan institutions. It would seem reasonable then, that later in the fifth century Christians would not offer objections to the introduction of the seven liberal arts of Capella as a part of a curriculum which was to be a preparation for the study of theology.¹

The tendency to use the elements of the pagan curriculum in Christian education was encouraged by Augustine himself (354-430), who wrote treatises on grammar, rhetoric, dialectic, arithmetic, geometry and music—six of the seven subjects of the mediæval curriculum.² His influence was potent in committing the church to a recognition of the arts as suitable subjects for Christian study. He sought to prove his contention by recalling the biblical sanction which justified “despoiling the Egyptians.” Although not the originator of the curriculum of the seven liberal arts, he, more than any one else, made possible

¹ For a full treatment of the subject of the period of the conflict between Pagan and Christian ideals, vide Schmid, *Geschichte der Erziehung etc.* II, pp. 48-83; Boissier, *La Fin du Paganisme* I, p. 233 et seq.; O. Denk, *Geschichte des Gallo-Frankischen Unterrichts und Bildungswesens* pp. 140-163.

² Augustine, *Retractationes* I, c. 6, Migne XXXII, col. 591; *Confessiones* IV, c. 16, *ibid.* col. 704. For a full discussion of Augustine's relation to the curriculum of the seven liberal arts vide Parker, *loc. cit.*, p. 427 et seq.

its general adoption by the Christian world of the west.¹

With the support of such eminent authority it was but natural that the position of the secular liberal arts as a part of Christian education in the mediæval curriculum should become secure.² As far as is known Cassiodorus (480-575) was the first Christian to use the term "seven liberal arts" in his *De Artibus et Disciplinis Liberalium Literarum*, a work designed to supplement his earlier work, *De Institutis Literarum Sacrarum*. He supports Augustine's contention as to the necessity of the study of the liberal arts as a preparation for sacred studies, and quotes biblical proof to show that the number of these studies must be seven. The text "Wisdom builded her house; she has hewn out her seven pillars" (Proverbs IX; 1) gives him conclusive scriptural authority for his views. This quotation is of significance. It means that at this time Christian leaders admitted the necessity of incorporating secular studies into the Christian curriculum, and since these secular studies had been definitely seven in number for over a century and a half, reasons were to be found from a Christian standpoint explaining scripturally that their number seven was divinely sanctioned. With Cassiodorus, the number and the subjects of the mediæval curriculum become definitely fixed. His authority, together with that of Augustine before him, was all sufficient to give to it that rigid character which it maintained for some nine hundred years. Their work marks the final transition of the pagan curriculum into the mediæval world of letters.

From the time of Cassiodorus the term "seven liberal arts" is the regular expression for the round of preparatory secular studies. Isidore of Seville uses it as well as the terms "trivium" and "quadrivium."³ So do Alcuin, Rabanus Maurus and the scholastic writers. Later, so definite did this term become, the seven liberal arts were not infrequently made the subjects of poems and paintings.⁴

¹ Augustine, *De Doctrina Christiana* II, c. 40, Migne, XXXIV. col. 63.

² His final judgment on the subject is found in one of his last productions, *De Doctrina Christiana*, a work in which are expressed his matured views on Christian training. Vide *loc. cit.*, especially II, c. 18, 40-42.

³ Vide Isidore, *Etymologiae Lib. XX*, I, 2, III, 1, Migne, *Pat. Lat.* LXXXII cols. 73, 153.

⁴ Theodulfi "Carmen de septem liberalibus artibus in quadam pic-

Summing up the results of this inquiry, the following generalizations may be made:

(1) In some five centuries of educational development the Greeks evolved a curriculum of seven liberal arts. The entire content of this curriculum was preserved for Rome in the Latin language by Varro.

(2) This curriculum was adopted by Rome in part only during the period of the later Republic and the early Empire.

(3) In the declining days of the Empire a marked reversion to the entire Greek curriculum took place, and its form and content were definitely fixed about the close of the third century of our era.

(4) Christianity, triumphant, found it necessary to appropriate to its own use the content of this curriculum as a preparatory discipline for its higher study of theology.

(5) Having settled upon the fundamental proposition of the value of secular studies, the Christian leaders followed the line of least resistance, and adopting *in toto* the pagan curriculum then in existence, hallowed their choice by scriptural sanction.

(6) The spirit of tradition, so characteristic of the period, was strong enough to maintain this curriculum in its rigidity throughout the middle ages.

tura depictis", in Duemmler, *Poetae Latini Aevi Carolini*, 544-7. Cf. Ebert, *op. cit.* II, p. 77. For a copy of a typical artistic illustration, vide Cubberley, *Syllabus of Lectures on the History of Education*.

The term *liberal* as applied to the arts was used as early as the days of Plato and Aristotle. It was to distinguish studies fit for freemen, which do not vulgarize and are not a preparation for gainful occupations. (*Polit.* VII, 2.) Later on the general preparatory course came to be known often as the ἐγκύκλια παιδεία. (Grasberger, II, 235). The Romans did not use the term till after their adoption of Greek ideas of education. Seneca used the term "Liberalia Studia" (*Epist.* 88). Cassiodorus used the adjective. Isidore of Seville introduces his brief treatise on the arts thus: "Disciplinae liberalium artium septem sunt" (*Etymologiae* I, 2, Migne LXXXII.)

CHAPTER II.

Grammar.

I. THE PRACTICAL STUDY OF LATIN.

Grammar as taught in the curriculum of the seven liberal arts had a far wider scope in the middle ages than the term at present implies. Introduced into Rome from Greece during the Hellenistic period, the word "grammar" came to have the same meaning in Rome which it had by that time acquired in the Greek world—the study of literature.¹ Naturally this view of the scope of the subject was adopted by the text book writers in the early middle ages.²

¹ Vide Sandys, *History of Classical Scholarship*, pp. 6–10, where the history of the word "grammaticus" is briefly traced. The wide scope given to the term is plainly seen from the definitions of the grammarians. The grammar of Dionysius Thrax (c. 166 B. C.), the oldest book extant on the subject, and the basis of grammatical texts for at least thirteen centuries, defines the subject thus: "Grammar is an experimental knowledge of the usages of language as generally current among poets and prose writers. It is divided into six parts: (1) trained reading, with due regard to prosody; (2) exposition, according to poetic figures; (3) ready statement of dialectical peculiarities and allusions; (4) discovery of etymologies; (5) accurate account of analogies; (6) criticism of poetical productions, which is the noblest part of the grammatical art." Quoted in Davidson's *Aristotle etc.* p. 214.

This broad definition of the scope of grammar by the famous Greek grammarian was adopted almost word for word by the Roman writers. Varro, one of the earliest Latin grammarians, assigns the following functions to the subject: "(1) emendatio—the correction of the text; (2) lectio—accurate reading; (3) enarratio—exposition; (4) iudicium—criticism." Vide Wilmanns, "*Varronis Fragmenta*," p. 208, (as quoted in F. A. Eckstein's "*Lateinischer Unterricht*" in Schmid's *Encyclopaedia des gesammten Erziehungs und Unterrichtswesens*, Bd. IV, p. 210 et seq.)

The definitions of Cicero and Quintilian are no less comprehensive. Vide Cicero, *De Oratore*, I, 42, Watson's translation, p. 198; Quintilian, *Institutes etc.* I, 4, Watson's translation, Vol. I., p. 29.

Diomedes, the grammarian of the fourth century, defines grammar thus: "Tota autem grammatica consistit praecipue intellectu poetarum et scriptorum et historiarum prompta expositione et in recte loquendi scribendique ratione." Vide Keil, *Grammatici Latini*, Vol. I, p. 426.

² Thus in Cassiodorus: "Grammatica est peritia pulchre loquendi ex poetis illustribus orationibusque collecta." Vide Cassiodorus, *De Artibus ac*

This fundamental subject of the curriculum of the seven liberal arts aimed therefore at a mastery of the universal language of the day, the language of the church and state, the bond of union of all the cultured classes of Western Europe.

But after all, our final estimate of the value of grammar in the mediaeval curriculum will not depend on the breadth which the grammarians gave to the definition of the subject, but rather on the actual aims and the results of the instruction. What then were the aims? They were, first, a practical mastery of the Latin language as the highroad to all knowledge; and second, an appreciation of its literary forms.

The present chapter will be devoted to an examination of the actual scope, content and method of the instruction. In other words, an attempt will be made to find out how and in what ways a boy in a mediaeval school was taught Latin, a language not his mother's tongue, so that after three or four years he was able to make it the language of daily intercourse in a scholar's environment. In the next chapter a more difficult task will be attempted. There the writer will endeavor to give a satisfactory answer to the mooted question: "To what extent did the instruction in grammar in the curriculum develop a capacity for literary appreciation?"

There can be no doubt that in the teaching of grammar, the practical aim—the acquisition of the language for purposes of intercourse—was always realized.¹ But in this work the mediaeval teacher was thrown on his own resources, for whatever method of teaching Latin grammar might have been in vogue in ancient Rome, such a method was plainly of no value when Latin was to be taught to the mediaeval boy as a foreign language. This fact lends additional interest to the problem.

Disciplinis Liberalium Literarum, Migne, LXX, col. 1152.

Isidore of Seville, writing in the sixth century, defines the subject as: "scientia recte loquendi et origo et fundamentum liberalium literarum" Vide Isidore, *Etymologiae Libri XX*, Lib. I, c. 5, Migne LXXXII, col. 81.

Rabanus Maurus in the ninth century, says: "Grammatica est scientia interpretendi poetas atque historicos et recte scribendi loquendique ratio. Haec et origo et fundamentum est artium liberalium." Vide Rabanus, *De Clericorum Institutione* III, 18, in Migne, vol. CVII, col. 395.

From these definitions it is clear that "grammar" in the middle ages was equivalent to our expression "language and literature."

¹ F. Paulsen, *Geschichte des gelehrten Unterrichts*, Bd. I., pp. 40-48.

While a complete authentic record of the method of instruction in Latin during the middle ages is not in existence, it is still possible to reconstruct to some extent the school life of that period.¹ From various sources it has been established that the mediaeval boy, when he began the study of grammar was already able to pronounce and write Latin words and to recite prayers in Latin. Three years' practice in singing Latin verse and in reciting psalms had already strengthened his memory and had taught him the elements of Latin quantity though he had as yet no knowledge of the meaning of any of the words.²

With this preparation, the mediaeval "grammaticus" began the task of introducing the boy to Latin grammar. The starting point was, of course, the parts of speech. For this an elementary text book was required, one in which the elementary rules of Latin grammar should be illustrated in simple words. The difficulties which the teacher encountered in this work would in our day be considered insurmountable. Books were scarce and frequently not even one pupil in the class possessed a copy of the text. How much progress would a modern teacher make in teaching a foreign language unaided by plentiful manuals? But undaunted, the mediaeval schoolmaster made the most of his own worn, often imperfect, copy of the text. Day by day he would read a selected portion of the book. This he would carefully explain, translating every difficult word into the mother tongue. Simple sentences

¹ The "autobiography" of Walafrid Strabo as printed in Karl Schmidt, *Geschichte der Paedagogik*, Bd. II., pp. 197-212, is neither an authentic account of that poet's school days written by himself, nor, as Wattenbach has erroneously claimed (*Deutschlands Geschichtsquellen im Mittelalter*, 6th ed. 1893, Pt. I, p. 279, note 2.) a forgery of the twelfth century. It is simply an imaginative *Darstellung* composed by Father Martin Marty and published anonymously by him as supplement to the *Jahresbericht über die Erziehungsanstalt des Benedictinerstifts Maria Einsiedeln 1856-1857* entitled "*Wie man vor tausend Jahren lehrte und lernte*;" vide S. Günther, *Geschichte des Mathematischen Unterrichts im Deutschen Mittelalter*, p. 63, note I; and M. Cantor, *Vorlesungen zur Geschichte der Mathematik*, Bd. I., p. 792.

² Vide Specht, *Geschichte des Unterrichtswesens in Deutschland* pp. 68-80, note—where copious citations of original sources covering every point of each assertion made in this paragraph will be found. This evidence as gathered by Specht is based mainly on a collation of incidental references found in the *Vitae* of eminent mediaeval personages.

illustrating the rules would be copied by the pupils on their wax tablets. This daily lesson the pupils would memorize. The next time, after reciting the lesson of the previous day, the boys were required to master another portion of the text in the same manner. Day after day the process would continue, varied only by frequent reviews, until after a time the pupil would have memorized a limited number of rules, and acquired a knowledge of the meaning of many Latin words in daily use. Meanwhile the teacher, noticing that the pupils were rapidly learning the meaning of new words, would gradually substitute explanations in Latin for those given in the mother tongue. To make the acquisition of words in daily use more easy, he would introduce some convenient "colloquium"—a sort of manual of conversation. If none was at hand, his ingenuity would probably help him to compile one of his own.¹

The task of the teacher only increased after his pupil had mastered this elementary knowledge, for then there arose the need of a suitable reading book for a child of eleven years of age. To provide such a book was, of course, simply a question of practical pedagogics. In our day, when the teacher, constantly suffering from a plethora of text books, is perplexed about the conflicting claims of the different ones from which he is to choose, the importance of this problem to the mediæval teacher can hardly be appreciated. Nevertheless the way in which the mediæval schoolmaster solved it challenges our admiration. Although every teacher was practically compelled to compile his own reading book, they all used similar material. They seemed to have had a common appreciation of a pedagogical situation which they all met in approximately the same fashion.

In their philological, antiquarian and *kulturgeschichtliche* investigations, modern historians have brought to light a vast number of these productions. Some, though representing the labor of a teacher's lifetime, probably never obtained popularity outside of the school in which the author taught; others enjoyed a wider celebrity and were often in use in widely scattered places; a few were known throughout Christendom, and no school could claim distinction unless such books were studied

¹ Thus in the *Colloquium* of Aelfric (cf. *infra* p. 49), a conversation is carried on in turn by a monk, peasant, huntsman, fisherman, cook etc., each one giving the names of objects used by him.

in them. These little books are real monuments of pedagogical skill. Coming from so many different parts of Western Europe with the chances for plagiarism reduced to a minimum they all show in a remarkable degree evidence of the same use of material and similar treatment of the subject. Fables, folklore, biblical proverbs, characteristic national proverbs, facts of every day interest, particularly to a schoolboy—all these make up the content of the host of these little mediæval readers.

But similar use of material is not the only remarkable fact about these text books. Far more significant are the progressive changes and adaptations made by the authors in the material used and in the methods of presentation to suit the spirit and temper of their respective generations. In our day we think it most natural that the historic *New England Primer* of the eighteenth century should give way to a reading book more interesting to the schoolboy of the twentieth century. We scarcely, however, look for such changes in the middle ages. And yet they were as marked as those in our time. Thus in the period immediately following the dissolution of the Roman Empire, the books used show a striking dependence on Roman methods; but with the progress of time, material and methods of presentation suitable to the succeeding centuries appear—until in the later middle ages the peculiar classical characteristics of the manuals disappear almost entirely. In modern educational terms we would say that the mediæval schoolmaster understood the “principle of apperception.”¹

As one would naturally suppose, the spirit of the age required that in the teaching of elementary Latin the subject matter should in no way overshadow the whole aim of education—right living according to Christian standards. Hence the didactic, ethico-religious purpose is always manifest in the choice of reading matter. In fact all these text books are uniformly based on two ideas: the teaching of elementary Latin, and incidentally, the inculcation of religious and ethical principles. But it appears, on the other hand, that the authors varied in their method. Some used material of an abstract character

¹ The fable and the proverb of the middle ages dealt with incidents of a mediæval character. Thus the story of the Wolf of the classical period became the story of the Wolf, the Pastor and the Monk. For instances showing this adaptation, vide *infra* pp. 17, 19 et seq.

while others preferred the concrete, and a combination of the two was not infrequently employed in the same book.

The typical text representing the tendency to present the reading matter so as to impart ethical instruction in abstract form was the so-called *Distichia Catonis*, dating as far back as the fourth century. The wide use made of these verses give a peculiar interest to an analysis of the contents of this unique little work. The one hundred and forty-three couplets of the version analyzed can be classified as follows:

1. Spirit of religion	17
2. Self-control	35
3. Duties to others	35
4. Prudential maxims	35
5. Miscellaneous information	21 ¹

This analysis shows plainly that the work was not, as is generally supposed, of pagan origin, an assumption which a perusal of the couplets breathing the spirit of Christian ideals makes uncertain. The text book may be considered as representing the period of transition—the early middle ages, when the book was in general use. The abundance of material of a Stoical character did not in the least diminish the popularity of the *Distichia*. Western Europe was in this period still too much in need of the classical heritage to dispense with such an eminently useful schoolbook. And so this age of faith put in the hands of its schoolboys—the budding churchmen—a reading book which so studiously avoided questions of orthodox church doctrine that the zealous suspected its author was a pagan.

¹ The classification is the writer's own. There is nothing in the text to indicate such a division. The relatively small portion of purely religious didactic matter probably accounts for the supposition that the author was a pagan. There is nothing in the text to justify this assertion. The analysis is based on the text in the English-Latin edition published in London in 1663, which is practically the same as the four versions of Cato edited by Zarncke and published in *Berichte über die Verhandlungen der Königl. Sach. Gesel. der Wissenschaften, Phil. Hist. Class.*, Bd. XV, pp. 23-78 (2 versions), Bd. XVII, pp. 54-103, and Bd. XXII, pp. 181-192. The fourth version is the text of the famous Cato manuscript in Cambridge University. A fifth version is edited by R. Peiper, "Beiträge zur Lateinischen Cato-Literatur," *Zeitschrift für Deutsche Philologie*, Bd. V, pp. 165-186. For lists of English versions vide W. C. Hazlitt, *Antiquary*, Vol. XXXV, p. 205 et seq., and Vol. XXXVI p. 138 et seq. For full bibliographical notes on the Cato literature vide Teuffel-Schwabe, *Geschichte der Römischen Literatur*, II, pp. 1002-1003.

The *Liber Proverborum* of Othlo was composed in the tenth century to supplant in a measure the *Distichia Catonis*. Othlo's generation would naturally find more interest in a reading book based on biblical and patristic sources than on profane authors.¹

The *Deliciae Cleri* of Arnulf, mark another step from the Roman towards the purely mediæval point of view. His work is based almost entirely on the Bible in general, and "Proverbs" in particular.² The *Proverbia Wiponis* the so-called *Scheftlarer Proverbia* and the *Proverbia Heinrici* represent a still later stage in the evolution of the text books of this character. In them the material is derived neither from classical nor Christian authors, but from national characteristics.³

The reading book which, however, appealed most both to the mediæval teacher and pupil, was the one in which fable literature abounded. The doctrine of the modern schoolmaster, that for children the concrete is better than the abstract, seems to have been the watchword of his mediæval prototype. On no other assumption can we account for the astonishing number of these collections.

Though passing under the different names of Phaedrus, Avian, Flavius, Novus Avianus and Romulus, all these collections were based mainly on the five books of Phaedrus, and on Avianus, the first a writer in the times of Tiberius, the second of the second century. In the tenth century the fables of Phaedrus were worked over by one Romulus, and as such served as the basis of all subsequent mediæval compilations in Latin as well as in the different vernaculars.⁴ Among the

¹ Text, Pez. *Thesaurus Anecdotorum*, III, 2, cols. 483-536.

² Text and introduction by J. Huemer ed., *Romanische Forschungen*, II, pp. 211-246. For a source critique of Arnulf vide *ibid.* pp. 383-390, and Bd. III., pp. 461 et seq. It is not to be supposed that there was a ecclesiastical desire on the part of the Church to suppress the supposedly pagan *Distichia*. The books which supplanted it were really more up-to-date. A boy studying in a monastery under Christian influences would surely be more interested in a reader based on the scriptures than in the *Distichia*.

³ For text and critique of the *Scheftlarer Proverbia* vide Anz. f. Kunde d. Deutsch. Vorzeit. N. F., Bd. XX, pp. 217-220. Of the *Proverbia Heinrici* four manuscripts have been found; vide Voigt, *Fecundia Ratis*, LXIII-LXV, Note 4.

⁴ The latest complete investigations on the subject are given in Hervieux, *Les Fabulistes Latins etc.* 5 vols., including criticisms and texts.

notable collections which were current in the later middle ages, the *Novus Aesopus* and the *Novus Avianus* of Alexander Neckham may be mentioned as being widely used. No less than seven manuscripts in verse of Neckham's work have been found by Hervieux.¹ The *Phaedrus* was paraphrased and put into prose by monks to be used for teaching purposes, and no less than forty-seven such prose manuscripts have been found in France, South Germany, England, Belgium and Italy, while of the versified manuscripts one hundred and twelve have been discovered and traced to France, Spain, Holland, North Germany, South Germany, England, Austria, Belgium, Italy and Switzerland. The number of manuscripts of various books of fables which have been shown to be directly derived from Phaedrus reaches two hundred and sixteen.²

The kinds of text books described indicate the general tendency in the method and manner of teaching elementary Latin. There were, however, teachers from the eleventh century on who aimed to combine the abstract and the concrete and to adapt the material to the tastes of their particular generation or nationality.³

¹ Text, edited with an historical introduction, of Aesop's Fables by Du Méril, *Poésies inédites du Moyen Age*, pp. 169-259; 262-276.

² Vide Hervieux, *op. cit.* vol. I, pp. 38-239; 334-347; 436-445; 461-602; 608-683. The computation does not include the translations in the vernacular, the consideration of which is not relevant to our subject. An analysis of the printed editions of the fables was likewise omitted since that topic also properly belongs to the later period of humanism. The subjects of the fables of Avian, as exhaustively treated by Hervieux, reveal another vast source of didactic material for elementary Latin school purposes. Vide Hervieux, *op. cit.* III, *passim*.

³ Of some importance as showing the methods of instruction in introductory Latin in the earlier stages, is the "Riddle Literature," originally a British product, which spread rapidly throughout Europe. Written in verse these riddles not only formed interesting reading matter, but also afforded easy exercises in Latin scansion. On this subject vide Ebert, *op. cit.*, I, pp. 628 et seq.; p. 650. Cf. *Forschungen zur Deutschen Geschichte*, Bd. XXVI, pp. 599-632. The most successful attempts to construct a reading book that should be distinctively written in the spirit of its time, are represented by two unique collections, both written by monks—who were teachers, whose books were used as readers by them and by others. Though there are not many manuscripts of these in existence, they are undoubtedly typical of other works of the same character. One is *Fabulae et Parabola* of Odo of Cirrington, a Cistercian monk (c. 1200). The work, like the Phædro-Avian

All these efforts find their best expression in the wonderfully skillful *Fecundia Ratis*, a book of two thousand four hundred and seventy-three lines of verse composed by Egbert, a priest and "submagister scholæ" at Liege, where he finished the work between 1022 and 1024.¹

In his exhaustive investigation of the subject, Voigt demonstrates that the object of the author was to create an ideal school reader, one that would embody what was best in literature and still present the same in a form not above the understanding of a boy. To accomplish this purpose, sacred and profane, concrete and abstract material was used; fables and proverbs were placed side by side with the best writings of the Church fathers. That Egbert bestowed sufficient thought upon his subject is seen from the large number of writers he quotes directly or indirectly.

The book contains a vast amount of information concerning student life and similar matters. In form the work is so arranged as to be progressively difficult, beginning with thoughts expressed in one line sentences, and reaching towards the end paragraphs occupying more than twenty lines. In content the two thousand four hundred and seventy-three lines of the *Fecundia Ratis* is likewise progressively difficult. The first five hundred and ninety-five lines (1-595) are single line proverbs and epigrams. The next four hundred lines (596-1008) are distiches containing more involved thoughts. The next seven hundred and sixty (1008-1768) include fables of varying length, while the remaining six hundred are devoted to theological and biblical topics.²

productions, was widely used both in the original and modified forms. The fables are full of allusions and references to incidents in the life and manners of the times. There are in existence some seventy-five of these more or less original stories. Internal evidence proves conclusively that the book was designed to be used as a text book. Cf. Voigt in *Quellen u. Forschungen zur Sprach. u. Kultur—Geschichte der Germanischen Völker*, vol. XXV, p. 47 et seq.; *Anzeiger für deutsches Alterthum und deutsche Literatur*, Bd. V, pp. 99-125. For complete text vide Hervieux, *op. cit.*, Vol. IV, pp. 173-248. The second is the collection of fables edited by Ernest Voigt, vide "Kleine Lateinische Denkmäler der Thiersage," in *Quellen und Forschungen zur Sprach- und Kultur-Geschichte der Germanischen Völker*, Vol. XXV, p. 16 et seq.

¹ *Fecundia Ratis*, XII-XXXIX. Text edited by E. Voigt; Cf. *ibid.* pp. XII-XXXIX.

² Voigt has identified among the writers cited in this work the following:

The material examined in this chapter certainly bespeaks an intelligent appreciation on the part of the mediæval school-master of the pedagogical difficulties involved in the problem of teaching the elements of a foreign language which was to become for the pupil the language of daily intercourse. The gradual modification and adaptation of the materials at hand to suit the varying needs of the changing times and also to enhance the pupil's interest, likewise show great skill on the part of the mediæval teacher, who, in many instances, was compelled to be the author of the reading book that his pupils were to study.

1. LATIN PROSE AUTHORS:

Varro	Curtius
Cicero	Capella
Cornificius	Boethius
Sallust	Isidore
Seneca	Latin Grammarians

2. LATIN POETS:

Plautus	Ovid
Terence	Phaedrus
Lucilius	Persius
Publius	Lucanus
Virgil	Juvenal
Horace	Avianus and others

3. THE BIBLE:

Genesis	Psalms
Exodus	Proverbs
Deuteronomy	Ecclesiastes
Kings	Prophets
Esther	Acts of Apostles
Job	Epistles

4. THEOLOGICAL WRITERS:

Lactantius	Prosper
Ambrosius	Petrus
Jerome	Gregory the Great
Chrysostomus	Isidore
Augustine	Beda
Sedulius	Rabanus
Cassian	Ratherius

Cf. Voigt, *op. cit.* LIII-LXV.

CHAPTER III.

Grammar.—(Continued.)

II. THE STUDY OF LATIN LITERATURE.

(A.) SCOPE.

In his introductory studies, the student, as we have seen, acquired an introductory knowledge of Latin through the study of collections of proverbs and fables in prose and verse. He also mastered to a certain extent the rules of Latin composition and prosody, and was consequently ready for the more serious reading of Latin authors.

As in the days of Greece and Rome, the poets were read first, poetry affording a better means for correct mastery of words and being more easily memorized. The choice of authors to be read always depended on the teacher's taste and the accessibility of copies of the authors. Religious bias had much less to do with the selection than has been supposed.¹

But granting that Latin authors were read by the mediæval pupils in their advanced study of grammar, a number of interesting questions still remain unanswered: To what extent were the classics studied? In what spirit were they studied? Which were the favorite authors? Did the study of the classics after the manner and method of the age really cultivate an appreciation of literary form in general and of Roman literature in particular?

It is not possible in this study to suggest definite answers to these questions which have been a source of controversy from the day of the Humanists. At bottom they involve the all-embracing problem of the use of the classics in the middle ages, —a subject of which an adequate history has still to be written.² Moreover this general subject of mediæval education has not unnaturally become a battle ground of Protestants and

¹ Sandys, *op. cit.* Chap. XXXII, p. 594 et seq. The difference between the study of the classics in the middle ages and in the Renaissance would then simply be the difference of the end in view.

² Cf. Comparetti, *Virgil in the Middle Ages*, Introduction; H. O. Taylor, *Classical Heritage of the Middle Ages*, p. 363.

Catholics. One party would have us believe that the age was one of general ignorance. It could not have been otherwise, we are told, since the church exercised such a baneful influence on learning. Isolated cases of barbarisms and mutilated quotations from the writings of a few prominent mediæval personages are cited and it is naïvely assumed that a case has been made out.¹ Others again, in their zealous defense of the old faith, not only vehemently assert that the classics were studied most assiduously during this period, but in their enthusiasm they claim that in the middle ages there was almost an approach to universal elementary education.²

The attitude of recent writers on this vexed subject of mediæval culture indicates a reaction from these extreme partisan positions. This new point of view has led modern investigators laboriously to compile lists of allusions to classical authors by mediæval writers. From these compilations we are assured that mediæval writers were acquainted in a greater or lesser degree with at least the following authors: Plautus, Terence, Catullus, Lucretius, Virgil, Horace,

¹ Vide Hallam, *Introduction to Literature of the Middle Ages* Vol. I, Ch. 1; Robertson, *History of Charles V etc.* pp. 14, 15, 514-516; Cf. Compayre, *History of Pedagogy* p. 68; Laurie, *Rise and Constitution of the Universities*, p. 73. These traditional Protestant contentions are controverted by Maitland, *The Dark Ages*, passim, esp. 9-52, 122-141; 171-187.

² Vide the chapter entitled, "The Primary School in the Middle Ages," in Brother Azarias' *Educational Essays*. The rosy view there presented is based on recent investigations into the educational conditions of a number of French districts before the Revolution. The fallacy in the arguments, it seems to me, lies in the fact that the middle ages are treated as one period. Facts which are true of the later middle ages are attributed to the whole period. The primary education afforded in the middle ages down to the fifteenth century at least, was only preparatory work for the advanced study of the liberal arts and was not popular education in our sense of the term. The polemical attitude on the subject has surely reached its absurd climax when the same shred of evidence is used by both sides to prove opposite contentions. Thus Oznam (*Des Écoles et de l'Instruction Publique en Italie aux Temps Barbares*, p. 39) cites Rather of Verona's refusal to admit to holy orders those who had not studied at an episcopal school or with some teacher, as proof that a preliminary education was considered essential for a priest even in the ninth century. The same fact is taken by Güdeman (*Geschichte des Erziehungswesens der Juden in Italien während des Mittelalters* p. 20) as a sure indication of the ignorance of the clergy of the time.

Ovid, Lucan, Statius, Martial, Persius, Cicero, Seneca, the Plinys, Quintilian, Cornelius Nepos, Cæsar, Sallust, Livy, Suetonius, Tacitus.¹ Now, while this generalization is certainly more in accord with the spirit of the modern historical standpoint, the evidence on which this assertion rests is far from being conclusive, for it has not as yet been proved that even a fair portion of these quotations are from first hand sources. Indeed, the facts point to an opposite conclusion. It is well known that the mediæval scholar had at hand a vast collection of quotations from all of the above enumerated authors in Priscian's elaborate manual of grammar.² How many of these learned citations really represent more than an acquaintance with the work of Priscian?

From the foregoing paragraphs it is clear that a complete answer to the questions raised cannot be given in this brief study. Nevertheless the following points are susceptible of proof:

(1) There was a general interest in literary culture in the schools of the middle ages.

(2) The classic Latin authors were studied to an extent surpassing even the expectations of the most sympathetic student of mediæval culture.

That there was literary culture in the schools of the middle ages may be inferred from the following facts:

A mass of good Latin writing—prose and poetry—was produced even in the same periods in which the stock specimens of wretched writings were composed.³

¹ For bibliography on the various classical authors and their use in the middle ages vide Taylor, *op. cit.*, pp. 363-365; Sandys, *op. cit.* pp. 606-638 gives a brief and inadequate summary of the subject with some detailed references. For an example of such studies showing the extent of the use of the various authors vide the series of articles by Manitius, "Beiträge zur Geschichte Römischer Dichter im Mittelalter" in *Philologus-Zeitschrift für das Classische Alterthum*, Bde. 47, 49, 50, 51, 52, 56, and supplement Bd. 7. For bibliography of manuscripts of the several authors extant vide Teuffel-Schwabe's *Geschichte der Römischen Literatur* under the several names.

² Cf. *Infra*, pp. 37 et seq.

³ The record of mediæval literary productions as enumerated in Ebert's exhaustive *Allgemeine Geschichte der Literatur des Mittelalters im Abendlande*, bristles with such evidence. Thus in the ages of Gregory the Great and Gregory of Tours, we find the excellent productions of Arator; Fortunatus and Aldhelm; vide Ebert, *op. cit.* I, pp. 490-616, II, pp. 144 et seq. For a sympathetic

Moreover part of the evidence of ignorance of the classics and indifference to them, belongs to the university period when scholasticism had changed the entire spirit of the pre-university education, and the church's influence on education was by far less potent than it had been in the earlier period. The complaints of John of Salisbury as to the neglect of the classics cannot be set down as characteristic of the entire period. In fact his remarks when read in their entirety show that the study of grammar in its widest sense—the reading of the classics—was in reality the old-fashioned idea, common before the advent of the scholastics.¹

If we look into the educational conditions of the middle ages as they obtained, not in one country but in Western Europe, it can be easily demonstrated that at no time were the classics neglected to any such extent as has been supposed. Even in the traditional "period of darkness"—(the sixth to the eighth centuries)—we find indisputable evidence that Italy was at that very period full of schools, monastic, cathedral, secular, public and private.²

There is sufficient evidence to show that the so-called restoration of Charlemagne did not mean a renaissance of literature, but simply a transference of the learning of the day from Britain and Italy through Alcuin and Paulus Diaconus into Frankland, a change which was not temporary but permanent. This culture remained in the restored Empire and is found at the court of the Ottos, in the many schools which were directly and indirectly influenced by the followers of Alcuin. The continuity of

appreciation of the worth of mediæval literary productions vide Guizot, *History of Civilization*, II, pp. 317-378.

¹ John of Salisbury, *Mythologicus*, I, c. 24; II, c. 9, 10, 17, *Polycraticus*, VII, c. 12; Migne, *Pat. Lat.* tom. CXCIX cols. 662, 663, 853-856, 866-869, 874-876. Cf. Poole, *Illustrations of the History of Mediæval Thought*, pp. 109-124, 200-208. Even in regard to educational conditions of his own time the strictures of John of Salisbury are not as severe as has been generally supposed. Vide Poole, "Excursus on the Interpretation of a Place in John of Salisbury's *Metalogicus*," *op. cit.*, App. VII, pp. 359 et seq.

² Vide Bursian, *Geschichte der Classischen Philologie in Deutschland*, I, pp. 9-76; Oznam, *Des Écoles et de l'Instruction Publique en Italie aux Temps Barbares in Documents Inédits pour servir à l'histoire littéraire de l'Italie*.

classical interest was not broken throughout the period.¹

Intensive studies of particular districts wherever they have been made, show a persistent interest in the study of grammar and literature in monastic as well as in cathedral and other schools.²

The attitude of the mediæval church towards secular studies has been misconstrued. Some have argued that because the official view of the church was against secular learning as an end in itself that therefore its influence was against any literary study whatsoever. The error is evident. In the first place, the rule which obtained in so many of the monastic orders ordaining the copying of manuscripts as a part of the monastic routine inevitably made for a study of the classics. The isolated instances of monks erasing classical texts to write legends of saints on the parchment certainly dwindle into insignificance when we consider that there are in existence today over one

¹ By tabulating into an educational genealogy the mass of material relating to the histories of the famous schools from which historical records have come down to us, as given in Wattenbach's *Deutschlands Geschichtsquellen im Mittelalter*, I pp. 241-487, I have traced the influence of Charlemagne's renaissance as far as the twelfth and thirteenth centuries—two centuries beyond the date reached by the investigations of West (*Alcuin etc.* pp. 164-179) and Gaskoin (*Alcuin etc.* p. 201 et seq.). The evidence shows the spread and persistence of the traditions of St. Gall, Reichenau, Corvey, and numerous other famous schools throughout Western Europe. It may be remarked here that the accepted view as to the decline of learning under Louis, the Pious can no longer be accepted. Cf. Wattenbach, *op. cit.*, I, p. 112 et seq.

² Among the typical studies of this character the following may be cited: M. Fayet, *Les Écoles de la Bourgogne sous L'Ancien Régime*; Ch. de Beaurepaire de Robillard, *Recherches sur l'Instruction publique dans le Diocèse de Rouen avant 1789*; Ch. Stallaret, *De l'Instruction publique au Moyen Age* in *Memoires Couronnées, Académie Royale au Brussels* tom. XXIII (treating of conditions in the diocese of Utrecht, Cambray and Tournay); Sack, *Geschichte des Schulwesens zu Braunschweig*; K. F. Krabbe, *Geschichtliche Nachrichten über die höheren Lehranstalten in Münster*; Meyer, *Geschichte des Hamburgischen Schul-und-Unterrichtswesens des Mittelalters*; C. Engel, *Das Schulwesen Strassburgs bis 1538*; D. A. Fechter, *Geschichte des Schulwesens im Basel*; F. Nettesheim, *Geschichte des Schulwesens im alten Herzogthum Geldern*; J. E. Struve, *Geschichte des Hochstifts Osnabrück*; H. Holstein, "Geschichte der Ehemaligen Schule zu Kloster Berge" in *Neue Jahrbücher für Philologie und Paedagogik*, Bde. CXXXII, CXXXIII; Baring, *Beiträge zur Han. Kirch. u. Schul. Geschichte*.

thousand copies of that important manual for teachers and advanced students—Priscian's Grammar.¹

The catalogues of monastic libraries of the middle ages such as have come down to us show that invariably nearly half of the books contained therein were grammars and pagan-classical authors.²

Furthermore, the very accounts of the lives of the supposedly official opponents of classical learning show that they had all received a good training in the classics.³

The number of centres of secular education, the existence of which can be definitely established, is constantly increasing. There is evidence to prove that in every land of Western Christendom not only was there an abundance of schools of minor importance but that there were also scores of centres of culture where the literature of the classical world was assiduously studied.⁴ Instances of approximations to modern standards have even been found in the brilliant classical teaching of Bernard of Chartres.⁵

¹ Keil, *Grammatici Latini*, vol. II, praef. p. XIII. On libraries in monasteries vide Montalembert, *The Monks in the West*, Vol. V, p. 136-156.

² Such lists are mentioned in F. A. Eckstein's article "Lateinischer Unterricht" in Schmid's *Encyclopaedia des gesammten Erziehungs-und-Unterrichtswesens* Bd. IV, p. 231 et seq.; Gottlieb, *Ueber Mittelalterliche Bibliotheken* pp. 17-273 passim. Cf. Wattenbach, *Schriftwesen im M. A.* pp. 481-500; Putnam, *Books and their Makers during the Middle Ages*, I, p. 133 et seq.

³ Vide summaries of the host of *Vitæ* in Ebert's work, passim.

⁴ From the days of Mabillon, whose *Traité des Études Monastiques*, Paris, 1691, set forth for the first time the great extent of secular studies among the Benedictines, this subject has received attention from the defenders of the middle ages. Among them may be mentioned, Hurter, *Tableau des Institutions et des Mœurs de l'Église au Moyen Age*, Vol. III, Chaps. XXXVI, XXXVIII, and the exhaustive treatment of the subject by Ziegelbauer, *Historia Rei lit. O. S. B.*, passim, especially Pars. I, Ch. 1, 2. An admirable scholarly summary of the topic is found in Montalembert's *Monks of the West*, Vol. V., pp. 102-169, where a long but incomplete list of the famous schools is given pp. 108, 109, 123, 124. Specht, *Geschichte des Unterrichtswesens in Deutschland*, pp. 296-394, has a lengthy account of the educational work of the most famous schools in Germany.

⁵ Vide John of Salisbury, "De usu legendi et praelegendi et consuetudine Bernardi Carnotensis," *Metalogicus* Lib. I, c. 24, Migne, *Pat. Lat.*, CXCIX, cols. 853-856. Cf. *supra* p. 24 note 1. Cf. Poole, chapters on the "School of Chartres," and "John of Salisbury," *op. cit.*, pp. 109-135; 201-225. For an admirable brief study based on sources of the educational work of

That the classics were much studied can be proved by an abundance of fragmentary evidence which indisputably establishes the fact that the standard Roman writers were taught by teachers of grammar in the mediæval curriculum.

Theodulfus, Bishop of Orleans, successor to Alcuin as the educational adviser of Charlemagne, whose interest in the traditional seven liberal arts is unquestioned, shows that he actually taught the following named classic and Christian authors in the eighth century: Virgil, Ovid, Pompeius, Sedulius, Rutilius, Arator, Fortunatus, Juvenius, Prudentius.¹

Walther von Speier, who was educated at Speier at the end of the tenth century, studied as part of his instruction in grammar, Virgil, the Latin Homer, Horace, Persius, Juvenal, Statius, Terence, Lactantius, Boethius and Constantius.²

In the same century, we find Gerbert, afterwards Pope Sylvester II, teaching among the grammatical subjects at Rheims, Virgil, Statius, Terence, Juvenal, Persius, Horace and the historian Lucan. All these he considered as introductory to the study of rhetoric. Now, as we know, Gerbert's interests were more mathematical than literary; hence his judgment as to the necessity of studying the above named classical authors as a part of the subject of grammar would seem to indicate and reflect the average view of the teacher of his time.³

In the next century we find Othlo expressly mentioning Horace, Terence and Juvenal amongst his school books.⁴

Bernard, see Barach's introduction to edition of Bernard's *De Mundi Universitate*.

¹ Theodulfi Carmen de Libris quos legere solebam, Duemmler, *Poetæ Latini Medii Aevi*, p. 543 et seq. For his general interest in the classics and for an estimate of his character vide Ebert, *op. cit.* II, pp. 70 et seq.

² Walter Spirensis, *Acta St. Christoph*; Lib. I, de Studio, vv. 96-109 in Pez, *Thesaurus Anecdotorum Novissimus* T. II, Pt. III, col. 39.

³ Cum ad rhetoricam suos provehere vellet, id sibi suspectum erat, quod sine locutionum modis, qui in poetis discendi sunt ad oratoriam artem perveniri non queat. Poetas igitur adhibuit, quibus assuescendos arbitrabatur. Legit itaque ac docuit, Maronem et Statium Terentiumque poetas; Juvenalem quoque ac Persium Horatiumque satiricos, Lucanum etiam historiographum. Quibus assuefactos locutionumque modis compositos ad rhetoricam transduxit." Richer, *Hist. Libri IV*, III, c. 47, (*Monumenta Germaniae Historica, Scriptores, Tomus III*, p. 617.)

⁴ Othlo, *Liber Metr. de Doctrina Spirit*, Pez, *Thes. Anecd.* III, 2, 442.

Hugo von Trimberg, the learned schoolmaster of Bamberg, in 1280, gives the names of the books he taught and mentions Virgil, Horace, Ovid, Juvenal, Persius, Statius, the Latin Homer, Sedulius, Juvenius, Arator, Prosper and Prudentius.¹

Early in the thirteenth century we find that Eberhard of Bethune, a teacher of grammar and rhetoric, mentions (in the third part of his reference book, the *Laborinthus*) as models of style the authors which Hugo von Trimberg taught.²

Johannes de Garlandia, gives an explicit list of the authors one should read in each one of the subjects of the seven liberal arts, mentioning among the works on grammar, Statius, Virgil, Juvenal, Horace, Ovid, Sallust, Livy, Seneca, Martial.³

In the twelfth century we find a schoolmaster mentioning Lucan, Persius, Terence and Horace among the general books for the study of grammar.⁴

A satirical poem of the thirteenth century shows that the following books were studied: Ovid, Juvenal, Terence, Horace, Persius, Plautus, Virgil, Lucan, Boethius, Maximianus.⁵

The study of Virgil was continued throughout the middle ages. He was considered the chief authority on grammar, as well as a master of style, hence the careful study of the *Aeneid*. The vast number of glosses on Virgil and other authors which could hardly have been used except for instruction in the classroom, seem to prove the wide use of these authors.⁶

¹ "Registrum Multorum Auctorum" in *Sitzungsberichte der Philol. Hist. Class. Kaiser. Akad. der Wiss.*, Wien Bd. CXVI, pp. 145-190; These 1030 verses critically edited by Huemer form a sourcebook of great significance for the facts as to the extent of the study of classical literature in the middle ages.

² Vide "Laborinthus, Tertius Tractatus" in P. Leyser, *Historia Poetarum et Poematum Medii Aevi*, p. 825 et seq. Cf. Saintsbury, *History of Criticism* I, p. 408 et seq.

³ Hauréau, *Notices et Extraits etc.* XXVII, Pt. 11, 1-86; cf. Sandys, *op. cit.* p. 528 note 5.

⁴ Honorius of Autun, *Libellus de animae exsilio et patria, alias de Artibus*, Migne, CLXXII, col. 1243.

⁵ Nicolai de Bibera, *Carmen satiricum* vv 35-45. Text critically edited by Th. Fisher in *Geschichtsquellen der Provinz Sachsen* I, p. 38. For more cumulative evidence of this character, vide Gottlieb, *Ueber Mittelalterliche Bibliotheken* pp. 439-449

⁶ Comparetti, pp. 119-134. For texts and criticisms of Virgilian glosses

It is significant that Henri D'Andeli, the author of the humorous *Combat of the Seven Liberal Arts*, should put among those fighting under the banner of Orleans—the side contending for the study of grammar—the following authors: Donatus, Priscian, Persius, Virgil, Horace, Juvenal, Statius, Lucan, Sedulius, Prudentius, Arator, Terence.¹

The foregoing data, then, sufficiently exemplify the scope and the spirit of the instruction given in Latin literature as a part of the study of grammar. To be sure the classics were not the sole material used by the mediæval teacher to train the students in literary form and appreciation. The spirit of the age demanded that Christian writers be read also. But even this fact supports the thesis; for on examining closely the evidence on this point we find that only those Christian authors were read whose literary excellence was beyond question. As between literature of Christian content and literature of Pagan content the schoolmaster not unnaturally chose the former, but by no means to the exclusion of the latter. Even the most famous of the Christian authors barely rivalled, but never surpassed, Virgil in popularity as a mediæval text book.

Among such Christian authors may be mentioned:

Juvencus, c. 330, the author of the metric *Historia Evangelica*, a gospel in Latin verse. The book was much used. Its pure style and fluent diction must have impressed the teachers with its value as a text book from which to teach grammar. Moreover its content was most satisfactory from a Christian point of view. The vast number of glossaries as well as manuscripts extant shows the wide use of this author for school purposes.²

in old German, vide E. Steinmeyer, "Die Deutschen Virgilglossen" in *Zeit. f. Deut. Alterthum*, Bd. III, pp. 1-96. Some Virgilian glosses and some on Juvenal, dating back to the 8th and 9th centuries, are included in *Corpus Glossariorum Latinorum* IV, pp. 427-470; V, pp. 652 et seq. Steinmeyer and Siever's *Althochdeutsche Glossen*, Bde. 2 and 3.

¹ Vide Henri D'Andeli, *La Bataille des Sept Arts*, Text in *Notices et Extraits. etc.*, V., Pt. 2, pp. 503-510. This famous poem is generally taken as an indication of the contest which was waged in the thirteenth century in France to supplant the study of the classics by scholastic logic. The protagonists in this combat are Paris, representing the "new"—logic, and Orleans, representing the "old"—classical studies.

² Ebert, *op. cit.* I, pp. 115 et seq.; Meier, *Die Sieben Freien Künste im M. A.* p. 20.

Of great fame as a text book was the *Carmen Paschale* of Sedulius (fl. 430). The book was based on the Old and New Testament. It became at once a standard for the teaching of Latin grammar, metre, and biblical history. Praise of it in the middle ages was very general.¹

But the most widely used Christian author, whose popularity almost equalled the fame of Virgil, was Aurelius Prudentius Clemens, better known as Prudentius (fl. 400). His works, especially the *Psychomachia*, and the *Cathemerinon*, represent the highest expression of Christian thought in classic form. His influence most naturally was great, and praise of him fulsome. He was called the "dissertissimus atque Christianissimus poeta." The extent to which he was used can be judged by the twenty-one different manuscripts of German glossaries of his works which are still extant.²

(B.) METHODS.

Turning to the methods of teaching the classics, we find them fully as elaborate in the advanced study of the classics as in the elementary study of Latin. The object now being a mastery of literary forms mainly, the method employed was to bring out the etymological meaning. Hence dictionaries were used in abundance, and mythological and historical explanations were given to enable the student to understand the text.³

¹ Cf. Ebert, *op. cit.* I, p. 373 et seq. Huemer has traced to Remigius of Auxerre the authorship of a glossary on Sedulius. Vide *Sitz. der Königl. Akad. d. Wiss. Philol. Hist. Class.* Bd. XCVI, pp. 505-551. This fact can be taken as a link in the chain of our argument. It shows that even in the days of Remigius of Auxerre when the literary enthusiasm of the Carolingian revival is supposed to have waned, there still was interest in the purely literary form of a Christian poet, whose claim to distinction was the fact that he treated gospel themes according to the strict classic tradition of poetic composition.

² Cf. Ebert, *op. cit.*, 2 Aufl. I, pp. 251-288, especially 280-288; T. R. Glover, *Life and Letters in the Fourth Century* p. 249-277. Texts: *Psychomachia*. J. Bergmann, ed.; *Cathemerinon*, edition of Th. Obbarius, Tübingen 1845. English metrical translation by F. G. Smith, London & New York, 1898.

³ The following few introductory lines from Priscian's *Partitiones duodecim versuum Aeneidos principalium*, a work designed to be used in teaching grammar from Virgil, give a fairly adequate idea of the method employed. The author takes the first line of each book of the Aeneid and asks the pupil to

As the study of grammar at this stage invariably meant a study of Virgil, commentaries on the author became a necessity. Of these there was a vast number. They can be classed into four groups:

(a) Those giving a literary commentary on the author. The most famous of these was that of Marius Servius Honoratus, which was used as the source and the type of subsequent works. Servius only gives such information as is absolutely necessary to explain the text literally.¹

(b) Those dealing with Virgil as a rhetorician. The work of Tiberius Claudius Donatus (fl. 400 A.D.) is the best known of this class. He comments on the Aeneid only as a masterpiece of rhetorical composition.

(c) Those treating of Virgil as a subject for eulogy. In this class of commentaries Virgil is exhibited as the universal mind. Macrobius, a fifth century author, may be mentioned as the most typical.

(d) Those treating of Virgil from the mystical and allegorical point of view. To these commentators Virgil is the book from which hidden things are to be revealed. As a type of this class the two works of Fulgentius may be mentioned—the *De Continentia Virgiliana* and the *Mythologicon*.²

explain every word, and to analyze it grammatically and metrically. In this way by going through the twelve introductory lines of the twelve books of the Aeneid he covers most of the rules of grammar and prosody: "*Arma* quae pars orationis est? Nomen. Quale? Appellativum. Cujus est speciei? Generalis. Cujus generis? Neutri. Cur Neutri? Quia omnia nomina quae in plurali numero in *a* desinunt, sine dubio neutri sunt generis. Cur singulare ejus in usu non est? Quia multas varias res hoc nomen significat. Cujus est figurae? Simplicis. Fac ab eo compositum. Armiger, armipotens, semermis, inermis et inermus. Cujus est casus in hoc loco? Accusativi." Text in Keil, *Gramm. Lat.* III, pp. 459-528.

¹ Text by Thilo and Hagen, Leipzig 1878.

² The topic is exhaustively treated in Comparetti, *op. cit.* pp. 50-119, where copious notes are given. Of these four methods of interpreting Virgil as represented in the commentaries, the first was really the one used in the schools in the study of grammar. An examination of the subject matter of the other commentaries shows conclusively that they were unsuitable for school use and could only have been read by adult students of the classics. The existence of a large number of glossaries on commentaries of the first class supports this assertion. While all these commentaries, like the vocabularies of the middle ages, show an amusing ignorance of etymology, there can be no

In the study of classical authors accounts of their lives and the times in which they lived were often given to the pupils. The material for such instruction was found in little books which were known as "accessus ad poetas."¹ While the literal explanation of the text was the minimum aim of instruction, much more than that was very often accomplished. The forms and idiomatic expressions were emphasized; selected portions were memorized; and the pupil was often tested as to his understanding of the text by being required to reproduce the thought of the author in correct prose without barbarisms or solecisms.²

The quality of such appreciative teaching depended mainly on the character of the teacher. Bernard of Chartres, for instance, who taught John of Salisbury, in the twelfth century, not only read the authors with his pupils but also explained constructions, pointed out mistakes and beauties of the text, elucidated matters of antiquity, asked pupils to judge and to criticize, made them memorize passages and write original exercises in prose and verse.³ While this work represents of course the best of grammatical teaching of the time, practice by the pupils in prose and in metrical composition was very common indeed in the schools of the middle ages. In fact the last of the three years devoted to the study of grammar was expressly given up to work of that character. Such instruction had its technical name—"dictare"—and the ability to make a "dictamen metricum" was a sort of *testamentum maturitatis* in grammar.⁴ The exercise was naturally based on some classical question of their utility for class-room purposes. For a few of these vide Comparetti, *op. cit.*, p. 129. An interesting anonymous commentary on Virgil, from Ms. of Verona, antedating the work of Servius, has been published in Mai, *Interpretes Veteres Virgilii Maronis in Classici Auctores e Vaticanis codicibus editi* T. VII, Cf. *ibid.* praef. VIII, IX.

¹ Specht, *op. cit.*, p. 103, has found some such manuscripts dating as far back as the twelfth century. For the numerous biographies of Virgil and other poets vide Comparetti, *op. cit.* pp. 135-155.

² Donatus' larger work, the *Ars Grammatica major*, was used for that purpose to bring out the niceties of Latin diction. This work cites Virgil about one hundred times; Text, Keil, IV, pp. 367-402.

³ *Metalogicus*, I, c. 9, 14, Migne, CXCIX col. 838,853 et seq. For proofs of Bernard of Chartres identity with Bernard Sylvester, vide Poole, *op. cit.* p. 114 note 9.

⁴ Specht has collected a number of references in the *Vitae* of eminent persons relating to their original dictamina; *op. cit.* p. 112.

author or some biblical incident. In rare instances an especially gifted youth would write an elaborate poetical production as his "dictamen."¹

The foregoing statements as to the study of the classics as a part of the course in grammar apply only to the period before the opening of the university era, for then the character of the instruction both in the study of the elements of Latin grammar and in the advanced study of Latin literature changed and took on a more dialectical aspect. Of this change the text books of that period will give ample proof. This period is the time of the contest of the "arts" against the "authors,"—the "arts" as we have seen meaning merely a logical treatment of all the seven liberal arts. Instead of the originals and commentaries on authors we shall find brief anthologies. The *Floretus* and the *Facetus* (the latter erroneously attributed to the prolific Johannes de Garlandia) are but typical of the kind of reading matter that was beginning to replace the classics in the study of grammar.²

In this change in the method of teaching the whole subject of grammar, is to be found the explanation of the decline of classical taste and Latin style in the thirteenth and fourteenth centuries. This situation is in marked contrast to the prevalence of classical taste which even Hallam admits existed in the tenth, and more particularly in the eleventh centuries,³ which was unmistakably due to the great amount of instruction in the classics given in the schools of that period.

In the period of scholasticism there is a shrinking of the

¹ Three most noted instances are, (1) the *Visio Wettini* by Strabo, containing some nine hundred hexameters, and written in the eighteenth year of his age; Text in *M. G. H. Poet. Lat. M. A.* II, p. 303 et seq., and Migne CXIV col. 1063. (2) The famous *Waltherius* of Ekkehardt I (c. 973) though written in German was also a dictamen exercise modelled after Virgil and Prudentius. Vide Ebert, *op. cit.* III, p. 265 et seq. (3) Walther von Speier's *Acta St. Christophori*, was composed by him at the close of his grammatical studies when he was eighteen years old, and occupied two months in the writing; the first book of the exercise is the "Liber de Studio poetarum" quoted in this monograph; Text, Pez, *op. cit.* III. Pt. II, cols. 30-94.

² For "*Floretus*" vide *Hist. Lit. de la France* VIII, pp. 92-94; cf. Sandys, *op. cit.* pp. 643-650; Norden, *Antike Kunstprosa* II, pp. 688-731. For a notice on mss. of these two works, vide Hauréau, *Notices et Extraits etc.* XXVII, Pt. II, 15-20, 25-27.

³ Hallam, *op. cit.* p. 7.

subject matter of grammar. After a most cursory study of the mere elements of Latin the pupil was hurried to the university and Aristotle. In this neglect of Latin literature, pagan and Christian authors alike suffered. The decline of the interest in the classics in time brought its own reaction and made possible the humanistic movement. The main purpose, however, of the present investigation is to review the typical instruction of the middle ages proper, consequently a detailed examination of this period of decline is foreign to our inquiry and forms more properly a part of the study of humanism.

CHAPTER IV.

Grammar—(Continued)

III. TEXT BOOKS.

Nowadays if we wish to form a tentative opinion as to the character of instruction in an educational institution, we look at the curriculum and at the text books used in the various subjects. But while in our day these give only a clue to the general character of the instruction, in the middle ages the text book was almost the sum total of the instruction given. An examination of the mediæval text books, dry though such a task may be, affords a true insight into the character of mediæval teaching.

The mediæval text book represented not only what the pupil studied, but also what in many cases the teacher knew, of a given subject. His teaching consisted largely of dictation from the text book. So general was this method that the words "legere" and "docere" were synonymous and interchangeable terms.

The texts were of two kinds: (1) *encyclopædic*, including treatises on all or nearly all of the subjects of the curriculum; and (2) *individual*, treating of specific subjects only. The latter class were nearly always either abbreviated or elaborated adaptations from the standard encyclopædic works, and were almost always the work of some teacher. Such adaptations not infrequently became in turn the basis for subsequent texts.

Among the standard encyclopædias were those of Capella, Cassiodorus, Augustine, Boethius, Isidore of Seville, the Venerable Bede, Alcuin, Rabanus Maurus and Remigius of Auxerre. The works of Boethius and Bede included only the subjects of the Quadrivium. Capella's was based on the earlier work of Varro. Those of Cassiodorus and Isidore of Seville were in turn based on that of Capella.¹

Passing on to the consideration of the text books on grammar, we find them all based mainly on the works of Donatus and Priscian. They may be divided into two classes, belonging to

¹ The amount of space occupied by each subject in the above mentioned

two distinct periods, with the end of the twelfth century as the time of transition.)

The characteristics of the grammatical works of the early period are: (1) A strict adherence to the plan of Donatus and Priscian; (2) A disposition to reason about the facts of grammar. The characteristics of the works of the later period are: (1) An exaggeration of the disposition to reason observable in the first period; (2) A paucity of illustrative material; and (3) a general, dry and deadening method of treatment. It is the period of scholasticism in grammar.

THE FIRST PERIOD.

The text books of this period are almost all from Italy.¹ Many of them are in the form of a dialogue, not in the Socratic manner, but rather in the form of question and answer. As to the intrinsic value of these grammars it may be said that while worthless when treating of derivations, they are very scientific in the portions dealing with syntax.²

The most widely known text book on grammar throughout the middle ages was the *Ars Grammatica Minor* of Aelius Donatus.³ It is not a reading book, as is sometimes asserted, but a brief outline of the eight parts of speech occupying some eight printed pages. The author's source was the now lost *Ars Grammatica* of Palæmon, an author of the first century.⁴

encyclopaedias is of interest as showing the quantity of knowledge imparted

Author	Gram- mar	Rhet- oric	Dia- lectic	Arith- metic	Geom- etry	Astron- omy	Music	Pages reduced to 16 mo.
Capella . . .	11	14	11	11	15	9	11	
Cassiodorus	25	5½	18	2	2	15	2	
Boethius . .				40	30		67	
Isidore . . .	50	14	14	2	1	3	12	
Alcuin . . .	54	26	25			23		
Rabanus . .	55					60		

¹ Sandys, *op. cit.* p. 638 et seq.

² Fr. Haase, *De Medii Aevi Studiis Philologicis* quoted in Paulsen, *Geschichte des gelehrten Unterrichts*, p. 46.

³ Little is known of him except that he was a teacher of St. Jerome. On the fantastic accounts of his life, current during the middle ages, see Meier, *Die Sieben Freien Künste im M. A.*, p. 12. The *Ars Grammatica Minor* is so-called to distinguish it from Donatus's *Ars Grammatica Major*, a more elaborate treatise on the subject.

⁴ Cf. F. R. Stolz, *Lateinische Grammatik*, p. 4.; Teuffel-Schwabe, *Ge-*

Altogether different from the book of Donatus, and all-
schichte der Römischen Literatur Bd. II., p. 1035; Text in Keil, *Grammatici Latini*, Vol. IV., pp. 355-367.

Because of the great part which this work played in the education of the middle ages, an analysis of the book is herewith given as the best means of conveying an idea of its contents:—

I. The Noun.

A. Definition.

B. Attributes.

1. Quality.

Common, proper, collective, appellative.

2. Comparison.

Positive, comparative, superlative. Illustrations.

3. Gender.

Masculine, Feminine, neuter, common.

4. Number.

Singular and plural.

5. Figure.

6. Case.

Nominative, genitive, dative, accusative, vocative, ablative.

Nouns illustrating the six attributes given.

Rules for genitive plural and dative and ablative.

II. Pronoun.

A. Definition.

B. Attributes.

1. Quality.

Definite, indefinite.

2. Gender.

3. Number.

4. Figure.

Simple and compound.

5. Person.

6. Case.

Illustrations—all pronouns declined.

III. Verb.

A. Definition.

B. Attributes.

1. Quality.

Modes.

Indicative, imperative, optative, conjunctive, infinitive, impersonal.

Forms.

Perfect, meditative, frequentative, incohesive.

2. Conjugations.

3. Gender.

embracing in its scope was the *Institutio de Arte Grammatica* by

Active, passive, neuter, deponent, common.

4. Number.

5. Figure.

Simple, compound

6. Time.

Present, past, future.

All forms of *lego* given.

IV. Adverb.

A. Definition.

B. Relation.

Place, time, number, negative, affirmative, denominative,
optative, hortative.

Illustrations.

C. Comparison.

D. Figure.

Simple and compound.

V. Participle.

A. Definition.

B. Gender.

Illustrations.

C. Case.

Illustrations.

D. Time.

Illustrations.

E. Signification.

F. Number.

G. Figure.

Illustrations.

All participial forms of *lego* given.

VI. Conjunction.

A. Definition.

B. Attributes.

Copulative, disjunctive, expletive, causal, rational.

Illustrations.

C. Figure.

Simple, compound.

D. Order.

Praepositive, subjunctive, common.

Illustrations.

VII. Prepositions.

A. Attributes.

B. Cases.

Accusative and ablative.

List of prepositions taking each case.

Illustrations.

VIII. Interjection.

A. Definition.

Priscian.¹ The aim of the work as indicated by the author in his dedicatory letter, was to put the study of Latin on the same scientific basis as the study of Greek. His work follows the scientific method of Apollonius Dyscolus, to whom he is indebted for much of his material. The book is divided into eighteen parts, the first sixteen dealing with accidence, the last two with syntax. The portion on syntax, thoroughly scientific, is often found as a separate volume. Most of the manuscripts in existence contain only the first sixteen books, showing that the syntax of Priscian was not as much studied as his grammar.

The book formed the most advanced grammar of the middle ages. There are today in existence one thousand manuscripts of the work. The text covers about five hundred and eighty-four octavo pages, of which one hundred and sixty-two are devoted to syntax. The author quotes no less than two hundred and fifty-five different authors, among them, Aristophanes, Aristotle, Julius Cæsar, Cato, Cicero, Demosthenes, Herodotus, Homer (78 times), Horace (158 times), Juvenal (121 times), Lucretius (25 times), Ovid (73 times), Sallust (80 times), Terence (225 times), the Aeneid (721 times), Virgil's other poems (146 times).² Many abstracts of Priscian's works were made for school use, among which the one by Rabanus Maurus may be mentioned as an example.³

B. Attributes.

Meaning.

Joy, grief, admiration, fear.

Such in brief is the content of this famous text book. Its elementary character and form made it a good outline for teachers to amplify and comment upon. Originally, however, the book was written for boys whose mother tongue was Latin. Naturally therefore it had to be modified to suit the needs of the Germanic nations. In this way a large number of grammatical text books based upon it were composed.

¹ Little is known of the author's life. His fame was great; his title in the middle ages was "Communis hominis praeceptor", "Latini eloquentiae decus." Cf. Keil, *op. cit.* Vol. II., Praefatio, p. XII; Text in Keil, vols. II. and III.; F. Stolz, *op. cit.* p. 4, characterizes the work as elaborate, containing much material but of no originality.

² These numerous quotations must have made it also a valuable anthology. To what extent are the learned quotations in which mediæval writers abound copied from Priscian instead of from the originals?

³ Keil, *op. cit.*, Vol. II, Praefatio, X.

Besides the grammars of Donatus and Priscian, the following may be mentioned in their chronological order, as of importance for this period.¹

(1) The *Instituta Artium* and the *Catholica* of M. Valerius Probus, one of the foremost grammarians of the latter half of the first century, who is responsible for the main outlines of traditional Latin grammar.²

(2) *Artis Grammaticæ Libri III*, of Diomedes, a fourth century grammarian, who did much to preserve the grammatical teaching of an earlier age. His book was used at St. Gall in the ninth century. It served as a basis for many mediæval grammatical text books.³

(3) *Institutionum Grammaticarum Libri V* of Flavius Sosipater Charisius, a contemporary of Diomedes. His material is taken mainly from Varro and from Seutonius's lost work *De Poetis*.⁴

(4) *Ars de Nomine et Verbo* of Phocas. The book is mentioned by Alcuin as being in the famous library of York in the eighth century. Glosses on the work have been found.⁵

(5) *Ars Grammatica*, of C. Marius Victorinus, a grammarian and rhetorician of the fourth century. His treatise in four books devoted especially to the study of metre is mentioned by Notker.⁶

(6) *De Nomine et Verbo*, of Consentius, a contemporary of the famous Sidonius Apollinaris.⁷

(7) *De Differentiis et Societatibus Græci Latinique Verbi* of

¹ The grammar of Isidore of Seville is a mere compilation from Donatus and Priscian, and shows no individual characteristics. But for the fact that it formed a part of his encyclopaedic work it would have been used but little. There are no separate texts of it in existence.

² Cf. Teuffel-Schwabe, *op. cit.* II. p. 729 et seq.; Haase, *Vorlesungen über Lateinische Sprachwissenschaft*, p. 11, Text Keil, IV., pp. 3-192.

³ Teuffel-Schwabe, *ibid.*; Keil, I, p. XVII; Text, Keil I, p. 7 et seq.

⁴ Teuffel-Schwabe, *op. cit.* II. p. 1057 et seq.; Specht, *op. cit.* p. 91; Text, Keil, I. p. 300 et seq.

⁵ Text, Keil V, pp. 410-441; Cf. Specht, p. 90; "Versus de Sanctis Eboracensis Ecclesiæ" vv. 1535-1561, quoted in West, *Alcuin*, p. 35; for Glosses, see Steinmeyer and Siever, *Althochdeutsche Glossen*, Bd. II, pp. 363-365.

⁶ Specht, p. 91; Teuffel-Schwabe, *op. cit.*, II, pp. 1033 et seq.; Sandys, p. 217; Text in Keil, Vol. VI. pp. 3-184.

⁷ Cf. Sandys, p. 235; Text in Keil V., p. 329 et seq.

Macrobius. This treatise on the difference between the verb in Latin and the verb in Greek, was a Latin adaptation of the Greek syntax of Apollinaris of Alexandria. The author was a famous scholar living at the end of the fourth and the beginning of the fifth century.¹

(8) *Commentarius in Artem Donati*, of Marius Servius Honoratus. The author was a famous commentator on Virgil. His grammar, a commentary on Donatus, was also in the famous library of York in Alcuin's school days.²

(9) *Commentarius in Artem Donati* of Servius Marus, a contemporary of Sidonius Apollinaris. For this period Marus is unquestionably the great commentator of Donatus.³

(10) *Ars de Verbo*, of Eutyches, a pupil of Priscian. There are in existence a number of glosses on this grammar.⁴

(11) *Ars (Grammatica) Breviata* of St. Augustine, a brief excerpt from his encyclopædia on the liberal arts.⁵

(12) *Institutio de Arte Grammatica* of Cassiodorus. The work is the author's brief treatise on grammar, which forms a portion of his encyclopædia of the seven liberal arts.⁶

(13) *Ars Grammatica* of Asper. The treatise gives a brief treatment of letters, syllables, parts of speech etc. It was found in the library of St. Gall in the ninth century.⁷

(14) *Ars Grammatica* of St. Boniface, the apostle to the Germans. His work is an adaptation of the grammar of Donatus, Charisius, Diomedes.⁸

(15) *Dialogus de Arte Grammatica* of Alcuin is based on Donatus, whom he avowedly follows. The name of the author gave the work great prominence. Notker of

¹ Cf. Haase, *op. cit.* p. 11; Sandys, p. 224 et seq.; Text, extracts only, in Keil, 599-655.

² Sandys, p. 218; Text in Keil, Vol. IV, pp. 405-448.

³ Cf. Sandys, p. 462; text in Keil, IV, p. 405-448.

⁴ Teuffel-Schwabe, *op. cit.* p. 1245; West, *Alcuin*, p. 35; Steinmeyer und Siever, *op. cit.*, II, p. 161; text in Keil V., pp. 442-489.

⁵ Specht, p. 91; Text in Keil, V., p. 490 et seq.

⁶ Text, Keil, VII, p. 214.

⁷ Cf. Specht, p. 91; Text, Keil, V., p. 525 et seq.

⁸ Cf. Sandys, p. 453; Ebert, I, pp. 653-659; Bursian, *Geschichte der Classischen Philologie in Deutschland*, I, p. 14 et seq.; Text, Mai, *Classic auctores e Vat. Cod.* T. VII, pp. 475-598.

St. Gall thought it even better than Donatus and Priscian.¹

(16) *Excerptio de Arte Grammatici Prisciani* by Rabanus Maurus, is a brief compendium of the elements of grammar, with numerous illustrations and some observations on prosody, apparently culled from the *Ars Grammatica Major* of Donatus.²

(17) *Tractatus in Partibus Donati* of Smaragdus, the abbot of St. Michael (ob. c. 830). His work is a commentary on Donatus, but the illustrations are from the Vulgate and not as was the case with most of these commentators from Virgil and Cicero. The avowed object of his innovations was that "the reader might be able to get the spirit of grammar and the thought of the holy scriptures at the same time."³

(18) *Expositio super Donatum*, of Remigius of Auxerre, (d. 908), a famous commentary on Donatus by a famous teacher of sacred and profane literature, who also wrote a commentary on the encyclopædia of Capella. The former was used till the Renaissance.⁴

THE SECOND PERIOD.

The important place which Priscian occupied as an advanced text book on grammar in the first period was later usurped by the *Doctrinale* of Alexander de Villedieu, composed in 1199.⁵

From the mediæval point of view there were important considerations to explain the phenomenal popularity of this

¹ Cf. Meier, p. 14; Text, Migne, Vol. CI, cols. 848-901.

² Text, Migne, CXI, cols. 613-678.

³ For manuscripts extant and extracts vide Thurot, "Notices et Extraits de divers manuscrits Latins pour servir à l'histoire des doctrines grammaticales au moyen âge," in *Notices et Extraits etc.* Tom. XXII, Pt. 2, pp. 4, 65, 68, 81. Cf. Meier, *op. cit.*, p. 14; *Hist. Lit. de la France* T. IV., p. 445 et seq.

⁴ Remigius was the author of commentaries on Donatus, Priscian, and Capella's *De Nuptiis etc.* His grammatical work on Donatus was printed at Basel in 1499 under the name *Remigii fundamentum scholarum*; Cf. Huemer in *Sitz. der Königl. Akad. d. Wiss. Philol. Hist. Class.* Wien Bd. XCVI, p. 538. For a description of his grammatical works, vide Thurot, *op. cit.* p. 94, note 2; Extracts, *ibid.* passim.

⁵ Text and Introduction by Th. Reichling, in *Monumenta Germaniae Paedagogica*, Bd. XII. The *Doctrinale* has received elaborate treatment by Thurot, in his *De Alexandri de Villa Dei Doctrinale*, 1850, and in *Notices et Extraits etc.* XXII, Pt. 2. More recently K. Julius Neudeker has introduced new material on the subject in his *Das Doctrinale*, 1885. But the exhaustive work of Reichling has superseded all previous studies.

book, which had undisputed sway in most of the schools of Western Europe for over three centuries. The number of manuscripts actually in existence at present reaches two hundred and fifty, (thirty-three of the thirteenth century, sixty-four of the fourteenth, one hundred and fifty-four of the fifteenth, and nine of the sixteenth,) while the total number of printed editions is no less than two hundred and ninety-five.¹

The book was used in the schools of Germany, France, England, Spain, Italy and Poland, as well as in the universities. The reasons for its wide use are the following:—

(1) The entire grammar was in verse. In an age when memorizing was so prominent a feature of instruction because of the scarcity of books, the versified form was certainly a boon to the teacher. This innovation was the original feature of the text book.

(2) In his treatment of the subject the author took into account the changes which the Latin language had undergone in the seven centuries since Priscian wrote his famous grammar. He incorporated in his book many words from the scriptures; also Latinized Teutonic words. It therefore was a grammar meeting the practical requirements of the living language of the day.

(3) The Syntax of Priscian was no longer adequate to the requirements of an age when the logical aspects of grammar were to be emphasized rather than the literary. Alexander's book was very thorough in the syntax; his clearness in the treatment of that part of his subject has been commended even by modern students of philology.²

(4) His treatment of prosody and figures was likewise an improvement on the work of Priscian and furnished much new material.³

In spite of these innovations, the work, which is an excerpt of the author's larger work on grammar, the computus and canon law, was after all merely a commentary on the older grammars, especially Priscian, whom he closely follows in his treatment of the etymological questions.

¹ For a complete critical bibliography of these vide Reichling, *op. cit.* CXXI-CCCIX.

² Vide Haase, *Vorlesungen etc.*, pp. 13, 15.

³ Reichling, *op. cit.* pp. VII-XX.

The treatise comprises three main divisions: (1) On etymology (1073 lines); (2) on syntax (476 lines); (3) on quantity, accent and figure (1095 lines). In its usual form it appeared with a prologue and twelve "capitalia." Being an advanced grammar for those who had already studied the Donatus Minor the book naturally omits in its etymological portion the treatment of numerals, regular verbs, adverbs, conjunctions, prepositions, and gives only a most cursory treatment of pronouns. In the syntax, likewise, tenses and moods are omitted, while the treatment of quantity, accents, grammatical and rhetorical figures is very thorough.¹

The general use and authoritative character of the *Doctrinale* gained for its author the title, "Aristotle of Grammar." The Humanists quite naturally therefore directed their first attack on the old order against this book, and after twenty years of vehement campaigning they succeeded (c. 1510) in discarding it. Nevertheless, as Reichling has shown, the Humanists themselves in the succeeding generation incorporated much of the work of Alexander in their own text books.²

¹ The following verses which treat in full of the first declension give a fair idea of the character of the entire book:

Rectis *as es a* dat declinatio prima,
Atque per *am* propria quaedam ponuntur Hebraea
Dans *ae* diphthongon genitivis atque dativis.
Am servat quartus; tamen *en* aut *an* reperimus,
Cum rectus fit in *es* vel in *as*, vel cum dat *a* Graecus.
Rectus in *a* Graeci facit an quarto breviari.
Quintus in *a* dabitur, post *es* tamen *e* reperitur.
A sextus, tamen *es* quandoque per *e* dare debes
Am recti repetes quinto, sextum sociando.
Primo plurali decet *ae* quintoque locari.
Atque secundum habet *arum*, nisi syncopa fiat.
Tertius aut sextus habet *is*, tamen excipiemus;
Quando *mas* fit in *us*, in *a* femineum sine neutro,
Femineis *abus* sociabitur ut *dominabus*
Sexum discernens; istis *animas* auferaddes.
Accusativis pluralibus *as* sociabis
Versibus his nota fit declinatio prima.

Text, Reichling, *op. cit.*, pp. 8, 9. It is evident from the foregoing typical verses that a commentary to the *Doctrinale* was not unnecessary. In fact, as the author explicitly states in his prologue, the teacher was expected to explain the text and to illustrate it in "lay language." Of such commentaries there were many. Some of these Reichling describes, *op. cit.* LX-LXXI.

² Reichling, "Der Kampf um das *Doctrinale*," *op. cit.* LXXXIII-CX.

Second only to the *Doctrinale* in importance for this period is Eberhard of Bethune's, *Græcismus* (c. 1212).¹ Like the work of Alexander it is in verse. Its name is derived from the fact that the book has a chapter on Greek derivatives, a subject quite unknown to the author. The main topics which he treats are etymology, barbarisms, tropes, solecisms,

Another popular work by Eberhard was his *Laborinthus*, a didactic poem on grammar and style. It was much used in the fourteenth century and helped further to displace Priscian. Luther considered it one "of the foolish, useless and harmful monkish books introduced by the devil."²

Among the text books of lesser importance may be mentioned:

(1) The *Summa* of Petrus Helias, composed about 1142. The book is a commentary on Priscian, and is among those quoted in Duns Scotus' work on grammar. Typical of its time, it is in verse and explains Latin grammar without any reference to classical authors.³

(2) The *Ars Rhythmica* of Johannes de Garlandia, was an elaborate work on prosody with complete poems for illustration. The author was a teacher of grammar at the University of Toulouse from 1229-1232. He also wrote a commentary on the *Doctrinale*.⁴

(3) A commentary on the first fifteen books of Priscian by Robert Kilwardy, Archbishop of Canterbury from 1272 to 1279.⁵

¹ Full title, *Græcismus sive liber carmine hexametro scriptus de figuris deque octo partibus orationibus*. Text in Edition of J. Wrobel, *Corpus Grammaticorum Medii Aevi*, I. The date of the *Græcismus* has been fixed by Reichling, *op. cit.* LXXIX-LXXXIII; Cf. Bähler, *Beiträge zur einer Geschichte der Lat. Grammat. in M. A.*, p. 95 et seq., where extracts are given; also Schmid, *Geschichte der Erz. etc.* II, p. 298 et seq.

² Text in Polycarp Leyser, pp. 796-854. Partly in G. Mari, "Trattati medievali di Ritmica Latina" in *Memorie del Reale Istituto Lombardo*, tom. XX, p. 453 et seq. Critique by M. Thurot in *Comptes rendus, Académie des Inscriptions et Belles-Lettres*, 2nd Series, T. VI, pp. 239-269.

³ Meier, *op. cit.*, p. 15. Helias' commentary forms a part of Vincent de Beauvais' *Speculum Doctrinale*, Cf. Thurot, *Notices et Extraits. etc.*, XXII, Pt. 2, p. 2.

⁴ Text ed. G. Mari, "Trattati Medievali di Ritmica Latina," in *Memorie del Reale Istituto Lombardo*, tom. XX., pp. 383 et seq.; Cf. Reichling, *op. cit.* LIV, LV.

⁵ Exists in the Cambridge University Library, Cf. Sandys, 641.

(4) *De Modis Significandis*, commonly known as the *Modista*, by Michael of Marbais, a Flemish writer of the thirteenth century.¹

(5) *Catholicon*, composed in 1286 by Johannes Balbus Januensis. The book was so named "quod sit liber communis et universalis." The work comprised five books: (1) De Orthographia; (2) De Accentu; (3) De Etymologia; (4) De Figuris; (5) De Prosodia. Added to it was a brief vocabulary.²

(6) *Artis Grammaticæ Institutio*, a grammar well known in its day, in which the examples were based on Sallust, Virgil, Horace, Ovid, Lucan, and Juvenal, was the work of Cæsar of Lombardy, another thirteenth century writer.³

(7) *Grammatica Speculativa de Modis Significandis*, was composed by Duns Scotus, the famous scholastic writer. It is typical of the philosophical treatment given to grammar in the later middle ages.⁴

(8) *Florista* was written by Ludolf of Lüchow, (c. 1236.) The author was a teacher at the famous school of Hildesheim. The work, a syntax in verse, was so named because the first line read "Flores grammaticæ propono scribere." It was widely used in Germany, Netherlands and France.⁵

VOCABULARIES.

I. GENERAL CHARACTER.

Supplementary to the text books on grammar proper were the "vocabularies" which occupied a prominent place in the instruction in grammar in the curriculum of the seven liberal arts. These were variously known as "vocabularii," "dictionarii," "glossæ" and "commentarii." Only with their aid could

¹ Meier, p. 15.

² Haase, *Vorlesungen etc.*, p. 14; For extracts of portions of the dictionary, vide *Le Catholicon de Lille*, A. Scheler, ed., *Memoires Couronnées de l'Académie royale de Belgique* T. XXXVII.

³ Text in edition of C. Fierville, 1886; Cf. Sandys, 584.

⁴ Schmid, *op. cit.* II, p. 439. The work was much used and was printed by Gutenberg; Cf. Hallam, *Lit. of M. A.* I, 80, 4th ed.

⁵ Meier, p. 15. Ludolf is the author of the *Ethica Ludolfi* an adaptation of the *Distichia Catonis*. Cf. R. Peiper, *op. cit.* as quoted *supra* p. 16, note. For some of the other grammars of the period vide Thurot, *op. cit.* in *Notices et Extraits etc.* T. XXII, Pt. II, pp. 1-58; extracts, *passim*.

the mastery of the Latin language both for practical and literary purposes be accomplished.¹

The characteristic "vocabularies" of the middle ages were more than mere working dictionaries for particular classical authors. They were of different kinds, some explaining the Latin words in easier Latin, some in the vernacular,—French, German or Anglo-Saxon. Some were arranged alphabetically, others were classified by subjects (heavenly bodies, animals etc.) Some were dictionaries pure and simple; others partook of the character of an encyclopædia, a sort of a vade mecum. The evidence as to their number indicates that every library and school, in so far as we have records of the schools, had at least one of these dictionareis.²

Besides these formal dictionaries there were special glossaries designed to translate the difficult words of a standard text book on grammar, or some particularly widely used author. The extent to which these special vocabularies were used can be seen, to cite but one example, in the vast collection of glossaries which Steinmeyer and Sievers have published. In that we find an alphabetical dictionary of some fifty-two hundred biblical words of the Vulgate, compiled by Rabanus Maurus; glossaries for nearly every book of the Old and New Testament; glossaries for Capella, Donatus, Priscian and the other authors studied in the schools.³

¹ The subject of mediæval lexicography has received treatment at the hands of a number of scholars, though all of these studies are more or less fragmentary. The latest and most scientific monograph on the subject is by F. Heerdegen "Lateinische Lexicographie" in I. Müller, *Handbuch der Klassischen Alterthumswissenschaft* II, Pt. 2, p. 497 et seq. The numerous collections of these mediæval dictionaries will be referred to in the text of this chapter.

² Thus the famous codex of glossaries of St. Gall contains no less than twenty-five hundred words in alphabetical order. Vide text, *Corpus Glossariorum Latinorum* IV, pp. 201–298. On extant mss. Cf. *op. cit.* Praefatio XV et seq. Cf. Sandys, *op. cit.* p. 584, note.

³ Steinmeyer and Sievers, *Althochdeutsche Glossen*, 3 vols., I, "Biblical Glosses", II, "Church Fathers, Grammarians," III "Miscellaneous." Though Steinmeyer and Sievers' work covers only the territory of the German Empire the character of the "glosses" which it contains is so typical of the general character of such books as to deserve a more careful examination. It contains four different "glosses" on Capella, used at St. Gall; some on Donatus; no less than ten large "glosses" on Priscian; a number on Cicero; five on Horace; two on Ovid; nine on Sallust; and fifteen on Virgil. The number of "glosses" on

2. SCHOOL DICTIONARIES.

The many formal vocabularies in vogue in the middle ages were, as we would naturally expect, revised editions of dictionaries of classical times.

The earliest dictionary of the middle ages is the one of Isidore of Seville, and forms a part of his famous *Etymologiæ*. It represents perhaps the most useful portion of his encyclopædia, being based directly on the authors of the later Empire.¹

The next important work of this character is the famous dictionary of Paulus Diaconus, which was dedicated to Charlemagne. It was an abstract of the work of Pompeius Festus, an author of the second century, whose work in turn was an abridgement of the *De Verborum Significatu*—one of the earliest Latin lexicons.²

Next in order of time may be mentioned the so-called *Glossarium* of Salomo III, Bishop of Constance (ob. 919). It was a compilation by an unknown monk of St. Gall. The work was in reality what we should call an encyclopædia. The words were alphabetically arranged, numerous extracts from famous authors were given. There are many manuscripts of the work in existence. It was first printed in 1470. As originally composed the text was in Latin but later, German glossaries were added and in this modified form the book was much used throughout the middle ages.³

literary Christian authors is not exceptionally large as compared with the number on pagan authors, if we exclude Prudentius, who, as we have seen, seemed to have enjoyed exceptional popularity. Thus Sedulius' *Carmen Paschale*, is represented in the collection by fourteen "glosses;" Juvenius by eight, while Prudentius has no less than forty-seven, covering some two hundred quarto pages. There are some forty "glosses" on names of animals, birds, fishes etc., showing again the extent to which the fable was used. Of equal significance are those on Aldhelm's "Saxon Riddles" and on the "Eight Principal Vices." Vide *op. cit.* passim. Among other interesting "glosses" may be cited one on Bede's *De Temporum Ratione* Cf. *infra* p. 97.

¹ Text, *Etymologiae* XX, Lib. X; Migne, LXXXII cols. 367-398. Other portions of his work likewise include material partaking of the character of a glossary.

² The work of Paulus has been published three times, the latest edition by A. Thewrewk de Poner, *Sexti Pompei Festi de verborum significatu cum Pauli epitome*.

³ Maier, *Die Sieben Freien Künste im M. A.* p. 16.

Aelfric, Archbishop of Canterbury, (ob. 1016), is the author of a famous Anglo-Saxon *Glossarium*. The writer cleverly grouped and arranged his material to explain the meaning of many Latin words. With this little book the acquisition of a Latin vocabulary must have been an easy task to the Anglo-Saxon boy.¹

Perhaps the most famous dictionary of the middle ages was the *Vocabulista* of Papias, the Lombard (c. 1053-1063). It consisted of two kinds of material: first, a glossary; second, information of value to the student of the seven liberal arts. In the glossary proper it is interesting to notice that the author apparently makes no distinction in quantity, gender and inflection, between classical and barbarous forms. The second portion of the book accounts in a large measure for its popularity. This may be judged from the large number of imitations of that portion by other authors.²

One hundred years after Papias, we find another important work by a monk, Osborn, of Gloucester, England. It is the famous *Panormia* which has been considered a worthy attempt to make a dictionary explaining etymologically the meaning of words.³

Hugutio, or Hugo of Pisa (Bishop of Ferrera, 1210), is the author of the *Liber Derivationum*. The work though very popu-

¹ Full title of the work; "Colloquium ad pueros linguae Latinae locutione exercendos." Text in Thorpe, *Analecta Anglo-Saxonica*, pp. 18-36. Aelfric composed a Latin grammar in Anglo-Saxon. Vide text in Zupitza, *Sammlung Englischer Denkmäler*. Aelfric is probably the author of the vocabulary which is ascribed to him in Wright's *A Volume of Vocabularies*. On the popularity of his works as text books as judged by number of manuscripts and his educational activity, vide White, *Aelfric*, pp. 119-124.

² The full title of the work is *Elementarium doctrinae erudimentum*, Text in edition 1491, Venice. For analysis and criticism of the work, vide G. Lowe, *Prodromus corporis glossariorum Latinorum* p. 235 et seq. Cf. Sandys, p. 501; Haase, *Vorlesungen etc.* p. 13; Heerdegen, *op. cit.* p. 498. The principal source of Papias it is supposed was the anonymous dictionary of the eighth and ninth centuries, which in its turn was partly derived from the work of a contemporary of Sidonius. Vide Sandys, pp. 235, 639 note 3.

³ For text vide "Thesaurus novus Latinitatis sive lexicon vetus e membranis nunc primum erutum", in A. Mai, *Classici Actores e vaticanis codicibus editi*, Tomus VIII. For proof of the identity of this elaborate dictionary (632 octavo pages) with the *Panormia*, vide Lowe, *op. cit.* p. 240 et seq. A dictionary by Petrus Helias, composed at this period, is lost. Cf. Meier, p. 15.

lar in its time has never been printed in full. In content it is based on Papias, and derives its name from the author's attempt to arrange words according to their derivation. The mistakes in Greek derivations proved excellent material for satirical attacks by the Humanists.¹

Of similar importance were the three *Dictionarii* of Johannes De Garlandia, one of common words, one of obscure words, and the third of "things in general." The work was still in use in the boyhood of Erasmus.²

The *Catholicon* alluded to before, was as famous as a dictionary as it was as a text book on grammar. For the meaning of words, the author used Papias as his source, and for his etymology, the book of Hugutio. This work was among the first Latin books printed. The vocabulary treated rather pretentiously of "orthographia, prosodia, significatio, origo, etymologia, . . . dictiones quae frequenter inveniuntur in biblia et in dictis sanctorum et poetarum."³

Alexander Neckham is the author of no less than three elaborate vocabularies: (1) a *Vocabularium Biblicum*; (2) a *Repertorium Vocabulorum*; (3) *De Utensilibus*. Of these the first two have never been published. The last one treats of the names of articles by means of a connected narrative, with interlinear glossaries in French.⁴

Marchesini of Regio (c. 1300), is the author of the much abused *Mamotrectus*, a biblical glossary. The work contains grammatical, orthographical and exegetical glossaries of the Scriptures.⁵

¹ For analysis and critique vide Lowe, *op. cit.* p. 243 et seq. For descriptions of many derivatives of Papias' and Hugutio's texts in ms., vide Lowe, *op. cit.*, pp. 247-259. For an extended account of "glosses" of the late middle ages vide Lowe, *op. cit.* p. 222 et seq.

² Text in Wright's, *A Volume of Vocabularies*, pp. 121-138. Cf. Scheler, *op. cit.*, pp. 18-83; Eckstein, *l. c.* The author also composed an *Opus Synonymorum* and an *Opus Aequivocorum*, which may also in a sense be classed as vocabularies. For the text of the former vide Migne, CL. cols. 1578-1590. For a critique on the many manuscripts of dictionaries extant vide Hauréau, *Notices et Extraits etc.* XXVII, Pt. 2, pp. 38-48.

³ Text printed sometime between 1460-1483. Cf. Heerdegen, *op. cit.* p. 499.

⁴ Text in ed. of Scheler; Cf. Sandy's, p. 527.

⁵ Full title: "Expositio in Singulis Libris Bibliae authore Marchisino." Cf. Schmid, "Geschichte der Erziehung," II, p. 299.

There were numerous vocabularies of lesser importance. Of such only a few may be mentioned: (1) The vocabulary of Martinus de Arenis (1307), an abstract of Papias; (2) the so-called *Vocabularius Theabonicus*; (3) *Vocabularius Optimus*; (4) *Gemma Gemmarum*; (5) *Lucidarius*; (6) *Summarium Heinrici*. All of these were based more or less on the elaborate works previously mentioned.¹

¹ Vide Haase, *Vorlesungen etc.* p. 13 et seq.; Meier, *op. cit.* p. 17. For a brief study on glosses in the middle ages with illustrative extracts vide "Bäbler," *op. cit.* pp. 170-188.

CHAPTER V.

Rhetoric.

A. TECHNICAL STUDY.

GENERAL CONDITIONS.

More than any other of the subjects of the mediæval curriculum, Rhetoric shows the distinctive characteristics of the age. While in the other branches, grammar especially, the methods and ideals of the later Empire were largely followed, being modified only to meet the changing requirements of new conditions, the study of rhetoric assumed an entirely new character. On the one hand the old practical rhetorical training of the Roman period was almost entirely discarded or was reduced to a mere mastery of the technical rules of the science. On the other hand, one insignificant phase of classical rhetoric—the study of the Epistle and “Dictamen”—was overemphasized and developed to such an extent as to displace in the curriculum the study of rhetoric proper.

This change was not made without reason. The decadence of the rhetorical schools of Rome was caused by the deadening formalism which resulted from perpetuating the ideal of training orators at a time when the world had no use for orators. In the days of Cicero the training in oratory was in harmony with the spirit of the age. Since its noblest use was then the defense of liberty, his seven works on eloquence were timely treatises indeed. But in the later empire, in spite of changed conditions, this formal training in oratory remained, as we have seen in a preceding chapter, the same as in the period of Cicero. Such an artificial and lifeless system of education could not but be debasing in its influence.¹ Now, as was also shown in a former chapter, while the Christianized Roman world permitted the pagan schools to die, it was not

¹ The decadent character of the rhetorical instruction of the later Empire has been fully treated by George Kauffman, “Rhetorenschulen und Klosterschulen” *Histor. Taschenbuch* 1869, pp. 1–94; O. Denk, *Geschichte des Gallo-Frankischen Unterrichts und Bildungswesens*, pp. 40–163, especially 140–163; Dill, *Roman Society in the last century of the Western Empire*, pp. 321–376.

averse to appropriating for its own purposes the essential elements of the culture which these various schools imparted. But it could not, from the nature of things, accept the educational ideal of the ancient world—the training of the orator. Therefore rhetoric, the basis of education in the Empire, lost its importance in the middle ages and could no longer be the keystone of the educational arch. How natural that the Christian schools should refuse to place much value on the “*Ars bene dicendi*”, as rhetoric was most generally defined.¹ For the puerilities of youthful declamations, for panegyrics in stiff Latin and apostrophes to mythological heroes, they of course could have no use. The ability to speak might be of some value to a priest who would preach occasionally, but excellence in that direction could certainly not be his greatest field of usefulness.

The opinion of Rabanus Maurus, the “*præceptor Germaniæ*” can be said fairly to represent the mediæval estimate of the value of the study of rhetoric. He says:—

“It is sufficient if youths give some attention to the study of rhetoric. Even then not all who expect to enter the priesthood, but only those who are not as yet obliged to devote their time to pursuits of greater usefulness should study the subject. At any rate one who wishes to acquire the art of eloquence can so do more advantageously by reading and hearing great orators than by studying the rules of rhetoric.”²

This representative view adequately accounts for the small amount of time given to the technical study of rhetoric as compared with that given to grammar. The lesser number of manuscripts of rhetoric which have been found and their elementary character indicate the relative indifference to this branch of the curriculum.³

¹ Raban. Maurus, *De Clericor. Institutione*, III, c. 19, Migne, *Pat. Lat.* Vol. CVII, col. 396.

² *Ibid.* Col. 397.

³ By calculating the number of codices which K. F. Halm used in his collection of the text books on rhetoric—*Rhetores Latini Minores*, 1863—the writer estimates that there are extant to-day about one hundred manuscripts of the text books on rhetoric used in the middle ages. Compare this with the number of manuscripts of text books on grammar—one thousand of Priscian alone. The relative amount of time devoted to these two studies in the schools of the middle ages can easily be judged from these facts.

TEXT BOOKS.

While in the technical study of rhetoric, Cicero and Quintilian were throughout the middle ages looked upon as models of style, their own works were rarely used as text books, possibly enough on account of their considerable size¹.

For school purposes, at any rate, the principles of these masters were transmitted to the middle ages through a number of manuals, all of which were composed in the latter half of the fourth century, and through the elementary treatises on the subject in the encyclopædic works of Capella, Augustine, Cassiodorus, as well as through the special work of Alcuin. These treatises range from a mere catalogue of the figures treated by Cicero and Quintilian to elaborate summaries of the essentials of their works. They are small in bulk and number.

The most elementary of these treatises, forming as it were, a connecting link between grammar and rhetoric, is the

(1) *De Schematis Lexeos* of Julius Rufianus. The author, who lived in the latter half of the fourth century, made a collection of forty-four figures of speech and illustrated them by copious citations from Cicero and Virgil.²

(2) Of similar character is the anonymous *Carmen de Figuris*, of which a number of manuscripts are extant.³ It contains sixty-three rhetorical figures, each illustrated by three lines taken from different ancient sources on rhetoric, including, of course, Cicero's *De Oratore* and Quintilian's *De Institutis*.

(3) Less formal and briefer in the treatment of the figures of rhetoric are the *Schemata Dianeos*. Some fifteen of the sixty-three figures in this work are merely named, and the

¹ The small number of manuscripts of the rhetorical works of Cicero and of Quintilian's *Institutes* which have come down to us at once create this presumption. Compare also *M. T. Ciceronis opera* ed. Orelli I., praefatio. According to the editor there are extant of his rhetorical works four Mss. of the *de Inventione* and three of the *de Oratore*, antedating the thirteenth century.

² Text in K. F. Halm, *Rhetores Latini Minores*, pp. 48-58. Virgil, as we have seen, was considered an authority on rhetoric in the narrower sense of the term, namely the use and choice of words. Cf. Comparetti, *Virgil in the Middle Ages*, p. 133. Cf. *supra* pp. 31, 32.

³ Text, Halm, *op. cit.*, pp. 63-70.

reference to that portion of Quintilian which treats of the figure is indicated.¹

(4) Similar in content and of unique interest is the *Liber de Schematibus et Tropis* by the Venerable Bede. In this work the figures are explained and copiously illustrated from the Vulgate. The author justifies his choice of illustrations by arguing that since the figures constitute the most important part of rhetoric, those of the most important of books—the Scriptures—are of greatest value, and should be used in preference to any others.²

(5) The typical text book of technical rhetoric as studied in the middle ages is the *Libri III Artis Rhetoricæ* by Chirius Fortunatianus, composed in the latter half of the fourth century. Based as it was on the works of Cicero and Quintilian, and written in the catechetical form, the book was widely used and many manuscripts of the text, some dating even as far back as the eighth century, are extant today.³ Fortunatianus's work as a school text book of technical rhetoric was of great value in spite of its defects from the broader stand-

¹ Text, Halm, pp. 71-77.

² Text, Halm, pp. 608-618. Cf. Saintsbury, *History of Criticism* I, p. 374 et seq. His statement that this work of Bede's is of "critical importance" is based on the erroneous assumption that Bede illustrates his figures with quotations now from the classics, now from the Scriptures, thus suggesting comparisons. The most authoritative text, that of Halm, *does not show a single classical illustration*.

³ Its representative character makes an analysis of its contents of interest. The forty quarto pages of actual text are about evenly divided into three parts. The first is devoted exclusively to a treatment of figures and other technical terms, some eighty different kinds in all. His definition of rhetoric is "bene dicendi scientia." The orator is "Vir bonus dicendi peritus." His function is "bene dicere in civilibus quaestionibus." His aim is "Persuadere." The Genera civilium quaestionum are: "Demonstrativum, deliberativum, iudicale."

The remaining portions of the book are devoted to the technical exposition of the "partium orationis officium;" (1) "Invention;" (2) "Explanation;" (3) "Confirmation;" (4) "Excitation;" (5) "Persuasion." The second book is devoted to a detailed treatment of invention and is based on Cicero's *De Inventione* and the third book of Quintilian's *Institutes* (especially Chapters 5 and 6). The definitions are simple and the examples brief. The third book treats briefly in a traditional manner of "explanation," "confirmation," "excitation" and "persuasion," the examples again chosen being from Cicero and Quintilian. Text, Halm, *op. cit.* pp. 81-134.

point of the art.¹ That it met all requirements of the age can be seen from the fact that succeeding writers, notably Cassiodorus, largely based their work on it.

(6) Based on the works of Cicero and Cicero's own teacher Hermagoras of Rhodes, is Augustine's fragmentary work *De Rhetorica*. It need hardly be said that the fame of the author made the book very popular. Its adaptability as a text book may be judged from the fact that the author himself had been a teacher of rhetoric.²

(7) Similarly incomplete is the *Institutiones Oratoriæ* of Sulpitius Victor, of the end of the fourth century. The fragment extant indicates an elaborate and well arranged commentary on the elements of rhetoric. The author being a practical jurist, his work naturally emphasizes the aspect of the art most closely connected with the work of the pleader. It was printed at Basle in 1521.³

(8) The most elaborate technical treatise on the subject, based on Hermagoras, Cicero, Quintilian and others, is the *Ars Rhetorica* by C. Julius Victor, a contemporary of his above mentioned namesake. Under twenty-seven main headings the author exhaustively treats all the essentials of the "ars." Logical subdivision and relative subordination are the marked characteristics of the work. It is replete with illustrations from Cicero and Quintilian. His work is the first to treat of the *Epistola* as a component part of the "*Ars Rhetorica*." The "*Epistle*" was later emphasized to such an ex-

¹ It is argued that the minute treatment of the figures and the over classification of Fortunatianus's work is suggestive more of the spirit of the Greek rhetoricians than of Cicero and Quintilian, both of whom protest against such over refinement. Vide Saintsbury, *op. cit.* I, pp. 87-89, 107, 108, 346 et seq. Cf. Quintilian, *Institutes* Bks. VIII-IX, Watson's tr. II, p. 71 et seq. Also Cicero's brief classification of figures in his *Topica*, *Opera* ed. Orellii, I, 45-479.

² Text in Halm, 137, 151. Migne, XXXII, c. 1439 et seq. The current belief that the works on dialectic and rhetoric ascribed to Augustine are spurious is based on the statements of his Benedictine editors in the edition of 1679, T. I. app. 152. This opinion is accepted by Saintsbury, *op. cit.* I., 377. The view has been disproved by W. Crecilinus, *Jahresbericht über das Gymnasium zu Elberfeld*, 1857. Cf. August Reuter, "Augustine's Rhetoric" in *Kirchengeschichtliche Studien*, 324-351.

³ Text in Halm, 318-352.

tent that the study of this theme came to embrace the entire subject of rhetoric.¹

(9) Briefer in treatment but equally well arranged is the *Liber De Arte Rhetorica* in the Encyclopædia of Capella. The treatment is technical and clear except for the introductory portions. It is illustrated in parts by examples from Cicero's orations. It can however not claim any special distinction as a text book. Its use was due to the fact that it formed a part of a complete encyclopædia on the seven liberal arts.²

(10) A handy text book and one even more popular than Capella's work was the brief abstract from Fortunatianus by Cassiodorus. His *De Arte Rhetorica* forms a part of the *De Institutionis Liberalium Literarum*. Brief as it is, the work contains a number of diagrams to bring out the logical relations of the different terms. The six pages of Cassiodorus on rhetoric are probably all that the average mediæval student of rhetoric memorized.³

(11) Isidore of Seville is another author whose *De Rhetorica* is a part of an encyclopædic work on the seven liberal arts. It is little more than a brief treatment of the *partes orationis*, which in his own work have somehow shrunk to four: (1) exordium; (2) narration; (3) argumentation; (4) conclusion. He gives a considerable portion of his treatise to a discussion of "rhetorical syllogisms" and "law." The addition of the latter is as significant as are the remarks of Victor on the "Epistola." One third of his work is devoted to figures, which are briefly defined and often illustrated by examples from many standard classical authors.⁴

(12) *De Rhetorica et De Virtutibus* of Alcuin is a dialogue between him and Charlemagne, based almost entirely on Cicero's *De Inventione* and on Isidore's excerpt. Lacking the good qualities of both it is not wrongly characterized

¹ Text in Halm, 373-448.

² Text, Eyssenhardt's edition, Liber V. pp. 138-193; Halm, 451-492. Cf. Saintsbury, I. 349 et seq., who considers it "one of the best of the Latin rhetorics."

³ Text in Migne, LXX. c. 1157-1167; Halm, 495-504.

⁴ Text, Halm, 507-522.

as devoid of originality, ill-digested, dull and puerile.¹ Nevertheless it is of considerable importance from our point of view. It is one of the works which mark the transition from the ancient technical treatment of rhetoric, as we have seen it expressed in the preceding treatises, to the later mediæval treatment of the subject, typified in the text books of "Dictamen".²

The later treatises on rhetoric such as those of Notker, Remigius of Auxerre, of Boncompagno, and of Bernard of Chartres follow closely the models which have been described.³ Besides these texts in which rhetoric is treated as a component part of the seven liberal arts, there exist remains of fragmentary

¹ Cf. West's *Alcuin*, p. 104; Saintsbury, I, 375, 376.

² This can be seen from the following fact. The work, as its title indicates, links two subjects, rhetoric and virtue. Written especially for the King, its object is not oratorical but judicial, as can be seen from the following introductory verses:

Qui rogo civiles cupiat cognoscere mores
Haec praecepta legat quae liber iste tenet.

The emphasis is thus changed. In Cicero the object of rhetorical study is to train the orator, the pleader in civil causes, who of course should be virtuous. In Alcuin the purpose of the treatise is to teach the king rhetoric so that he may be able to judge and decide. Virtue for Alcuin too, is necessary for the trained orator but for a different purpose. Hence he included a discussion of the four cardinal virtues—prudence, justice, fortitude and temperance,—in what is avowedly a text book on rhetoric. The genera causarum of the old rhetoricians, demonstrativum, deliberativum, judicabile, become honestum, admirabile, humile, etc. The change is rather curious, but easily explained, when the purpose of the work is considered. Text, Halm, pp. 520–550; Migne, vol. CI, col. 101 et seq.

³ For text of Notker's *Rhetoric*, vide H. Hattemer, *Denkmäler des Mittelalters*, III, 560–585. On the *Rhetoric* of Remigius of Auxerre, vide, *Hist. Lit. de la France*, VI, p. 119. The text of Bernard of Chartres is lost. Cf. Barach, *op. cit.* XIII. The text of Boncompagno is found in the library of Munich, *Cod. Lat. ZZ.*, 499 fols. 1–58; cf. *Sitzungsberichte der Königl. Bayerisch. Akad. zu München*, 1861 p. 134, et seq. An *Antiqua Rhetorica* of Boncompagno, who was a teacher of rhetoric at Bologna, was read before the professors and students of canon and civil law on April 26, 1215, and the author was crowned for the work. It is strictly a rhetorical work. His other work, the *Novissima Rhetorica*, of some thirteen parts, has much legal matter in addition to the traditional material of technical rhetoric, showing how in his day rhetoric and the study of law were being mixed. His other works on Dictamen will be spoken of later.

treatments of portions of the rhetorical art.¹ All these, however, may be considered in the nature of commentaries rather than text books.

The reading of the text books on rhetoric was ordinarily all that was attempted in the technical study of this subject. Sometimes, however, certain schools, either because of the special interest of the teacher or because of the accessibility of books, or because of certain traditions, devoted some time to the reading of Cicero's works on rhetoric, particularly his *De Inventione*, *De Oratore*, *Topica*, and the pseudo Ciceronian *Ad Herennium*.² In some instances the works of historians and classical and Christian prose writers, notably the *Consolation of Philosophy* of Boethius, were also read as a part of the study of rhetoric. Such a practice, however, cannot be said to have existed as a general rule.³

In general it may be asserted that wherever conditions were favorable, Cicero's and Quintilian's works were read after the technical study of rhetoric, as excellent illustrations of Latin rhetorical style.⁴

¹ E. g. *De Ethopoeia* of Emporius, Halm, 561-574; *De Statibus* of Clodianus, *ibid.*, 590-592.

² The catalogues of libraries of the eighth and ninth centuries often show the existence of a copy of the *Topica* and *De Inventione Rhetorica*. Of frequent occurrence is the *Ad Herennium*. Cf. Mullinger, *Schools of Charles the Great etc.* p. 84.

³ "Huic urbi (sc. Rhetorici) subjacent historiae fabulae libri oratorie et ethice conscripti." *Honorii Augustod. Libell. de animae exsilio*, c. 3, Pez, *Thesaurus Anecdotorum*, II, 1, p. 229, quoted in Specht, *op. cit.* p. 116.

⁴ Johannes de Garlandia, in the thirteenth century, for instance, in enumerating the authors which a student should read, in his study of rhetoric, mentions Cicero's *De Inventione*, *De Oratore* and the *Ad Herennium*; also Quintilian's *Declamations* and *Institutes*. Cf. Leyser, *op. cit.* p. 339; Sandys, p. 525. That Cicero's rhetorical works, particularly the *De Inventione*, were considerably used is amply proven by the elaborate commentaries on them, of which that of Q. Fabius Laurentius is an example. (Vide text, Halm, pp. 155-310). For other manuscripts of commentaries, some even dating back to the seventh century, cf. Halm, *op. cit.* p. 593 et seq. Quintilian was probably used only in excerpts. Only in this way can the existence of many such selections be explained. These generally consisted of passages taken seriatim from the *Institutes*. They are often found as appendices to texts on rhetoric. For an example of one vide Halm, *op. cit.*, pp. 501-504.

B. THE STUDY OF THE DICTAMEN.

GENERAL CHARACTER.

While the subject of the theoretical and logical form of rhetoric received comparatively little consideration in the study of the seven liberal arts, two practical forms of rhetoric were assiduously cultivated. These were the epistle and the document, lay or ecclesiastic.

We have seen how, even in the early rhetorical text books, the *Epistola* was considered one form of rhetorical expression.¹ In the course of time the demands of the age forced the writers to lay more stress on the mastery of the practical side of rhetoric than on the theoretical conceptions of invention, excitation, etc. In an age when ability to read and write was the accomplishment of the few, it was surely a great achievement to be the master of a fair prose style. The ability to write a letter to a lay or ecclesiastical official, to make a contract, a will, or to reduce to writing any other formal act of every day life, of which a record had to be kept, was surely of greater importance to laymen and priests alike than the skill to prepare a well balanced oration or a literary composition.

Hence in the very beginning of the middle ages, letter writing and the preparation of ordinary documents began to occupy the attention of the student of rhetoric in his course of the seven liberal arts. Indeed these studies came gradually to comprise substantially the whole of the rhetoric of the middle ages. Charlemagne's laws on the education of the clergy ordered that they should be able to write "*cartas et epistolas*."² It can be asserted with a fair degree of accuracy that from the Carolingian period the study of rhetoric actually became the study of prose composition. The art was called "*Ars Dictandi*", "*Ars Prosandi*", or "*Dictamen*;" the teacher was the "*Dictator*" and "*dictare*" meant the ability to write private and official prose.

Now such a study naturally presupposes the use of models. But the ancient literature could not afford illustrative ma-

¹ Cf. *The Ars Rhetorica* of C. Julius Victor; Halm, p. 447 et seq. Cf. *supra*, p. 56.

² "*Capitula de Doctrina Clericorum*," *Mon. Germ. Hist. Leges*, I, pp. 107-108.

terial for this new subject. Cicero and Quintilian could only serve as models for oratory, but not for prose composition. Therefore in teaching this mediæval form of rhetoric the schoolmasters, as in the teaching of grammar, were largely thrown on their own resources, and were compelled to look for new models.

The amount of material which the teacher of the middle ages gathered for this form of instruction is vast indeed. Incidentally, besides showing the manner and method of instruction, these collections give us a comprehensive view of mediæval life and institutions which very few other sources afford. We find there glimpses of the growth of national law, the gradual introduction of the Roman law, the all-embracing relations in which the church came in contact with the affairs of life. In short many important phases of the life of the people are revealed to us through these modest school books. The material in these so-called dictamen treatises has attracted the attention of students of the history of Roman law. Following Savigny, who traced the origin of the study of the Roman law in Western Europe to the study of the dictamen in Italy as a part of rhetoric, investigators of this subject have been carefully sifting these formularies. Many a document here found serves to cast light upon the *Kulturgeschichte* of Europe.¹

¹ Vide Savigny, *Geschichte des Römischen Rechtes*, III, c. XXI, p. 152 et seq. The intimate relation of the study of rhetoric with the study of the elements of law has been established by the original investigations of Specht, who gathered data going as far back as the age of Gregory of Tours; vide Specht, *op. cit.*, 120, 121. The subject of Dictamen has been treated by the following special investigators: Savigny, *Geschichte des Röm. Rechts. im Mittelalter* Cap. 21; Eugène de Rozière, *Recueil Général des Formules usitées dans l'Empire des Francs du Vme au Xme siècle*, 3 vols. 1859-1871; L. Röckinger, *Ueber Briefsteller, und Formelbücher in Deutschland während des M. A.* Many *formularii* exist in the various libraries of Europe. These are often described in the periodicals devoted to mediæval history. Thus many important texts have been edited and published because of the historical value of the illustrative material. Many such codices have been published in the *Archiv für Kunde Oesterreichischen Geschichtsquellen*; in the *Neues Archiv für Ältere Deutsche Geschichtskunde* and in the *Notices et Extraits des Manuscrits de la Bibliothèque Nationale*. Cf. e. g. Wattenbach, "Ueber Briefsteller des Mittelalters" introduction and texts *Archiv etc.* Bd. XIV, pp. 29-67; Otto Stobbe, "Ein Formelbuch aus der Zeit König Rudolfs I und Albrecht's I," *ibid.* pp. 305-378; Johannes Voigt, "Das Urkundliche Formelbuch des Könighchen Notars aus der Zeit der Könige Ottokar II und Wenzel II von Böhmen," *Archiv etc.*, Bd. XXIX, pp. 1-185; Karl Zeumer, "Über die Älteren frankischen Formelsamm-

It is from this standpoint of *Kulturgeschichte* that these documents which have yielded so rich a harvest to the student of jurisprudence will be examined anew in this chapter. The evidence sifted with that object in view leads to four important generalizations. As space does not permit an elaborate examination and analysis of the sources on which the conclusions are based the succeeding paragraphs will contain only the typical data supporting the theses.

(1) In the study of rhetoric the preparation of formal documents was introduced in the early middle ages. This "ars dictaminis," which was assiduously cultivated in northern Italy, gradually developed into a special study of the preparation of documents, and eventually into the study of law, which soon became an independent subject of study. The history of the rise of the University of Bologna and the history of literary conditions in Italy in the middle ages fully support this thesis.¹

lungen," *Neues Archiv etc.* Bd. VI. pp. 1-115; E. Dümmler, "Zu den Carolingischen Formelsammlungen," *Neues Archiv etc.*, Bd. VII, pp. 401-403; Karl Zeumer, "Ueber Fragmente eines Formelhandschrift des IX Jahrhunderts" *Neues Archiv etc.*, VIII, pp. 601 et seq.; Id., "Neue Erörterungen über ältere frankische Formelsammlungen" *Neues Archiv etc.*, XI, pp. 311-358. Various collections of *formulae* are described in *Forschungen zur Deutschen Geschichte*, Bde. VIII, pp. 327-366; X pp. 642-653; XV pp. 213-238; others are described by Ch. V. Langlois, "Formulaire de lettres du xii au xiii et du xiv siècles," constituting a series of articles in *Notices et extraits etc.*, Tom. XXXIV, Pt. I., pp. 1-32; Pt. 2, pp. 1-29; Tom. XXXV, pp. 409-434. Mss. of epistolary formulae are described by M. Leopold Deslisle, *Notices et Extraits etc.* XXVII, Pt. I. p. 87 et seq. The most complete of all the general collections of dictamina is that of Ludwig Röckinger in *Quellen zur Bayerischen und Deutschen Geschichte*, Vol. VII, pp. 1-312, Vol. IX, Parts 1 and 2. In all, his collection contains about twelve hundred separate pieces of formulae and letters covering from the Carolingian period to the fourteenth century. De Rozière's collection, in spite of the fact that it contains the largest single collection of epistles (190, of which Röckinger has included only 78), is in the opinion of the writer of more value to the student of the history of law than to the investigator of general social conditions. The formulae, however, are well classified. A third of the work, containing a series of excellent tables, offers to the student a valuable concordance of the whole subject. For some bibliographical notes on this topic vide Langlois, *op. cit.*, *Notices et Extraits etc.*, XXXIV, Pt. I, p. 7, notes 1 and 2.

¹ Vide Rashdall, *Universities in the Middle Ages*, Vol. 1, Ch. iv, pp. 90-127; Oznam, *Documents inédits pour servir à l'histoire littéraire de l'Italie*, Paris, 1850; and more especially "Ars dictandi in Italien," *Sitzungsbericht der Königl. Bayer. Akad. der Wiss. zu München*, 1861, pp. 98-151.

(2) The need which the secular official world and the ecclesiastical chancelleries had for a knowledge of the art of drawing up documents, made the study of the "ars dictaminis" inevitable in the schools of the middle ages. The manifold relations of church and state, the complex inter-relations of the feudal society and the prelates required standards and models of prose composition. This need the mediæval teacher of dictamen was called upon to supply. In these formal model letters and documents the affairs of state and church were discussed, sometimes by quoting actual letters written by famous personages of the time, sometimes by writing imaginary replies composed by teachers or able students. A few instances may be cited in illustration. The *Breviarum de Dictamine* of Alberich of Monte-Cassino contains the original document of the controversy of Henry IV and Gregory VII.¹ Of the same character though belonging to an earlier age—the end of the ninth century—is the so-called *Formularius* of Bishop Salomon III of Constanze.² The *Summa Prosarum Dictaminis* of Saxony contains actual documents from the chancellery of the Archbishop of Magdeburg.³ The *Summula Dictaminis* of the Cistercian monk Bernold of Kaiserheim, similarly owes its material to the archives of Kaiserheim. Lastly the models in the *dictamen* of Arnold of Protzan are taken from the archives of the Bishopric of Breslau.⁴

¹ For a brief description of this "breviarum," vide Röckinger, "Briefsteller und Formelbücher des elften bis vierzehnten Jahrhunderts" in *Quellen zur Bayerischen und Deutschen Geschichte* Bd. VII, p. 5 et seq.

² Text in Röckinger, *Quellen etc.*, Vol. VII, pp. 189–256.

³ Text and Introduction, Röckinger, *op. cit.*, IX, pp. 201–346.

⁴ Text of Bernold's *summula dictaminis*, Röckinger, *op. cit.* IX, 2, pp. 849–924. The text of Arnold of Protzan's *Formularius* is printed in Wattenbach, *Codex Diplomaticus Silesiae*, Vol. V, Breslau, 1862. Cf. Röckinger, *Quellen etc.*, Vol. VII, p. xxxviii et seq. In the earlier centuries of the middle ages the *Variae* of Cassiodorus were used for such purposes. Later on Alcuin's *Epistolae* were also very popular. The collection of the Dictamina as based on the "Epistolarum libri sex de rebus gestis Frederici" by Petrus de Vineia, and the collection of the *Summa Dictaminis* of Thomas of Capua, representing the two sides of the controversy between the Emperor and the Pope could not, in the judgment of the writer, have been used as text books as Meier (*Sieben Freien Künste etc.* p. 26) asserts. Their controversial character would have barred them from the school room. I have been unable to find the Dictamen of Petrus in the *Collectio Historica* of J. Georg. Habnisch.

When such contemporaneous material was not at hand, famous documents of former days, often with appropriate imaginary replies by schoolboys, were incorporated in the collection. Of such interpolated texts we have many; as a notable example, the *Formularius* of Baumgartenburg, written in 1302, may be mentioned. This collection takes more than half of its illustrative material from a previous work of Ludolf of Hildesheim. Of his indebtedness due acknowledgment is made by the author in the following terms: "Multas elegantis styli præsenti operi inserui epistolas quae de magnorum dictatorum formulariis excerptas."¹

(3) The every day affairs of life requiring official records—wills, commendations, contracts, immunities, manumissions of slaves, etc.—called for a knowledge of the preparation of documents of a less formal character; this the schools of the seven liberal arts supplied in the teaching of rhetoric.

By far the largest portion of the material examined does not relate to *haute politique*, temporal or spiritual, but to every day exigencies of private life at their point of contact with public affairs. All the collections of the dictamina have models of *privilegia*, *commissiones*, *citationes*, *procuraciones*, *donatoriae*, *petitiones*, *sententia*, *confirmationes*, *appelationes*, *executoriae*, *dispensationes*, *indulgentiae*, *exemptiones*, *visitationes*, *inquisitiones*, *formatae*, *obligatoriae*, *testimoniales*, *testamenta*, *emancipationes*, *permutationes*, *adaptiones*, etc.² If the larger portion of these documents relate to matters ecclesiastical, it is only because the church touched the average man of the middle

There are extracts of the *Summa* of Thomas of Capua in the above named collection.

¹ For text of Ludolf of Hildesheim's *Summa Dictaminis* see Röckinger *Quellen etc.* Vol. IX, pp. 347-402. For the Baumgartenburger *Formularius*, vide *ibid.* Pt. 2, 713-838. On the introduction of school exercises in the collections, vide, Röckinger, *op. cit.* Vol. VII, p. XLI, where many instances in the texts given are referred to. Jaffé und Wattenbach in the *Codex Diplomaticus Silesiae* V, pp. 60-65, have proven that a number of important Lombard letters of the reign of Lothar and some bearing on the controversy between Frederick I and Pope Hadrian are mere dictamen exercises of schoolboys. Cf. Röckinger *op. cit.* VII, p. LII. The following question naturally suggests itself: How many spurious documents, later branded as pious frauds, were originally the innocent work of the imaginative schoolboy?

² Vide Röckinger, *Quellen etc.* IX, passim. especially 214-259, 385-398, 749-758.

ages at so many points. The part played by the mediæval schoolroom in the transmission of more or less technical legal knowledge through instruction and written practice is apparent.

More direct proof that these collections of dictamina were not, as might be supposed, mere reference books of the chancelleries of prelates and princes but were also used as books of instruction is found in the fact that all of them have prefixed to the models brief treatises on the "ars prosandi" or "ars dictaminis" or "summa dictaminis" or "summa prosarum dictaminis" or "summa dictaminum," as these are variously called. This didactic introductory portion is as characteristic a part of the collections dealing with great affairs of church and state as of the more modest collections treating of subjects of humbler life.¹

(4) The schools of the middle ages evolved a special form of composition, the *Epistola*, and much of the instruction was in this art of letter writing, which of course developed with all the minute distinctions of which the mediæval mind was capable.

That the instruction in letter writing was made an important part in the study of rhetoric can be seen from the fact that all of these collections in their didactic portions lay great stress on teaching the "partes epistolae." "salutatio;" "exordium" or "captatio benevolentiae;" "narratio;" "petitio" and "conclusio," are carefully explained, defined and illustrated. The relation of the "epistola" to the entire "ars dictaminis" is duly emphasized.² Much attention is given to the "salutatio" in the treatment of which examples of forms of address suitable to all conceivable grades of ecclesiastical and lay officials are given. Thus in the brief *Summa Dictaminis* of Orleans, composed about 1180, in which the didactic portion of the book occupies seven printed pages, the treatment of the "salutatio" takes up more than four and a half pages.³ In the elaborate Baumgartenburger *Formularius*, composed in 1302, the didactic portion, "de modo prosandi," devotes nearly the whole of the first of the six parts of the work to the treatment of the salutatio.⁴

¹ E. g. the *Sächsische Summa Prosarum Dictaminis* and that of Dominicus; Röckinger, *Quellen etc.* IX, pp. 209-346, and 525-592.

² Two other parts of the *Epistola*, the "superscriptio and the subscriptio," were rarely treated; Cf. Röckinger, *Quellen etc.* Vol. IX, p. X note.

³ Text in Röckinger, *Quellen etc.* IX, p. 103 et seq.

⁴ *Ibid.* pp. 725-748.

No less significant are the variations in the examples which many *dictamina* furnish. A form is given with two or three others as alternatives showing again that the models which we have before us were used for purposes of instruction, since it is not likely that the routine worker of a chancellery would care to vary his formulae. Thus in the models for a "salutatio ad patrem," Hugo of Bologna gives in his *Rationes Dictandi* the following variations: (1) venerabile et delecto patri; (2) reverendo ac diligendo patri; (3) a genitori dulcissimo eius dilectus filius perennem cum fidelitati seruitium; (4) quiquid patri peramens filius; (5) quiquid domino subditus servulus.¹ Still more significant is the form of salutatio given by the same author in the model "ad amicum" in which no less than sixteen variations are given.² The numerous glosses which many of these manuscripts exhibit show again that they were used as text books.

TEXT BOOKS.

The foregoing portions of this chapter have established the important position which the study of the "ars dictandi," a modified form of rhetoric, occupied in the mediæval curriculum of the seven liberal arts. It remains to name the leading text books in the subject. Those whose content was entirely legal, in the narrow sense, will, however, be omitted.

(1) While the "ars dictandi" was studied in Italy as early as the sixth century, the earliest collection of formulæ, of the use of which as a school book there can be no doubt, is the so-called *Salsburg Formularius*, written not later than 821. Its arrangement, content, incidental references and glossaries prove conclusively that it was designed as a school text of model letters, briefs and similar documents. Such a deduction is inevitable in spite of the fact that the volume does not contain the didactic introductions on the "ars dictandi." As the collection is the earliest of its kind, a brief description of it will be of interest.³

¹ *Rationes Dictandi Hugutionis Bononiensis*; Röckinger, *Quellen etc.* Vol. IX, p. 64.

² *Ibid.* pp. 64, 65.

³ *Drei Formelsammlungen aus der Zeit der Karolinger*. This latest critically edited text is in *Quellen und Forschungen etc.* pp. 47-168; for the evidence of its use as a school book vide, *ibid.*, "Einleitung" pp. 24 and 43 et seq.

The book consists of one hundred and twenty-six separate compositions occupying together about seventy pages of actual text. It contains about twenty-five formulæ relating to lay transactions of every day life, such as wills, sales, etc. About ten are of a strictly legal character; thirty others are on official relations between king and king, king and vassal, archbishop and king, and the like; only about ten are of a distinctly ecclesiastical character. The collection contains eight authentic letters of Alcuin to his pupil, Archbishop Arno. The rest of the documents form a miscellaneous collection of minor importance which cannot be classified.

(2) Similar in character in all particulars is the collection of *Formulæ Salomonis III*, composed about the year 900. It contains forty-eight documents, occupying some fifty pages of actual text; its distinctive characteristic is the fact that one-half of the collection consists of contemporaneous material written between 864 and 884. In content, it is of much greater importance than the previously mentioned collection, consisting as it does of documents on the relations between high ecclesiastics and kings.¹

(3) The earliest collection of "Epistolæ" proper for school use are the *Epistolæ Alati*, composed in the second half of the ninth century. This contains seven letters. The name of the author and the place of composition cannot be ascertained, but of its use as a text book in a monastic school the contents of the work give adequate proof. The letters are mainly an imaginary correspondence between a teacher and a pupil.²

(4) In the *Rationes Dictandi* of Alberic of Monte Cassino, we have the first text book in which the theory of letter composition is laid down. Alberic, a teacher of the eleventh century, was the author of two other more extended works on dictamen, the *Flores Rhetorici* sive *Dictaminum Rarii* and the *Breviarum de Dictamine* both of which are unpublished.³

¹ Text, Röckinger, *op. cit.* Vol. VII, pp. 189-256.

² Text, *ibid.* pp. 171-185; Cf. *ibid.*, "Einleitung" pp. 21-29.

³ As his *Rationes Dictandi* may be considered epoch-making in the teaching of rhetoric in the middle ages, an extended analysis of the work will be given here. The volume consists of some twenty octavo pages of actual text; it is divided into thirteen unequal parts. After a brief introduction the author proceeds to define the Dictamen and its parts leading up to the Epistola which is also defined as a form of the Oratio. The five parts of

(5) *Rationes Dictandi Prosaici*, was composed between 1119 and 1124 by Hugo of Bologna. A number of complete manuscripts of this text book are in existence today. The work is in every respect based on that of Alberic.¹

(6) The *Summa Dictaminis* of Orleans composed in the last decade of the twelfth century, is an anonymous composition. It is the briefest text book on dictamen extant. It follows Alberic in every detail and adds four chapters on as many different kinds of privilegia. Its brevity is remarkable when we consider the fact that Orleans at this time was the best known centre of classical literary culture. The cursory character of the text seems to indicate that at Orleans more attention was paid to the classical idea of rhetoric, i. e., reading of the ancient rhetoricians and historians, than to the mediæval study of dictamen and epistles.²

(7) *Summa Dictandi* of Guido Faba, a teacher of Bologna, composed about 1225, is a theoretical treatment of the "ars" with special reference to the *Epistola*. His *Dictamina Rhetorica* is a practical companion volume with excerpts from many writers.³

the *Epistola* are then named. The "Salutatio" as the first and most important part is explained and illustrated at great length. Indeed more than one third of the entire work is devoted to the consideration of the nature of the address; all possible forms are given by illustration, as forms of salutation by king to pope, subordinate to prelate, pope to the world, the emperor to the world, bishops to their charges, noble to noble, friend to friend. From the treatment of the salutatio the author proceeds to the consideration of the "benevolentiae captatio;" then the "narratio;" the "petitio" and the "conclusio" are treated very briefly. Having discussed the five formal parts of the *Epistola*, the author proceeds to the consideration of the question as to the possible diminution of these parts when occasion requires, and reaches the obvious conclusion that the "salutatio" and "narratio" cannot be omitted. From the consideration of the possible omissions he comes to the question of interchanging the different parts of the letters and gives various arrangements of interchangeable combinations. After some minor considerations of a rhetorical character on the "issue of letters," he takes up the discussion of the varieties of letters, treating of the personal, impersonal and other forms, concluding with some observations on the use of connectives for rhetorical effect. This brief treatment of the practical side of mediæval rhetoric is typical of all the dictamina which I have examined. Text, Röckinger, *op. cit.* vol. XI, Pt. I, pp. 9-28.

¹ Text, Röckinger, "*Quellen etc.*", Vol. IX, pt. 1, pp. 53-94.

² Text, Röckinger, *op. cit.*, Vol. IX, pp. 103-114.

³ The work has been printed and many manuscripts are still in existence.

(8) While the last four text books on dictamen owe their origin to Italy, where, as we have seen, the "ars dictandi" was fully developed in the eleventh century, we find even more elaborate collections compiled on German soil. The first and fundamental text book of this kind is the *Summa Prosarum Dictaminis* which goes under the name of the "Sächsisches Formelwerk." The author is unknown. We are certain though, that the date of the composition is not later than 1230. The models are based on the illustrative material used by a teacher named Gernard, who in 1222 became bishop of Brandenburg. It is a thorough treatise on the "ars" with special illustrations of high official "privilegia" or "litteræ missiles." Its contents begin with an exhaustive treatment of no less than twenty kinds of privilegia. Following this is a collection of one hundred and nine authentic documents to illustrate the principles enunciated. Its use as a text book is attested by the number of manuscripts still in existence, by the marginal notes found on them, and by the fact that at least two other famous German collections are based on this work.¹

(9) The *Summa Dictaminum* of Ludolf of Hildesheim, is of special interest. It contains, as is evident from even a cursory examination of the text, a traditional brief treatment of the *Epistola* prefixed to an extended discourse on "litteræ missiles," and almost the entire set of illustrative letters of the Saxon collection described above. Here again the content of the volume shows unmistakable evidence that it was designed as a text book. This is moreover strengthened by the existence of commentaries on the text.²

(10) *Summa de Arte Prosandi* by Conrad of Mure, a "rector puerorum" at Zurich, is but one of his many other school treatises. It is avowedly a compilation arranged in 1275. The characteristic of this treatise is its thoroughgoing dialectical character. Distinction upon distinction is drawn,

Munich alone has seven mss. of each one of his texts. Cf. Röckinger, *op. cit.* Vol. VII, p. 178.

¹ Text, Röckinger, *Quellen etc.*, IX, pp. 209-346.

² Text, Röckinger, *Quellen etc.*, Vol. IX, pp. 359-398. For the comparison of the illustrative letters of the two works vide, *ibid.*, p. 206, 207. For text of commentaries on Ludolf by Master Simo vide *ibid.* Pt. II, pp. 973-984.

divisions and subdivisions are made, until all possible cases are exhausted. Thus the author treats under different headings "Who sends the letter," "To whom," "For whom," "How," "When," etc. Endless lists of all possible persons who could be addressed, including even the ancient imperial Roman officials, are given. Of these he mentions the titles of ninety. Likewise one hundred and thirty-eight secular and fifty ecclesiastical titles are named and recombined under different headings. Yet there is hardly an illustration in the entire work. Conrad's book may be said to typify the scholastic character which at this time pervaded even such an avowedly practical study as the "ars dictandi."¹

(11) *Poetria de Arte Prosaica Metrica et Rithmica* by Johannes de Garlandia, the Englishman, was written about 1270. The portion, "De Arte Prosaica" contains the material of the instruction in rhetoric and dictamen, dialectically treated in fantastic fashion.²

(12) The so-called Baumgartenburg *Formularius de Modo Prosandi*, is a compilation dating as late as 1302. It anticipates the form which these treatises came to have in the latter years of the fourteenth century. Its size, elaborate and exhaustive, though not entirely dialectical treatment, the wealth of its illustrative material (much of it borrowed from the Saxon collection) stamps the work as more of a book of reference in the schools than as a manual which teacher and pupil used in the study of the dictamen. Not the least interesting portion of this book is the brief summary of the legal principles added as a "*sexta pars*" to the work.³

(13) The *Summula Dictaminis* collected by Bernold of Kaiserheim in 1312 may similarly be classed as a teacher's reference book. Omitting the elementary treatise on dictamen, the author devotes a brief chapter to the salutatio of which copious examples are given. His second book has classified collections of hackneyed "proverbia and sententia" for insertion in the letter. These are culled, as the author tells us, from the Liber Decretalis, Seneca, Solomon, and

¹ Introduction and text, Röckinger, *Quellen etc.* IX, pp. 405-482.

² Text edited by G. Mari, in *Romanische Forschungen*, Bd. XIII, 1902, pp. 183 et seq.

³ Text, Röckinger, *Quellen etc.* Vol. IX, Pt. 2, pp. 725-838.

various other sources. The other portion of the work is devoted to illustrative models of different kinds.¹

With the spread of the universities in the fourteenth century, and the students flocking to them to take up the study of logic, the attention to dictamen naturally decreased. The technical part became stereotyped into "tabulæ."² The other portions were taken up in connection with the study of law. Very few universities offered lectures on the dictamen.³

¹ *Ibid.* pp. 845-948.

² For examples of such "Tabulae," vide the *Practica usus Dictaminis* of Johannes Bondi of Aquilaia where schematic arrangements of tables of different kinds are given. Röckinger, *op. cit.* Vol. IX, Pt. 2, pp. 956-966.

³ At some universities lectures on a *libellus de arte epistolandi* were prescribed. Cf. Prantl, *Geschichte der Univ. Ingolstadt*, II, p. 76, as quoted in Rashdall, *op. cit.* I, p. 442, note 1.

CHAPTER VI.

Logic.

A. PRE-UNIVERSITY PERIOD.

SCOPE AND TEXT BOOKS.

Logic in a modern college curriculum definitely connotes the mastery of a propaedeutic to philosophy. Examining a modern text book on logic, we find therein some theory of knowledge, a few remarks on the difficulties concerning the relation of general names to thought and to reality, a few definitions of terms and propositions, some "laws of thought," and the formal treatment of the syllogism. To this are added some chapters on the modern scientific method of induction. Such a volume is truly an introduction to the study of philosophy.

From this it would appear a simple matter to determine the scope of logic in the mediæval curriculum.¹ But such is not the case. In the first place, the mediæval schoolmen themselves, in spite of their discriminating distinctions, often confused logic with philosophy. While admitting that logic was only a foundation stone on which mighty edifices of thought were built, almost in the same breath they glorified the subject as the "ars artium," "scientia rationalis," "disciplina disciplinarum," "scientia scientiarum."² Secondly, scholasticism in its manifold treatment of the problems of philosophy was so logical in its nature as well as its method that even modern scholars have found it all but impossible to differentiate between the logical philosophy and the purely didactic formal logic.³

¹ In connection with this inquiry it may be necessary to state that logic and dialectic were synonymous throughout the middle ages. We find this idea in Isidore of Seville vide D. E. Rémusat, *Abelard*, I, p. 300 et seq., where the point is established.

² Albertus Magnus inveighs against this confusion. Vide "De Praedicabili," Liber I. Tract. I, Chap. II c. 104 *Opera*.

³ "It is a difficult task," says Blakey, "to keep always in view the radical distinction between the science (logical philosophy) and the purely dialectic forms or systems which are ever obtruded on our notice in the abstract speculations of the middle ages." (R. Blakey, *Historical Sketch of Logic*, p. 121 et seq.)

From the standpoint of the historian of philosophy this confusion is not serious. But for the purposes of the present inquiry it seems essential to clear up this misconception. An appeal to the text books for a solution of this vexed problem produces abundant proof to support the following theses:

(1) *That the metaphysical problems were not a part of the study of logic proper in the curriculum of the seven liberal arts during the entire period of the middle ages.*¹

(2) *That the only point of contact between philosophy and the logical teaching of the middle ages lay in the fact that the question of the term "substance"—the first of Aristotle's categories—has a mixed character, partly ontological and partly logical.*²

(3) *That logic as taught in the curriculum of the seven liberal arts was nothing more than what we would call today the formal, technical study of logic.*

In the succeeding paragraphs these propositions will be supported in two ways: First, a direct analysis of the text books will be made to show that these actually contain only material for the formal teaching of logic; secondly, by an analysis of other logico-philosophical treatises, it will be shown indirectly that those logical questions which touched the field of philosophic speculation were invariably treated by these writers in their metaphysical works, and not in the logical text books.

Our point of departure will be an inventory of the exact

¹ The whole round of philosophical problems with which the scholastics busied themselves cannot be better summarized than in the historic words of Porphyry: "Mox de generibus et speciebus illud quidem sive subsistant sive 'in solis nudis intellectibus posita sint, sive subsistentia corporalia sint an 'incorporalia, et utrum separata a sensibilibus an insensibilibus posita et 'circa haec consistentia dicere recusabo; altissimum enim negotium est 'hujus modi, et majoris egens inquisitionis.'" (Text, Boethius, *Commentarius in Porphyrium a se translatus*, Migne, Vol. LXIV, col. 82.) We have here the scholastical problem in its threefold aspects: (1) The ontological problem; (2) The problem of realism versus idealism; (3) The epistemological problem. These in one form or another were of sufficient importance to occupy the attention of mediæval thinkers for some six hundred years. Cf (Hauréau, *De la Philosophie Scolastique*, I, pp. 30-46; Grote, *Aristotle*, II, App. I.)

² On the subject of the limits of logic in its relation to philosophy vide Baldwin, *Dictionary of Psychology and Philosophy*; Braniss's *Die Logik in ihrem Verhältniss zur Philosophie geschichtlich betrachtet*, pp. 13-16, 44-46; Grote, *Aristotle*, I, pp. 84-96.

quantum of knowledge of logic which was possessed during the period. The exact limits of this have been definitely established by the exhaustive researches of Jourdain,¹ Cousin,² De Rémusat,³ Hauréau,⁴ and Prantl.⁵

According to these investigators only the following logical treatises, from which others were derived, were known and used in Western Europe until the end of the twelfth century.

- I. Capella's section on Logic in his *Encyclopædia*.
- II. Augustine's, *Principia Dialecticæ* and the pseudo Augustinian *Categoriæ decem ex Aristotele decreptæ*.
- III. Cassiodorus's section *De Dialectica*.
- IV. The numerous logical works of Boethius, classified as follows:
 - A. Translations,
 1. Porphyry's *Isagogue*.
 2. Aristotle's *Categories*.
 3. Aristotle's *De Interpretatione*.
 - B. Commentaries,
 1. On Victorinus's translation of Porphyry's *Isagogue*.
 2. On his own translation of the *Isagogue*.
 3. On Aristotle's *Categories*.
 4. On Aristotle's *De Interpretatione* (2 editions).
 5. On Cicero's *Topica*.
 - C. Original works.
 1. An introduction to the categorical syllogism.
 2. On the categorical syllogism.
 3. On the hypothetical syllogism.
 4. *De Divisione*.
 5. *De Definitione*.

¹ Jourdain, *Recherches Critiques sur l'Age et l'Origine des Traductions Latines d'Aristote*, Paris, 1845.

² Victor Cousin, "Introduction aux Ouvrages inédits d'Abélard" in *Collection des Documents Inédits sur l'Histoire de France*, Vol. XXIV, 1836.

³ De Rémusat, *Abélard*, 2 Vols. 1845.

⁴ Hauréau, *De la Philosophie Scolastique*, 1850.

⁵ Prantl, *Geschichte der Logik im Abendlande*, Vol. 2, 1861.

6. *De Differentiis Topicis.*V. Isidore of Seville's section *De Arte Dialectica* in his *Etymologiæ*.

Jourdain and the other investigators have also proved conclusively that Aristotle's own *Prior and Posterior Analytics*, the *Topics* and the *Sophistici Elenchi* though translated and commented upon by Boethius in the fifth century were absolutely unknown before the end of the twelfth.

The material enumerated can be reclassified for our purpose into two groups: (1) A group consisting of those works of Boethius in which the subject of logic is treated with due regard to its philosophical relations, and (2) the group containing the purely formal matter of logical instruction, comprising the works of Boethius, Capella, Augustine, Cassiodorus, and Isidore of Seville.

Turning first to the words of Boethius, it appears that while the philosophical problems of logic were of more importance to him, he nevertheless clearly distinguished between what he called formal practical philosophy,—i. e., logic,—and theoretical philosophy,—i. e., physics and metaphysics.¹ Formal logic, which he defines as that philosophy whose object is to discern true from false reasoning, he really considers an aid to theoretical philosophy. His original works on definition, division and the syllogism, are thorough from the didactic standpoint, though they may appear schoolboyish to one who is looking for a theory of logic as Prantl does. Yet even this hostile critic admits that his treatment of the hypothetical

¹ "Est enim philosophia genus, species vero ejus duae; una quae theoretice dicitur, altera quae practice id est speculativa et activa." *Dialogus in Porphyrium Dialogia Victorino translatus*, Migne LXIII, col. 11. It is evident that his writings on logic were not for mere formal logical instruction. Its very bulk, some nine hundred columns of original matter, would have made that impossible. Boethius's plan aimed at more than the mere writing of a text book on logic. It was to hand down to posterity, in a form accessible and intelligible, the investigations of earlier philosophers. His work did in fact become so to speak, the store-house for logical and philosophical material to be worked up into text books. (Cf. Ueberweg, *Geschichte der Philosophie*, Vol. I, p. 354.) It does not appear that Boethius had any decided view on the philosophical disagreements of Plato and Aristotle. The keenest investigations of Hauréau have failed to disclose any leaning or partiality towards either view. (Cf. Hauréau, *op. cit.* I, pp. 92-99.) For exposition of the Boethian position on the philosophical problem, vide also Cousin, *op. cit.* pp. 67-75.

sylogisms is "remarkably thorough."¹ It seems clear then that he distinguishes between the formal logic of the school room and the logico-philosophical conceptions which form the point of departure for the study of metaphysical subjects. While gathering material for logical instruction, he also collected and made available a portion of that from which scholasticism was to build its monumental system. This material he presented not as logic but as philosophy. Unfortunately some of his didactic material, as the *Analytics*, *Topics*, and *Sophistici Elenchi*, remained unknown and only reached the mediæval school room in the twelfth century through other sources.²

This distinction which Boethius makes between the scope of what we shall call "school logic" and "logical metaphysics" is carried out by other writers who followed him. Examining the purely didactic works on "school logic" we shall find that the metaphysical element, namely, the extended consideration of the categories and their implications seems to be entirely omitted.

The text book of prime importance was undoubtedly Porphyry's *Introduction*. Glosses upon glosses, and commentaries upon commentaries, were made on it.³ How then, we naturally ask, did Porphyry treat the question of the philosophical problems in his work dealing with formal logic? He distinctly dismisses them as being beyond the scope of his introduction to the categories of Aristotle.

Again if we turn to the logical works of "The Philosopher" himself we look in vain for a statement of his position on what later became the scholastic problem. The science of logic which he created was to him only an instrument, an "organon," not a system of philosophy. It is true, as philosophers have seen, that in the treatment of the first category-substance—the position of "universalia in re" is assumed, but his consideration of this category, involving as it does the ontological problem, in no way implies from the standpoint of logic proper, the acceptance of this anti-Platonic position on

¹ Prantl, *op. cit.* I, pp. 682–700.

² Prantl, *op. cit.* II, pp. 98, 99.

³ Boethius, the pseudo Rabanus Maurus (super Porphyrium) and Abelard, are among the noted commentators.

the subject of universals. The problems of realism versus nominalism, of ontology and of epistemology were plainly not logical questions to Aristotle.¹

Turning to the text books of the mediæval period proper we find that the oldest of these is the *De Arte Dialectica* of Capella. Not counting the fantastic introduction and conclusion, we have some thirty duodecimo pages of text, divided into four parts: (1) *predicables*, definitions, division, homonyms, synonyms, paronyms; the *categories* and the doctrine of contradiction (12 pages); (2) *de nomine et verbo*, and the *proposition* (3 pages); (3) the *quantity* and *quality* of the *propositions* and *conversion* (5 pages); (4) the *categorical* and the *hypothetical syllogisms* (6 pages). We have thus before us the essential elements of the logical text books of the middle ages anterior to the university period. While the proportion of space devoted to the treatment of the categories is relatively large, amounting to one-third of the treatise, it is wholly formal. No philosophical implications are to be found, although *substance*, *quantity* and *relation*, are naturally more fully explained and illustrated than the other seven categories. Our first typical mediæval text book then does not reveal a philosophical bias in the treatment of these portions of the logic which touch the borderland of metaphysics.²

The *Principia Dialecticæ* of Augustine (sometimes called *Tractatus de Dialectica*), defines logic as the "scientia bene disputandi." It is absolutely silent on the questions which the schoolmen later deemed so vital. This much used fragment on logic does not even discuss the categories,

¹ Trendelenberg, *Geschichte der Kategorienlehre* p. 250. This omission to touch upon the problems alluded to in Porphyry's introductory paragraph is characteristic of all the *formal* text books on logic. Capella, Cassiodorus, Isidore of Sevilla, Alcuin, Rabanus Maurus and others all avoid the issue. None of them take any position on these philosophical subjects in these treatises. In fact they do not even seem to realize the importance of the question. They speak of first and second substance apparently without realizing the metaphysical implications. (For a summary of the problem of universals and the antithesis of the Platonist and Aristotelian positions which has its echo in the first category, vide Hauréau, *op. cit.* I Chap. 3, pp. 47-73; Grote, *Aristotle*, II, App. 243-268; De Rémusat, *Abélard*, I, pp. 275-367.)

² Text in Eyssenhardt's edition, Liber IV, pp. 98-137.

the author confining himself mainly to the elucidation of the noun, the verb and the preposition.¹ Likewise the pseudo Augustinian, *Categoriæ decem ex Aristotele decreptæ* which partakes of the character of a translation, paraphrase and commentary on Aristotle's categories, does not in any way touch the philosophical implications of the categories.²

The *De Dialectica* of Cassiodorus,³ whatever may be its defects from the standpoint of historical criticism, no matter how much evidence it may give of being "a crude and senseless compilation," as Prantl characterizes it,⁴ was beyond doubt one of the most popular and influential text books for the study of logic. Examining these fifteen folio pages, what does one find? Much about terms and definitions (seven of Boethius's fifteen attributes of definition are given); much about the moods and figures of the syllogism; a great deal about probable reasoning (taken from Boethius's topics; even matter relating to rhetorical argumentation is introduced), but of subjects discussed by the schoolmen hardly a word. Why? The inference is clear—logic to him also was simply a means—a means "ad disserendas res," not an elementary treatise on philosophy. For this reason he barely mentions the predicables of Porphyry, explaining them in a few lines. All the intricate relations of genus, species, differentia, accidentia, and proprium in which Porphyry fairly revels are entirely omitted. The categories themselves fare no better than the predicables. In Cassiodorus's mind, apparently, the boy who would study logic need know little more about the predicables than their logical distinctions. The relation of these categories to the

¹ Prantl, *op. cit.* I, p. 665 et seq. Cf. Augustine, "Retract." I, 6, *Opera Migne*, XXXII, p. 591; text in Migne, XXXII, cols. 1409-1419. For the question of the authenticity of the first named work, vide Prantl, *l. c.* and *supra* p. 56, note 2.

² Prantl establishes its spurious character. There is no doubt however that it was composed towards the end of the fourth century and that it was used as a text book in the middle ages, a fact to which the large number of copies and glosses bear ample proof. Prantl goes so far as to think that it formed the basis of Isidore's and Alcuin's treatises on logic. Vide Prantl, *op. cit.* I, p. 669 et seq. Text, Migne, XXXII, col. 1419-1439.

³ Text, Migne, LXX, cols. 1167-1203.

⁴ Prantl, *op. cit.*, I 722.

eternal verities the student was to study later when he began theology.¹

Turning to the other great encyclopædists, we find in Isidore of Seville, section, *De Dialectica*, the same conception of the scope, limits and functions of the study of logic.² He defines it as the art "quae disputationibus subtilissimis vera secernis a falsis" and again as "disciplina ad discernendas rerum causas inventa." "Id est species philosophiae . . . rationalis definiendi, quaerendi et discernendi potens." To him, as to Cassiodorus, it is the syllogism "ubi totius eius artis utilitas et virtus ostenditur."³

In Isidore's treatise, the *Isagogue* and the *Categories* are treated more diffusely but hardly more thoroughly than in Cassiodorus's work. There is nothing to indicate that he includes in his treatment of formal logic any philosophical, metaphysical or theological problems. Evidently to his mind also these questions belong to another field. He says, in fact, that theology embraces logic as well as everything else.⁴

So far the analysis of the text books of the early middle ages has shown that the writers who defined the scope of logic in the curriculum, have one and all limited the formal instruction to a mere mastery of the rudiments of the science and have carefully omitted anything of a metaphysical character. This same attitude towards the scope of the subject is maintained by succeeding text book writers whose works were entirely based on the labors of these predecessors.

In *De Dialectica* of Alcuin we have an example of what the teaching of logic was in the period of the so called

¹ Cassiodorus takes pains to tell us, in introducing the subject of the syllogism, of the overshadowing importance of that portion of logic: "Nunc ad syllogisticas species formulasque veniamus in quibus nobilium philosophorum jugiter exercetur ingenium." (Migne, Vol. LXX col. 1174.) He builded better than he knew. It was the practical elements of logic that his and the succeeding generations wanted, and these not in the exhaustive form in which Aristotle left them. That is why Cassiodorus's section was so much used, while half of the elaborate apparatus which Boethius preserved for the world was soon lost sight of not to be recovered till the end of the twelfth century.

² Text, Migne, LXXXII, *Etymologiae etc*, Lib. II, cols. 140-154.

³ *Ibid.* cols. 73, 128, 146.

⁴ *Ibid.* *Etymologiae*, Liber II, c. XXIV, col. 140.

Carolingian Renaissance. The influence of that work was great; some believe that it was greater even than that of all the other text books, an assumption not unreasonable when one considers the far-reaching educational importance of Alcuin and of his pupils, and how vast was the number of schools in the eighth, ninth, tenth and eleventh centuries which can properly be said to have been influenced by the traditions of Alcuin.¹ The work is of course not original. It is based in its plan and subject matter almost entirely on the pseudo Augustinian work on the categories, on Isidore's work, and indirectly on Boethius.² We cannot therefore expect anything more than what we found in his sources; but the relative importance he assigns to different portions of the subject can be seen from his compilation, giving us precisely the clue we are seeking. He does devote, it is true, a relatively large space (about one-half of the text) to the discussion of the categories, but as his text on the categories is but an excerpt from the pseudo Augustinian treatise, we again look in vain for any statements on the philosophical problems involved in the categories. That he should assign so much space to these purely verbal distinctions of his source can be explained by the literary and subtle bent of Alcuin's mind. Apparently then he also holds that metaphysical problems are to be treated in a work on formal logic.

The opinion of Rabanus Maurus on the subject is of importance. Though the works on logic passing under his name in the succeeding generations were not his own, his influence as the greatest pupil of Alcuin and as the leader of the educational work of Fulda makes his utterance on the subject of great value to our inquiry. The teachings of logic at Fulda exerted a most favorable influence on the cultivation of that branch of the seven liberal arts.³ Through Rabanus and through Haimon of Halberstadt, his colleague, the study of logic spread rapidly through Brittany, Gaul and Germany, and the Flanders, so much so

¹ Thus the education in logic which Walther von Speier received in the tenth century tallies with the amount we find in Alcuin's book. Cf. *Acta St. Christophoris*, Liber I, De Studio, vv. 114-147, edition Pez, *op. cit.* Cf. Specht, *op. cit.*, p. 24.

² Text, Migne, Vol. CI., col. 949-975; for critique Cf. Prantl, *op. cit.*, II p. 14 et seq.

³ Prantl, *op. cit.* II, p. 40 et seq.

that in the teaching of theology in the schools of the tenth and eleventh centuries logical disputations and theological discussions were often carried on.¹

Rabanus Maurus's views on logic are given in his famous treatise, *De Clericorum Institutione*.² He defines it in words which we have met before as, "disciplina rationalis quærendi, definiendi et disserendi, etiam vera et a falsis discernendi potens."³ His enthusiasm for that subject of the curriculum of the seven liberal arts was due to the fact that to his mind logic enabled one to penetrate with subtlety into the craftiness of the heretics and to confute their opinions by magical conclusions of syllogisms. By illustrating how a fallacious syllogism might, for instance, disprove the resurrection, he showed the necessity of studying the true modes of the syllogism in the school, which knowledge was "to be applied to find the truth in the scriptures."⁴

¹ E. g. Thaugmari, *Vita S. Bernwardi*, C. I.; Othlonis, *Vita St. Wolfgang*. C. 28, M. G. SS. IV., pp. 538-578, quoted in Specht, p. 125. The works on Logic by Notker Labeo give an indication of the character of instruction in logic which was given at St. Gall, in the tenth century, while the large number of mss. extant afford evidence of the popularity of the subject. For an edited text of his commentaries on the categories, on *De Interpretatione* and on Capella, as well as of his original work on the syllogism, vide Hattemer, *Denkmäler des Mittelalters*, Bd. III, pp. 257-559.

² Migne, vol. CVII, III, c. XX, cols. 397, 398.

³ *Ibid.*

⁴ "Cum ergo sint verae connexiones, non solum verarum; sed etiam falsarum sententiarum, facile est veritatum connexionum etiam in scholis illis discere, quae praeter ecclesiam sunt, sententiarum autem veritates in sanctis libris ecclesiasticis investigandae sunt." *l. c.* These views of Rabanus Maurus, as to the transcendent importance of formal logic to theology, can be considered epoch-making in the history of the mediæval curriculum. Aristotle had considered logic only as a means of clear and consistent thinking, especially in disputation, requiring a knowledge of the ambiguities of language, a power to detect identity of meaning under different expressions, and an appreciation of all that a proposition implies and all that may be deduced from it, and requiring no philosophical conceptions or predilections. In the same spirit, Boethius, Cassiodorus, and Isidore of Seville had considered logic merely an insignificant part of philosophy, which they considered the "disciplina disciplinarum." Up to the time of Erigena no direct connection between the study of logic, as it was taught in the schools, and theology or metaphysics, can be found. Rabanus Maurus marked the departure by calling logic itself the "disciplina disciplinarum," and dwelt on its great importance in the study

Passing on to the tenth century, we have an exact account of the great Gerbert's extensive teaching of logic in the cathedral school at Rheims, an institution acknowledged to have been one of the best of its generation. For his text books he employed portions of the works of Boethius; the *Isagogue*, the *Categories*, the *De Interpretatione* and the *Topics* of Cicero,—

of the scriptures. It needed but the experience of the heresy charges against Erigena and Berenger of Tours to show what dangers to dogma lurked in the use of the syllogism. The average mediæval educator, controlled as he was by the conceptions of the church, appreciated the significance of Rabanus Maurus's dictum on the value of logic. But while the latter's ideas on the subject stimulated its formal study they in no way affected the scope of the subject in the curriculum. If, as is often asserted, this fervid pursuit of dialectic was unproductive of new scientific results, it is nevertheless true that the increased study of logic prepared the way for the intellectual renaissance of Abelard's age. In fact, it was with the instruments of the schoolroom logic that the early schoolmen, Roscellinus, William of Champeaux, Anselm and Abelard, so successfully attacked the mediæval philosophical problems. The great use which these schoolmen made of the dialectical method of reasoning in their metaphysical and theological discussions has led to the impression that at this period when scholasticism blossomed out, the teaching of logic and metaphysics became identical. ("Ainsi la dialectique devint la philosophie première ou la métaphysique." Hauréau, *op. cit.* I, 32. With this compare the following: "La scolastique n'a donc pas été la philosophie réduite à la dialectique, mais aux formes de la dialectique." De Rémusat, *Abélard etc.* I, p. 303.) It was not so. What did happen was that the study of logic usurped the important place hitherto occupied by grammar in the curriculum of the seven liberal arts. The proof for this statement, dissenting from the supposition of Hauréau, can be found in the writings of these very schoolmen. Whether in the scholastic writings or in the text books of the period the idea of the separation of logic from metaphysics seems clear as far as the formal study of logic as one of the seven liberal arts is concerned.

The well known satire of the combat of the seven liberal arts, is a later indication of changes which began about the opening of the twelfth century in France, in and about the schools of Paris, where the influence of Abélard was so potent. It was just about this time—the beginning of the twelfth century—that logic became the second subject of the trivium. The oft quoted couplet

"Gramm loquitur, Dia vera docet, Rhet verba colorat.

Mus canit, Ar numerat, Geo ponderat, Ast colit astra,"

does not date earlier than the tenth century, and cannot be taken to indicate the order of studies of the trivium before the beginning of the scholastic period. This is furthermore proven by the fact that with the exception of Capella, the order of the other writers of text books is invariably, Grammar, Rhetoric and Dialectic.

all, as has been shown, dealing with the formal technical side of logic.¹

In the next century, Anselm, theologian and philosopher, as well as teacher of the Trivium, wrote a didactic logical work. The categories is the text, the aim is didactic formal logic. We look in vain for metaphysical distinctions in the *Dialogus de Grammatico*. The book in short is merely an introduction to logic.²

The same view of the scope of formal logic is found in Abelard. His *De Dialectica*, a voluminous text book on formal logic written in his old age, simply defines logic as "discretio veritatis seu falsitatis." His failure to state in that work his position with reference to the problem of nominalism and realism is most significant when we consider the character of the man. Evidently he also thought such questions did not form a part of a treatise on formal logic.³

The examination of the pre-university text books on logic may be fitly concluded with the mention of Abelard's treatise. As has been shown, these texts one and all—from Capella to Abelard—prove that the teaching logic of the mediæval curriculum up to the thirteenth century in no way involved instruction in metaphysics. Retracing our steps and examining over again the works of the same authors we shall find additional indirect proof supporting this view. The very problems which belonged to the borderland of logic and metaphysics were elaborately treated not in the text books on logic, but in the other philosophical and metaphysical works of these same authors.

Beginning with Boethius we find that philosophy with him had two sides: (1) the *formal, practical* side, that is, logic; (2)

¹ Richer, *Histor. Libri etc.* III, c. 46, 47 (M. G. H. SS., III, p. 617).

² Anselm, *Dialogus de Grammatico*; text, Migne, CLVIII, cols. 561–581. Cf. Weddigen, "Essai Critique sur la philosophie de S. Anselmo" in *Mémoires Couronnées de l'Académie Royale etc. de Belgique*, T. XXV, no. 3, pp. 18 et seq. Anselm undertakes to solve the logical paradox of the syllogism "Omnis homo potest intelligi sine grammatica; nullus grammaticus potest intelligi sine grammatica; ergo nullus grammaticus est homo," and on this basis the whole subject of formal logic is developed in his little treatise.

³ Cf. Ueberweg, *op. cit.* I, 392. Hauréau, I, pp. 267–286. Text in Cousin, *Ouvrages d'Abélard* p. 173 et seq., Prantl, II, pp. 163–204; De Rémusat, *Abélard* I, p. 361 et seq.

the *theoretical*, speculative side, which concerns itself with philosophy in its broader sense, that is, physics and metaphysics. The theological and metaphysical problems being of greater importance than formal logic to one of Boethius's temperament, it is of course natural that even in his discussion of purely logical problems, the philosophical implications of these questions should always come to the front. He could emphasize these easily in the treatment of those portions of this practical subject which touch the philosophical problem. Porphyry's *Isagogue*, Aristotle's *Categories*, and the *De Interpretatione*, afforded this point of contact. In this way all those philosophical ideas that were worth preserving from the standpoint of the mediæval ages were here brought together in the consideration of the problems raised and not solved by Porphyry in the discussion as to the nature of genus and substance. They were not introduced as parts of a text book on logic, but rather as logico-metaphysical material worthy of transmission for later generations.¹

Alcuin is the next writer from whose pen we have both a formal treatise on logic and metaphysical writings. A comparison of these shows that while in his *De Dialectica*—the treatise on logic—the categories received but brief treatment, they are analyzed most exhaustively in his great work on theology—*De Fide Sanctæ Trinitatis*. The reason is obvious; as logical terms for school use the importance of the categories is slight, hence their scant consideration in the *De Dialectica*. In metaphysics the categories have a direct bearing on the very essence of faith, hence his elaborate treatment of the subject as a part of his theological writings.²

If we pass on to Erigena, we find this acknowledged founder of scholasticism, although using the syllogistic form of reasoning in arguments maintaining the identity of religion and

¹ Cf. *supra* pp. 75, 76. If we are to judge the relative interest which Boethius showed in the two aspects of his "Philosophia" by the amount of commentary he produced on each of the two kinds of logic—the formal and the speculative—we find that two-thirds of his writings under consideration are devoted to those portions of logic which verge on metaphysics. He writes two commentaries on Porphyry's *Isagogue*, one on the *Categories*, and two on the *De Interpretatione*.

² Vide Alcuin, *De Dialectica*, Migne CI, cols. 949 et seq. *De Fide Sanctæ etc.*, *ibid.* Lib. I. col. 9 et seq.

philosophy, himself despised formal logic as an unfit subject to engage the attention of thinkers. He clearly rejected the idea that logic or dialectic was an introduction to philosophy.¹ But in spite of his low estimate of dialectics, he gave an extended discussion of the categories in connection with his theological and metaphysical inquiries. In his very first book of the *De Divisione Naturæ*, the Aristotelian ontological implications of the categories are exhaustively analyzed and criticised. Here again we see that the theme of the author was metaphysics and not formal logic.²

Gerbert, whose eminence as a teacher and writer on formal logic is well established, was of course interested in the philosophical implications of the logical problems. His *De Rationali et Ratione Uti*, apparently a treatise on logic, is in fact a metaphysical work—a contribution to the ontological problem. The text is the categories, the treatment is metaphysical. That he did not use this work in his teaching of logic in the school room (and of this we have proof) is again evidence that to him also the philosophical implications of the categories did not form a part of formal logic.³

¹ *De Divisione Naturæ*, Migne, CXXII, col. 870 et seq. Cf. Hauréau, *op. cit.* I, 12, pp. 118–119.

² *De Divisione Naturæ*, Lib. I, c. XXII, *ibid.* cols. 469. Cf. Ueberweg, *op. cit.* pp. 358–366. From Erigena's time we see the same questions discussed apparently under the name of logic. But even a cursory examination of these works shows that they were not the logic books of the school room and that it was not the formal logic of which they treated, but metaphysics pure and simple. It is true that among the commentators on logical treatises in the latter half of the ninth century, the nominalistic position shows itself first slightly and then gradually more distinctly. Hauréau has shown this to be the case in Erigena's and Remigius of Auxerre's commentaries on Capella. These brief references to the philosophical problem are stated simply as accepted facts and foreshadow the coming nominalistic position. As such they are important only as tracing the history of scholasticism, but when viewed in connection with the text, they do not prove that the philosophical question was in any way a part of the formal elementary logical instruction. For Erigena's commentary, see *Notices et Extraits etc.* T. XX. Pt. II, pp. 8–39, where realistic implications are shown in contrast with the Aristotelian view of Capella. For Remigius's commentary, text, introduction and criticism vide *Hist. Lit. de la France* VI, p. 120; Hauréau, *op. cit.* I, p. 144 et seq; and *Notices et Extraits etc.* XX, Pt. II, p. 20; Cf. also Ueberweg, *op. cit.* I, pp. 367–388.

³ Vide Richer, *Histor. Libri etc.* III, c. 46, 47, (M. G. H. SS. III, p. 617.)

Again the great Anselm, of whose logical treatise mention has been made, discussed the categories at great length not, as we have seen, in the *Dialogues de Grammatico*, but in his justly famous philosophical works the *Monologium* and *Prosologium*.¹

In Gilbert de la Porée's *De Sex Principiis*, composed about 1150, we have an entire work devoted to the logico-metaphysical problem. The work treats of the metaphysical aspects of the categories, and the author endeavors to reduce Aristotle's classification of the *ten* to *six* categories. The treatise was not a text book on formal logic. Had it been so he surely would have incorporated the material of Aristotle's "Nova Logica"—the two *Analytics*, *Topics* and *Sophistici Elenchi*—since it is known that he possessed a knowledge of these works.²

The foregoing analysis of the logical and philosophical works of the authors that influenced the course of teaching in logic in this period tend to but one conclusion, that there was an absolute separation of philosophy from logic as it was taught in the curriculum of the seven liberal arts during the pre-university period.

B. UNIVERSITY PERIOD.

The generations following that of Abelard and Gilbert de la Porée witnessed the advent in Western Europe of all of Aristotle's works. It forms no part of the present inquiry to write the history of this important epoch in the intellectual history of Europe. Equally foreign to our subject would it be to trace the hostile attitude of the church towards this "new" philosophical material, or to indicate how finally the church united with "The Philosopher" when it became evident that Aristotle could very well aid it in its endeavor to prove theism by reason.³ For our purpose it is sufficient

For text of *De Rationali* etc., vide Migne CXXXIX cols. 157 et seq.

¹ For text of the *Monologium* and *Prosologium* vide Migne, CLVIII col. 141 et seq., 223 et seq.

² Prantl, *op. cit.*, II, p. 215 et seq.; Cf. Hauréau, I, p. 298 et seq.; Ueberweg, *op. cit.* I, p. 399.

³ The subject is treated fully in the works of the authorities referred to in notes 1-5 p. 74, especially in Jourdain. Cf. *supra*, p. 75. For some notes on the literature of this topic, vide Ueberweg, *op. cit.* I, p. 430.

to state the established fact, that his complete logic was welcomed into the university curriculum even before his philosophy was accepted as orthodox. Indeed while his philosophical works were condemned no less than thirty-seven times, the study of his works on logic was never forbidden.¹

What was the content of this "complete logic?" It consisted of the "Vetus Logica," the material used by the text book writers of the pre-university period, to which was added the "Nova Logica"—the two *Analytics*, the *Topics* and *Sophistical Refutations*. The whole body of this knowledge was known as the "Antiqua Logica," and thus comprised all that is known today as the Aristotelian Logic. The possession of the complete *Organon* together with the other metaphysical works of Aristotle made clear to the schoolmen the exact relation which logic occupied to the whole of philosophy—logic was to be an instrument and introductory study to be taken up before the study of what was termed "The First Philosophy."² The era of the universities broadened the scope of the Seven Liberal Arts to include the philosophy of Aristotle. Grammar, i.e., literature, was neglected. But while the quantum of the instruction in the Arts was greatly increased, the general character of the instruction in Logic did not change its formal aspect. The teaching of logic in this period no more included metaphysics than it did in the earlier period. As in the pre-university period, the text books in this epoch clearly show it. The change was rather in another direction: all teaching became infected with the spirit of the formal logical methods. But logic as one of the seven liberal arts still occupied its former position of a propaedeutic.³

¹ Vide Launois, *De variis Aristotelis in Academia Parisiensis fortunis*, quoted in Prantl, *op. cit.*, III, pp. 9, 10.

² This idea is expressed in his *Metaphysics*, IV, 3, vii, 12, Cf. Ueberweg, *System of Logic and History of Logical Doctrines*, p. 33.

³ That this was the view taken of the subject even during the period of the widest and most complete extension of scholasticism may be seen from the opinion of Albertus Magnus: "Logica," he says, "una est specialium scientiarum; sicut in fabrilis in quae specialis est ars fabricandi maleum." Again he defines it as "sapientia contemplativa docens qualiter et per quae devenitur per notum ad ignoti notitiam." Logic to him is not a part of metaphysics or philosophia prima. Cf. Albertus Magnus, "De Predicab." Lib. I, Tractatus I, Chaps. I, II, III, *Opera*, Cf. Ueberweg, *History of Philosophy*, I, p. 435.

One would suppose that in this period Aristotle's own treatises would have formed the only text book on logic in use at the universities. But when we bear in mind the age of the mediæval student at the time he began the study of logic—barely fourteen years—it becomes evident that the *Organon* was too difficult a text to begin the subject. Here, therefore, as in the teaching of the other subjects of the curriculum, the work of the commentator and adapter came into play. The logic to which the student was introduced by means of these derivative text books was hardly more than a simplified treatment of the mere elements of logic—a brief survey of the “Logica Antiqua.”

As time went on and subject after subject of the curriculum became enmeshed in the maze of logical distinctions, even Aristotle's elaborate treatise was no longer a sufficient source for the text book writers. The spirit of the age demanded more subtle verbal distinctions than Aristotle afforded. To meet this demand there arose a “logica moderna” or “parva logica.” This “modern” material consisted of chapters on the subject “de terminorum proprietatibus.” This characteristic mediæval logical material soon became a regular addition to text books which formerly contained only the Aristotelian logic.¹

The number of these up-to-date text books was very large. But in character they were all similar. They represented the attempt of the practical teacher to utilize the increased knowledge of a subject that for some two centuries had been gradually growing in importance and was encroaching upon the traditional claims of Grammar in the Trivium. Because of this similarity it will serve no useful purpose to enter upon a careful examination of these different texts.² Only the earliest and most typical of them all need detain us, the far-famed *Summulae Logicales*, by Petrus Hispanus, who, as Pope John XXI, died in 1277.³

¹ Cf. Ueberweg, *op. cit.* I, pp. 453, 454; Prantl, *op. cit.* III, p. 50 et seq.

² Among the more famous writers in this field the following may be mentioned: Michael Psellus, Johannus Italus, William Shyreswood, Lambert of Auxerre. Prantl, *op. cit.* Vol. II, III, enters into a discussion and analysis of the texts of these and other authors. For our purposes his discussions are of no value, because Prantl is ever in quest of logical theories. The point of view of a modest text book he frankly considers of no interest to him. Vide Prantl, *op. cit.* II, Abschnitt, XIII, Anm. I, and III, Absch. XIX, Anm. I.

³ The writer has been unable to secure a copy of the texts and was there-

The treatise was used almost exclusively as a text book of formal logic throughout Western Europe for some three hundred years. Though often appearing under different names, it was the same text of Petrus.¹ The book is divided into seven parts, six of which treat of the material contained in the *Organon* and Porphyry's *Isagogue*. The last part is devoted to an elaborate treatment of grammatical and logical distinctions—the vague “De terminorum proprietatibus.”

In the present chapter the aim has been mainly to determine the exact limits of the subject taught under the name of logic, throughout this period. To the elucidation of this point all other matters were subordinated. The prevailing confusion in regard to the scope of logic in the curriculum of the seven liberal arts and the relation of the subject to mediæval philosophy seemed to call for this special treatment. From the material presented here it certainly seems incorrect to assert, as Rashdall does,² that it was a morsel of logic that started scholasticism on its career. Logic had nothing whatever to do with the metaphysical questions which both Porphyry and Boethius had set aside as being outside of the domain of logic. That this position was consistently maintained throughout this period has been amply proved in the foregoing pages.

fore obliged to rely on the analysis of the work by Prantl, which is exceptionally full. Vide Prantl, *op. cit.* III, pp. 33–74. Cf. *Hist. Lit. de la France* XIX, p. 326. The priority for the content of the *Summulae* is claimed by Prantl for a *Synopsis* of Michael Psellus, but Thurot and others hold that the so-called *Synopsis* attributed to Psellus is really a translation of Petrus Hispanus's work. Ueberweg leans towards the latter view. Ueberweg, *System of Logic etc.* p. 41. For a full bibliographical note on the controversy, vide Ueberweg, *History of Philosophy* I, p. 404.

¹ Prantl alone has used no less than forty-eight printed editions.

² Rashdall, *Universities in the Middle Ages*, I, p. 38.

CHAPTER VII.

Arithmetic.

A. THE GENERAL CHARACTER OF THE QUADRIVUM.

While it is undoubtedly true that the Trivium—grammar, rhetoric and logic—occupied the greater portion of the time devoted to the study of the seven liberal arts, the traditional opinion that “the real secular education of the dark ages was the trivium,” and that the quadrivium, or mathematical, subjects were hardly pursued, is far from being historically correct.¹ Such a statement could not be made with an approach to accuracy even in speaking of the university era when logic and philosophy were admittedly the essential studies. This misconception as to the scope of the mathematical subjects of the curriculum is, however, natural enough. The actual amount of mathematical knowledge during the period before the twelfth century was so small that until recently it had been almost overlooked. Historians of the mathematical sciences have considered the age “barren of interest.” It is even claimed that the mediæval mind had no aptitude whatever for mathematical science.²

But the absence of creative work in mathematics during the greater portion of our epoch does not of itself argue a lack of instruction in the subject. Quite the contrary is the case.

Looking at the question from all points of view, the evidence seems to point to but one conclusion—an extensive pursuit of the subjects of the Quadrivium throughout the middle ages. In the first place, those personal experiences which give accurate illustrations of the study of the seven liberal arts, invariably include the subjects of the Quadrivium as well as of the Trivium.³ Again, if we examine

¹ Rashdall, *Universities in the Middle Ages*, Vol. I, p. 35. Laurie, *Rise and Constitution of the Early Universities*, 61 et seq. Both hold this traditional opinion.

² H. Hankel, *Geschichte der Mathematik*, pp. 304–359, especially pp. 334 and 358.

³ E. g. Walther von Speier's life of St. Christopher; Othlo's St. Wolfgang; Bruno's St. Adalbert. Cf. Specht, *op. cit.* pp. 89–149, especially p.

the facts bearing on the position of the church on the question of mathematical instruction we find that from the days of Charlemagne synod after synod enforced upon the clergy both a knowledge of the methods of Easter computation and of music. It is certain that in England, to cite but a single example, from the eighth century to the days of the Norman conquest, no one was ordained to the priesthood who could not compute the date of Easter and teach it after the manner of the writings of the Venerable Bede.¹ Beyond doubt the church was interested directly in at least three of the studies of the Quadrivium—arithmetic, astronomy and music. We should therefore expect no hostility to the instruction in the Quadrivium in mediæval schools.

Moreover, if we examine the general condition of the schools of Europe in the Carolingian period and later, down to the intellectual revival of the thirteenth century, we can easily trace an unbroken record of continued interest in mathematical studies in all the famous monastic and cathedral schools of Europe. We find this existing at the schools of Fulda, Heresfeld, Reichenau, St. Gall, Augsburg, Mainz, Hildesheim, Speier, Cologne, Stablo, Munster, Verdun, Corvey, Ratisbon, St. Emmeran, Passau, Ranshofen, Klosterburg, Reichersburg, Wessobrunn, Metten, Benedictbeuern, Polling, Niederaltaich, Kremsmünster, St. Florian, Admont, and many other educational centres of the Holy Roman Empire. Still greater for these subjects was shown by many institutions of France and the Netherlands, such as those of Rheims, Liege, Lobach and others.² We furthermore find that the great teachers of this period were nearly all famous both for their teaching of mathematics and for contributions to the science of that subject. To cite but a few examples, we may mention, Rabanus Maurus, Heiric of Auxerre, Remigius of Auxerre, the three Notkers, Ratpert, Ermenrich, Heilpric, Tatto, Hermannus Contractus, William of Hirschau, Herard of Landsberg, Odo

127 et seq., for references to original sources containing fragmentary data relating to the education of eminent mediæval persons.

¹ Günther, *Geschichte des Mathematischen Unterrichts im Deutschen Mittelalter* p. 14.

² Specht, *op. cit.* pp. 297-394; Wattenbach, *Deutschlands Geschichtsquellen im M. A.* 7 Aufl. pp. 241-487, passim. Vide Ziegelbauer, *Historia Rei. Lit. O. S. B.*, Pars 1, passim.

of Clugny, Gerbert (afterwards Pope Sylvester II), Engebert of Liege, Bishop Gilbert of Lisieux, Odo of Tournay, Abbo of Fleury, Hucbald, Othlo, Conrad of Nurnberg, (brother of the famous Anselm,) Siegfried and Reginbald. Researches have established the fact that all these possessed mathematical ability, were active in teaching mathematics, and in most cases even produced original works of merit on some of the quadrivial studies.¹ The lack of scientific value in most of these treatises on the Quadrivium accounts for the neglect to publish the large number of them found in the libraries of Europe.² But even the incomplete records of text books of the Quadrivium sufficiently indicate that there was a great deal of teaching done in these subjects.

Much is generally made of the fact that up to the twelfth century, the knowledge of the subject matter of mathematics was very slight. But even a most cursory examination of the contents of the text books in actual use during the period proves the incorrectness of the assertion that only the most elementary propositions in geometry, the method of calculating Easter and the use of the abacus, were the sole themes occupying the attention of the mathematicians of the age. We must be on our guard against estimating the achievements of the middle ages from the standpoint of our own time, when projections, the calculus and theories of the composition of sound are the commonplaces of mathematical studies.

In the succeeding chapters, which are to deal with the several subjects of the Quadrivium, the writer will endeavor to maintain the following theses:—

(1) The amount of knowledge imparted to the student of the Quadrivium was large in proportion to the amount of knowledge which Europe possessed of mathematical subjects at that time. This does not mean that the mediæval teachers knew much mathematics, but that the schools ful-

¹ Cantor, *Vorlesungen über Geschichte der Mathematik*, Bd. I, pp. 771-797; Günther, *op. cit.* pp. 39-61, where specific references to the mathematical activity of each one of the schools and persons are given. For lists of such works, vide Ziegelbauer, *op. cit.*, IV, 304-411.

² Thus *Codex Vaticanus* 3896 contains no less than 26 treatises on Arithmetic in manuscript; cf. Günther, *op. cit.* p. 67.

³ As claimed by Hankel, *op. cit.*, p. 334.

filled their mission and transmitted all the mathematical knowledge they possessed to future generations, and that the student was obliged to master this knowledge before he took up the advanced study of philosophy.

(2) The standard of mathematical teaching in the middle ages in the well known schools was very high. Although there is no evidence of creative mathematical ability in the early periods, the later centuries show advance in assimilation of new material.¹

(3) The quantity and character of the mathematical instruction throughout the age improved *pari passu* with the advance of mathematical knowledge in the several subjects.

(4) Even after the thirteenth century, when in the university period the Quadrivium was merged in the general course of philosophy in the school of arts, mathematical studies were far from being neglected. Even then when scholasticism held sway, the amount of instruction in mathematics kept pace with the gradual advance of the sciences.²

B. EXTENT OF KNOWLEDGE.

The arithmetical knowledge of the middle ages can be classified into three periods. During the first, ending with the tenth century, Europe possessed a very small amount of knowledge of the kind of arithmetic which was cultivated so much by the Greeks in the so-called Alexandrian age. What was known comprised chiefly the contents of the text books on arithmetic of the Neo-Pythagorean Nichomachus, composed about the close of the first century.³ During this time the study

¹ Günther, *op. cit.* pp. 81-121, 146-207.

² The traditional view as to the neglect of these studies before the Renaissance is based on an erroneous assumption. Paris, being the mother of the universities, it is supposed that its *special* neglect of the mathematical studies was characteristic of the entire period. In opposition to this belief it might be urged that the University of Vienna made much of the teaching of these very subjects. As a matter of fact there was a mean position between the two extremes—Paris and Vienna—taken by most of the universities of the middle ages. These afforded a reasonable and ample amount of instruction in mathematics. Cf. Rashdall, *op. cit.* I, pp. 440-443; Günther, *op. cit.* p. 207 et seq.

³ It was through Boethius who translated and adapted this text that this peculiar form of arithmetic came to be known as, Boethian arithmetic. For text of Nicomachus, vide edition of R. Hoche Leipzig, 1866. For analysis of Nicomachus, vide Gow, *Short History of Greek Mathematics*, pp. 89-95.

arithmetic was confined on its practical side to the computation of Easter, and on its theoretical side to a study of the properties of numbers. In the operations, the Roman abacus, a crude instrument, and Roman numerals are used.¹

During the second period, embracing the age from the end of the tenth century to the end of the twelfth, a considerable advance can be noted. The use of the abacus as modified by Gerbert was extended; and complementary division and columnal computation, methods which obviated much of the digital computation of the earlier age, were common.² During the third, the so-called algoristic epoch, embracing the years to the end of the middle ages, still greater progress can be noted. Arabic numerals and the zero came into use while much of the old Greek mathematics was reclaimed through Arabian translations.³ Although each period had its characteristic arithmetical method, quantum of knowledge and range of instruction, it is not to be supposed that any definite lines separating these stages could be traced. We shall find in our examination of the text books that these overlapped one another, and that text books characteristic of an earlier age continued in use to some extent in the succeeding age.⁴

THE FIRST PERIOD.

I. GENERAL CHARACTER.

Arithmetic in this stage is essentially the art of computation. It is almost exclusively devoted to the work of computing Easter so that during this age the terms "computus" and "arithmetic" became synonymous. Still, aside from this practical phase, a theoretical treatment of properties and relations of numbers was not absolutely wanting. In the

¹ Cf. Ball, *History of Mathematics*, p. 137.

² H. Weissenborn, *Gerbert-Beiträge zur Kenntniss der Mathematik des Mittelalters*, Berlin 1888, pp. 208-251. Cajori, *History of Mathematics*, p. 114 et seq.

³ Günther, *op. cit.* Bd. I, pp. 797-809. The word Algorithm is derived from *Alchwarizmi*, the first and most important writer on Arabian mathematics known to Europe.

⁴ There is in existence a *Computus* composed in 1395. It is in the interesting collection of mediæval text books on arithmetic, belonging to Mr. George Plimpton of New York.

theoretic arithmetic the mystical and symbolic elements are very prominent; this was due, of course, to the fact that Nicomachus, whose book was the source of Boethius and the Christians—Isidore of Seville, Alcuin, Rabanus and others—made much of such speculations. The method of reckoning was crude; the abacus was rarely used, while the cumbersome Roman notation made computation with large numbers well nigh impossible. In fact no records of actual computations beyond three figures have been found.¹ Roman fractions wherever used necessitated the aid of special multiplication tables, based on the “half system.” If the text books in use afford any test, the total amount of arithmetical knowledge possessed by Western Europe during this period was small indeed;—so small in fact as to make the later re-discoveries and translations of the arithmetic of the Alexandrian school of mathematics seem like an addition of entirely new material to the stock of knowledge on this subject.²

2. TEXT BOOKS.

Strange as it may seem, the text books of the period do not treat of methods of operation. The few instances in which operations are incidentally given show plainly that a laborious mental process and the digital method were employed.³ The following are among the typical texts in use:

(1) Capella's chapter on *Arithmetic* in his *De Nuptiis Philologiae et Mercurii*, is nothing more than the briefest abstract of the arithmetic of Nicomachus. Besides the allegorical introduction, the chapter consists of material on the properties and the mystic significance of number according to Pythagorean

¹ Hankel, *op. cit.*, p. 309 et seq.

² Cf. Günther, *op. cit.* pp. 64-78.

³ Thus in Alcuin's work on the computus, *De Cursu et Saltu Lunae ac Bissexto*, Migne, vol. CI, col. 979 et seq, CCXXXV is multiplied by IV in this manner:

$$\begin{array}{r}
 \text{CC} \quad \times \text{IV} \text{ — DCCC} \\
 \text{XXX} \times \text{IV} \text{ — CXX} \\
 \text{V} \quad \times \text{IV} \text{ — XX} \\
 \hline
 \text{DCCCCXL}
 \end{array}$$

For a similar example of the method of division ($6144 \div 15$) see the pseudo-Bedan *De Argumentis Lunae*, Migne Vol. XC col. 719.

notions. The treatise owed its popularity to the fact that it formed a chapter of a convenient general text book on the seven liberal arts.¹

(2) Boethius's *De Institutione Arithmetica Libri Duo* was used as the source of arithmetical knowledge in the middle ages for some thousand years. The book was a standard text book, even after the introduction of the Hindu system of notation and computation. Abridged, commented and edited numberless times, it was still worth printing in the sixteenth century.² What, then, were the contents of this remarkable work?

Looking through the eighty folio columns and the one hundred diagrams, one is surprised not to find a single rule of operation, only an almost endless classification of the properties of numbers. Of these there are triangular, perfect, excessive, defective, amicable, etc. There is a variety of kinds of even and of odd numbers; proportion and progression are not overlooked. The contents of the work would seem to indicate that Boethius's text was not for the use of pupils. It formed a teacher's guide on the subject. It also formed an adequate introduction to the study of the mystical interpretation of scriptural numbers from which moral lessons were not infrequently deduced.³

(3) Cassiodorus's brief *De Arithmetica*, is at best a condensed section of the work of Boethius. Nothing new is added. In four diagrams the properties of numbers are classified and each kind is defined and illustrated. One finds nothing in the work in the way of information on practical methods.⁴

¹ Text, Eyssenhardt's edition, Liber VII, pp. 254-296. For a fuller account of the mystical interpretations on Capella, vide Gow. *op. cit.*, p. 69 et seq.

² Vide A. De Morgan, *Arithmetical Books*, pp. 3, 4, 10, 11, 13, where references to books on Boethian arithmetic printed in Paris and Vienna are given, the latest date of an edition being 1521. It also bore the simpler name *De Arithmetica Libri Duo*; Text, Migne, *Pat. Lat.* LXIII, cols. 1079-1168.

³ Cf. Günther, *op. cit.*, p. 82, et seq; Text in Migne, Vol. LXIII, cols. 1079-1166, special edition by Friedlein, 1867. For an amusing example of mystical interpretation of numbers vide Rabanus Maurus, *De Institutione Glecicor.*, Migne vol. CVII, col. 400, where the mystical meaning of the number 40 is explained.

⁴ Text in *De Artibus etc.*, Migne LXX, cols. 1204-1208

(4) Isidore of Seville's brief chapter on Arithmetic follows the same lines as that of Cassiodorus. It is a mere fourfold classification of numbers in regard to their properties and relations. The author adds some absurd remarks on the derivation of the Latin arithmetical nomenclature and some rapturous expressions about the importance of numbers.¹ In this work also we look in vain for any single sentence about methods and rules of operations.

(5) The Venerable Bede's *De Temporum Ratione*, an elaborate text on the method of computing Easter, is the first book of this period to touch upon the subject of practical computation. It is therefore not surprising that this work should have become a standard for many centuries to come.²

(6) The same author's *Liber de Ratione Computi* is of similar character but more condensed in form.³

(7) Alcuin's *De Cursu et Saltu Lunæ ac Bissexto* is likewise a work on the computus. Its content, however, is astronomical rather than arithmetical.⁴

(8) Rabanus Maurus's *Liber de Computo*, is perhaps the most complete and the most typical book of the period under discussion. The ninety-six chapters of the work treat fully and concisely of all that is necessary for a complete knowledge of Easter reckoning. To be sure the inevitable multiform classification of the properties and relations of numbers is there, but less than one column is given to the subject. The rest of the text is devoted to the Greek system of notation, the divisions of time, the names of Greek and Roman months, names of planets, facts about the moon, the solstice, the equinox, indications, epacts, and similar astronomical phenomena involved in the study of the computus. The lunar cycles and the method of finding Easter are explained after the plan of

¹ "Tolle numerum rebus omnibus et omnia pereunt. Adime saeculo computum et cuncta ignorantia caeca complectitur nec differi potest a ceteris animalibus qui calculi nesciunt rationem." *Etymol. Lib. XX*, Liber III, c. IV, 3. Text of entire work, Migne, Vol. LXXXII, cols. 154-163.

² Text in Migne, vol. XC, cols. 294-578. Deducting glosses and scholiae, there remain some eighty folio columns of actual text, making a moderate sized text book.

³ *Ibid.*, cols. 579-606.

⁴ Text, Migne, vol. CI, cols. 679-1002.

Bede. As one would expect, the principal portion of the whole work is the part devoted to the method of Easter reckoning.¹ It is significant that in the introduction to the work, a chapter on digital reckoning and practical Roman symbols is given. Still more significant is the omission of the rules for the four operations. From this it would seem, then, that the computation was done mainly in the head, perhaps with the aid of an elaborate system of finger reckoning; and that a knowledge of the four elementary operations with whole numbers was assumed in the study of the computus. In all respects the work of Maurus may stand as typical of the arithmetical knowledge and teaching of this period. The great influence of the "prae-

¹ Migne, CVII, 669-727. As this was the essential part of the contents of all the text books of this period, a brief statement of the problem involved in the Easter reckoning is appropriate. The computation was for the purpose of finding the date on which the first Sunday after the first full moon after the vernal equinox would fall. The problem was solved by finding the so-called "golden number" and "Dominical Letters," thereby determining their relative positions in the tables of the metonic cycles. To be able to do so implied a knowledge of (1) the vernal equinox of the sun, (2) the day of the following full moon, (3) the correction of the error involved in the metonic cycle. As early as in the days of Abbot Dionysius Exiguus (c.525) the astronomical problems involved were solved and successive tables were constructed by Dionysius, Abbot Felix of Cyrilla, Isidore of Seville and Bede, those of the last named going as far as 1063. With the aid of two rules involving simple operations and the use of the tables named, Easter day could be readily determined. The rules were: (Rule 1) To find the golden number, add one to the numeral of the year (in the table) and divide the sum by nineteen, the remainder is the golden number; if there be no remainder nineteen is the golden number. (Rule 2) "To find the Dominical Letter, to the numeral of the year add its quotient on dividing by four, add also four, divide this sum by seven and subtract the remainder from seven; the remainder will determine the place of the Dominical Letter in the table." By referring the two answers obtained by the operations of the two rules to the tables, Easter day is easily determined. The demands upon the arithmetical knowledge of the pupils who aimed simply to solve this problem were not very great, it is true, but from what has been said before it is safe to assume that after the Carolingian revival every priest who had studied the liberal arts actually understood not only the method but the principles involved; this implied not a little arithmetical and astronomical knowledge in addition to clear mathematical thinking. Vide Smith and Chetham, *Dictionary of Christian Antiquities*, article "Easter"; F. J. Brockman, "Die Christliche Oesterrechnung," *Systeme der Chronologie*, pp. 53-83. For a modern simplified rule for calculating the date of Easter, vide Ball, *Mathematical Recreations and Problems*, p. 238; Cantor, *op. cit.* I, p. 532 et seq., p. 780.

ceptor Germaniae" of itself argues for a widespread use of his work, and numerous text books on the computus, anonymous and otherwise, were based on his treatise.¹

Besides these text books, all of which show the characteristics ascribed to this period, there are some exceptional works on arithmetic composed in this epoch, but their existence and use in no way weaken the conclusions which have been formulated as to the general character of the instruction in arithmetic in this early period of the middle ages.²

Of unique interest as showing methods of division and some fractional work are the following short works, mistakenly attributed to Bede:

1. *De Numerorum Divisione Libellus.*
2. *De Loquela per Gestum Digitorum et Temporum Ratione Libellus.*
3. *De Unciarum Ratione.*³

¹ Günther, *op. cit.* p. 66. Among the mediæval teachers who based their works on the computus entirely on the works of Rabanus, the following are noteworthy, as showing both the wide extent of its influence as late even as the thirteenth century, such are Heilpric, a monk of St. Gall, Marianus Scotus, Hermannus Contractus, William of Hirschau, Notker Labeo and Johannes de Garlandia. It should be noted, however, that the works on the computus by the authors named, although composed after the age of Gerbert, thus apparently belonging to the second period of our classification, can in no way be taken as an index of the methods used in teaching arithmetic in their day. When these were composed the study of the computus had come to mean simply the technical study of Easter reckoning. It did not mean, as in the days of Rabanus Maurus, the study of Arithmetic.

² Thus the "propositiones (arithmeticae) Alcuini ad acuendos juvenes," a collection of difficult problems with appended solutions, rightly ascribed to that famous teacher, while of a special interest from other points of view, can in no way be taken to imply that such deep problems were usually studied under the head of arithmetic during this period. The fact that they were known by Gerbert at the end of the tenth century, is equally inconclusive proof as to their use in school work, since Gerbert was the mathematical genius of his age. Such problems belong to the same class with mathematical games which were known to the select few. Vide Cantor, *op. cit.* I, p. 784, who establishes the authenticity. Text in Migne, Vol. CI, col. 1143. Cf. Hankel, *op. cit.* p. 310, note. For full references on these games in the middle ages vide Günther, p. 88, note 1.

³ Texts, in Migne, vol. XC. cols. 682-709. For critique vide Karl Werner, *Beda der Ehrwürdige und seine Zeit* p. 107 et seq. Wien, 1875, as quoted in Günther, *op. cit.* p. 5.

The origin of these little treatises cannot be traced back later than the tenth century.¹ It can therefore be assumed that they indicate a slow advance in arithmetical knowledge. This material was to serve as a foundation for Gerbert's achievements in the field of arithmetic.

THE SECOND PERIOD.

I. GENERAL CHARACTER.

The point of departure for tracing the progress of the study and teaching of arithmetic during this period can be found in the epoch-making mathematical achievements of Gerbert. The question as to the precise value of Gerbert's contributions to the development of the science of arithmetic is still an open one. According to some, he is to be credited only with the introduction of columnal computation into Western Europe.² Others go so far as to attribute to him the introduction of the Arabic system.³ On the other hand Cantor, the Nestor of the historians of mathematics, maintains that Gerbert was absolutely unfamiliar with the Arabic system.⁴

There is, however, substantial unanimity on the following points: (1) that Gerbert and his disciples, notably Bernelinus, improved the abacus and extended its use by the introduction of special kinds of *apices*—place symbols at the top of each column of the abacus; (2) that he and his disciples did not employ the zero; (3) that in Gerbert's book we have the first work on the method of computing with the abacus; (4) that Gerbert, first to use the method of complementary division, made possible all the four operations on the abacus. For the purposes of our inquiry one other fact in regard to Gerbert is pertinent: he taught the Quadrivium with marked success at the school of

¹ Hankel, *op. cit.* pp. 307-310.

² A. Nagl, "Gerbert und die Rechenkunst des 10 ten Jahrhunderts" in *Sitzungsberichte der Hist. Philol. Class. der Kais. Akad. der Wiss.*, Wien, Bd. CXVI, pp. 861-922. Friedlein, "Das Rechnen mit Columnen vor dem 10 ten Jahrhundert" in *Zeitschrift für Math. u. Phys.*, Bd. IX, 297-330, esp. 320 et seq.

³ H. Weissenborn, *Gerbert-Beiträge zur Kenntniss der Mathematik des Mittelalters*, pp. 209-239, esp. 233.

⁴ Cantor, *Vorlesungen etc.*, Bd. I, p. 797 et seq. where the historical points in this controversy are summarized.

Rheims from 972 to 982 and an extended account of his methods is in existence.¹

Gerbert's two works, the *Regulae de Abaci Numerorum Rationibus* and the fragment *De Numerorum Abaci Rationibus*, can thus be taken as typical of arithmetical text books from the tenth to the beginning of the thirteenth century. Examining these treatises,² we find that the processes employed in addition, subtraction and multiplication are much like those used in modern times, while the process of division (the subject matter of his second and lesser work) is entirely different from the modern method. Compared with the Arabian system, Gerbert's methods of division have been considered not improperly "about as complicated as human ingenuity could make them." This opinion is confirmed by the name "divisio ferrea" given to his method after the introduction of the Hindu system, which was called the "divisio aurea."³

In the days of Gerbert writers on arithmetic were generally called "abacists." With the introduction of the Hindu methods through Arabian influence, this term came to be restricted to those who clung to the older methods, namely, (1) the use of the abacus; (2) Roman notation; (3) Roman duodecimal fractions; (4) the absence of zero in the calculation; (5) the failure to deal with the method of extracting the square root.⁴ The better methods of the algoristic school, as the writers of the next period were called, did not necessarily supplant the

¹ Richer, *Hist. Lib. etc.*, ("Pertz M. G. H. SS." III, 618 et seq.)

² Latest critically edited texts by N. Bubnov, *Gerberti opera Mathematica*, Berlin, 1899. The extent of the influence of these works can be inferred from the vast number of mss. still in existence. These the editor enumerates, *op. cit.* pp. XVII-CXI passim.

³ Cajori, *History of Mathematics* p. 117. Its mechanical character is revealed by some of the rules which Gerbert laid down thus: (1) The use of multiplication was to be restricted as far as possible and was never to require the multiplication of two digits by two others. (2) Subtraction was to be avoided as far as possible and be replaced by addition. (3) The operations were all to proceed in a purely mechanical way without requiring trials. Vide Hankel, *op. cit.* 319 et seq., where concrete examples of division by this method are given. The most complicated illustrative example is given in Friedlein, *Die Zahlzeichen und das elementar Rechnen der Griechen und Römer und des Christlichen Abendlandes vom 7 ten bis 13 ten Jahrhundert*, pp. 109-124.

⁴ Cajori, *op. cit.*, p. 119.

works of the abacists. There was indeed a contest waged between the abacistic school, sometimes erroneously known as the Boethian school, and the new Arabian school.

The intrinsic superiority of the newer system did not necessarily cause the immediate disappearance of text books based on the older. As text books of Boethian arithmetic were printed even as late as the sixteenth century, so editions of works on the old abacal system continued in use long after the triumph of the Algorists.¹

2. TEXT BOOKS.

Turning to a consideration of the text books of the period, we find many printed editions of these. Only the typical ones, those which in their day were considered famous, need however be mentioned here.

(1) Hermannus Contractus, a monk teaching at Reichenau in the first half of the eleventh century, is the author of a *Liber de Abaco*, a briefer treatise than either of Gerbert's works, but avowedly based upon them.²

(2) Rudolph of Laon, in the twelfth century composed a similar work.³

(3) Johannes De Garlandia, the author of the treatise on the computus, also composed a text book on the abacus. The preparation of these two separate texts by the same writer is significant. It shows that in his day the scope of arithmetic had widened and that there was a complete separation of the computus from the arithmetic proper.⁴

¹ For others Cf. Günther, *op. cit.* pp. 99-110. Cantor, *Mathematische Beiträge zum Kulturleben der Völker*, pp. 330-340.

² Text in Boncompagni, *Bulletino di Bibliografia e di storia delle scienze matematiche e fisiche*, X, pp. 643-647. It forms one of a collection of seven such texts on the abacus. *l. c.* 595-647. Two similar texts of the twelfth century are printed *op. cit.* XV, 135-162. For other abacists of the period Cf. Cantor, *op. cit.* I, pp. 831-836.

³ *Hist. Lit. de la France*, VII, p. 89 et. seq. Text and critique by A. Nagl in Supplement to *Zeit. für Math. u. Phys.* vol. XXXIV, pp. 129-146, 161-170.

⁴ Text Boncompagni, X, 593-607. For other abacists of the twelfth century, vide Cantor, *op. cit.* pp. 843-848 and Günther *op. cit.* pp. 92-106.

THE THIRD PERIOD.

I. GENERAL CHARACTER OF MATHEMATICAL KNOWLEDGE.

The consideration of the third, the algoristic period arithmetic, brings us almost into the midst of the later university era. Although the Quadrivium was merged into the general course offered under the faculty of arts, yet arithmetic, both theoretical and practical, was studied more than in the earlier period. This was, of course, due to the increase in the quantum of knowledge of the subject.

The characteristics of the algoristic arithmetic are: (1) the use of the Hindu-Arabic system of notation; (2) the system of local value; (3) the use of the zero; (4) the entire discarding of the abacus; (5) the combined use of symbols and numbers (in reality a combination of algebra and arithmetic, as these terms are understood today); and (6) the introduction into Western Europe of a vast amount of arithmetical material from the East by means of Latin translations from Arabian sources. While the general tendency of this period was to approach the study of arithmetic from its practical and scientific sides, the mystical aspects of the subject—so popular in the earlier periods—are by no means neglected. The fantastic treatment of the properties of numbers is still common in this age.¹

Thus the beginning of the thirteenth century marks the introduction of the Arabian system of notation and its adoption in place of both the Roman notation and the abacus. This fundamental revolution was brought about only gradually. The transition between the period of the abacus, and that of the algorism can be traced in the translated literature of the Hindu-Arabian arithmetic of the following well-known mathematicians of the twelfth century:

(1) Adelhard of Bath, who wrote *Regulæ Abaci* about 1130. To him also is ascribed the Cambridge anonymous ms. *Algorithm de Numero Indorum*.²

¹ Cantor, *Beitrage etc.* p. 338; Cajori, *op. cit.* p. 119.

² Texts, Boncompagni, *Bullettino etc.* XIV, pp. 91-134 and Boncompagni, *Trattati di Aritmetica* pp. 1-23. For a fragment of his work on multiplication and division vide *Zeit. für. Math. u. Phys.* XXV, Supplementheft, pp. 132-139.

(2) Abraham Ibn Ezra, whose treatise on arithmetic was written early in the twelfth century.¹

(3) John of Seville, who, about 1140 composed his *Algorismus*.²

(4) Gerard of Cremona, who, in the latter part of the twelfth century, prepared an *Algorismus*.³

(5) The brief anonymous work on algorism, composed about the year 1200 in South Germany.⁴

These transitional works, however, composed before the rise of the universities and during the period of the decline of the monastic and cathedral schools, while establishing the historical sequence in this particular study, cannot be taken as typical text books of our period. Their consideration therefore need not detain us. They only served one admirable purpose—they introduced the Hindu-Arabian system among the mathematicians of Europe, and thus paved the way for the later works.

It is true, nevertheless, that in the history of arithmetical as well as of all mathematical studies, the early years of the thirteenth century are truly epoch making, for after that time there was a constant stream throughout the century of translations and adaptations of both Arabian and Greek mathematical books.⁵ In the field of arithmetic, the introduction into Europe of this new knowledge produced two different effects: (1) its utilization and extension as applied to commerce; and (2) the adoption of the Arabian system of notation in the academic study of mathematics. The first resulted in an extraordinary development in the practical aspects of arithmetic and of

¹ For a critique of this work vide M. Steinschneider in *Zeit. für. Math. etc.* Bd. XXV, supplementheft pp. 59-128.

² Text, Boncompagni, *Trattati d'Aritmetica* pp. 25-136.

³ Cantor, *op. cit.* I, p. 853.

⁴ For text and critique vide Cantor, *Zeitsch. f. Math. u. Phys.* X, pp. 1-16. A similar text of an algorismus from a monastery near Regensburg, composed in the same century is edited by M. Curtze in *Zeitsch. etc.* XLIII, Supplementheft, pp. 1-23. The existence of these manuscripts shows that even in the declining days of the monastic schools some of them at least kept pace with the advance in arithmetical knowledge.

⁵ Wüstenfeld "Die Übersetzungen Arabischer Werke in das Lateinische seit dem 11 ten J. H." *Abhand. König. Gesel. d. Wiss. zu Göttingen*, Bd XXI, passim., esp. pp. 20-38; 50-96.

portions of algebra in the commercial centres of Italy, England and Germany during the next three centuries.¹ The chief representative of this tendency was Leonard of Pisa, the son of a merchant. His voluminous *Liber Abaci* composed in 1202, presented to the world an amount of practical and theoretical knowledge of arithmetic and algebra which may be considered remarkable, even from the standpoint of our modern age.² But the influence of his book on the study of arithmetic in the universities was not perceptible—even in his own country, Italy.³ In spite of its importance from the standpoint of the general history of mathematics, it need not be considered here.

The introduction of the study of arithmetic as a science into the universities was obviously of the greatest importance. Here also a great representative writer can be named. Jordanus Nemorarius, a Dominican monk, did as much to make the science of arithmetic acceptable within the tradition-worshipping mediæval schools as did Leonardo of Pisa to make that convenient accomplishment, popular among the merchants and traders of Europe.⁴ Nemorarius's treatises which concern us here are, (1) *Algorithmus Demonstratus*, a brief elementary treatise on practical computations.⁵ (2) *Arithmetica Demonstrata*, a work on the theory of numbers.⁶ (3) *De Numeris Datis*, a treatise on algebra.⁷ Their abstract and scientific character is fully established by the fact that general symbols are used. All business methods being rigidly excluded, we have in these works the material exactly suited for the academic study of the subject.⁸

¹ Günther, *op. cit.* pp. 131-141; Cantor, *Vorlesungen etc.* Bd. II, p. 110, 216 et seq.; F. Unger, *Die Methodik der practischen Arithmetik* pp. 1-33.

² Cantor, *op. cit.* Bd. II, pp. 3-35.

³ *Ibid.*, 167, 205.

⁴ Cantor, *op. cit.*, II, p. 86, has found manuscripts of Jordanus at Basle, Cambridge, Dresden, Erfurt, Munich, Oxford, Paris, Rome, Thorn, Venice, Vienna, and many places in Southern Germany.

⁵ Printed in 1534. This work was for a long time wrongly ascribed to Regiomantus. Cf. Cantor, *op. cit.*, II, p. 49-61. A. De Morgan, *op. cit.* p. 16.

⁶ Printed in 1514. Cf. A. De Morgan, *op. cit.* p. 10; Cantor, *l. c.*

⁷ Best text edited by Treutlein, in *Zeit. für. Math. u. Phys.* XXXVI, Supplementheft, pp. 127-166.

⁸ The first work mentioned is a brief treatise on practical arithmetic, of

2. SCOPE.

After this survey of the field of arithmetical knowledge which Western Europe possessed at this time, the question suggests itself: How much of this material was actually used in the teaching of arithmetic? An examination of the records of arithmetical instruction in the universities should give the answer.

Turning then to these records we find the following typical conditions: At Paris, mathematics was very much neglected. The requirements for the M. A. degree in 1366 mention vaguely "that the student attend lectures on some mathematical books."¹ Nevertheless, from the fact that Sacrobosco, the author of an algorism based on Jordanus, lectured on mathematics at the University of Paris before 1256, it may be assumed that at least the material contained in the arithmetic of Jordanus was most likely taught before 1366, the date of record referred to.

In Bologna, where mathematics was cultivated much more than at Paris, there was in the faculty of arts a chair in arithmetic. A course on the "algorism de minutiis et integris,"

some fifty-seven pages, explaining the Arabic system of notation, and the methods of operation. Among the latter the author includes the following nine: numeratio, additio, subtractio, duplicatio, multiplicatio, mediatio, divisio, progressio, radicum extractio. The representative character of this book helped to fix this classification, which we find so often in the popular arithmetical books both in Europe and England. Considerable space, about two-fifths of the entire work, is devoted to a consideration of the two kinds of fractions, the "minutiae philosophicae" or "minutiae physicae" i. e., the astronomical fractions, and the "minutiae vulgares," or common fractions. In its treatment of the former the book is especially full. There are in addition a few pages on proportion. The second work is of an altogether different character. The ten books treat successively of the properties of numbers, relations of numbers, of prime and perfect numbers, polygonal numbers, solids, redundant numbers, proportions and similar overrefined classifications. Here, then, numbers are treated very much in the same way as in the work of Boethius. There is, however, as Cantor (*op. cit.* II, p. 61 et seq.) has pointed out, a distinctive scientific value to the work: *It is the first book to use letters as general symbols instead of concrete numbers.* The third work consists of four books of problems in algebra and arithmetic with solutions involving simple and quadratic equations of one or more unknown quantities. Proportion is also involved in the problems.

¹ Rashdall, *op. cit.*, I, p. 437, note 1.

the material of Jordanus's *Algorismus Demonstratus*, was definitely prescribed.¹

The statutes of the University of Prague for the year 1367 require for the M. A. degree the "Algorismus," which, according to a schedule of lectures for that year could be learned in three weeks. Evidently this provision refers to the study of some such book as Sacrobosco's; that is, the elements of practical arithmetic.² At the same university, we find among the requirements for the M. A. degree the study of some arithmetic. Other records show that at this university, the "Algorismus" and the "Arithmetica Accurata" were studied. Obviously the distinction here is between the elements of practical and of theoretical arithmetic.³

The University of Vienna was throughout its career in the middle ages as renowned in mathematics as was Paris in philosophy. Even disputations on mathematical subjects were held there. While data relating to the study of arithmetic at Vienna could therefore not be taken as typical of the conditions obtaining at other universities, nevertheless the information we possess on the subject when taken in connection with the other evidence is illuminating. The lecture schedule for the years 1391-1399 shows that there were given at Vienna different sets of lectures on: (1) "Algorismus de Integris"; (2) "Algorismo de minutiis"; (3) "Computus physicus"; (4) "Astronomical fractions"; (5) "arithmetica et proportiones"; (6) "arithmetica."⁴ When we consider the care that was taken to avoid competition by the duplication of courses we may assume that courses were given on elementary and theoretical arithmetic and algebra—the subject matter of Jordanus's work.

The same distinction between the *Algorismus*, that is, the practical elements of arithmetic, and *Arithmetica*, the theoretical, is further emphasized in the records of the conditions in the fifteenth century. It appears that there were then grades of compensation paid to the lecturer for different kinds

¹ Rashdall, *op. cit.* p. 249.

² Cantor, *op. cit.* II, p. 140.

³ *Mon. Univ. Prag.*, I, 1, p. 56 and p. 77 as quoted in Rashdall, 442 note 3.

⁴ Compiled by Günther, *op. cit.* p. 209, from J. Aschbach, *Geschichte der Wiener Universität im Ersten Jahrhundert ihres Bestehens*, Vide *ibid.* I, pp. 137-168 *passim*.

of lectures on arithmetic. The "Arithmetica" lectures were worth twice the amount of the lectures on the "Algorism," though the number of lectures required in both courses was the same. Again it seems significant that the honorarium for lectures on arithmetic was equal to the compensation for the same number on a kindred theoretical mathematical study—music.¹

At Leipzig, the daughter of Prague, the same conditions prevailed.²

Most significant perhaps is the fact that in the University of Cologne, which was founded in 1389 on the Paris plan, the requirements for the M. A. degree in 1398 are identical with those of Vienna.³

Conditions substantially similar to those described existed at Erfurt, Heidelberg, Freiburg, Oxford, and even in the Italian universities, Padua and Pisa, where the work of Leonardo of Pisa had no influence even in the fifteenth century.

The evidence produced indicates pretty clearly the scope of the instruction in arithmetic in the European universities. Since the material of this instruction was apparently identical with the contents of the three described books of Jordanus Nemorarius, we are justified in inferring that the scientific knowledge of arithmetic at that time was fully represented in the university instruction and nearly all of it was prescribed for graduation in the M. A. degree.

3. TEXT BOOKS.

Having determined the character of the contents of arithmetical text books of this period, we may proceed to an examination of the texts used at the universities.

(1) First among these in importance since it was the most widely used of all for some three centuries, is that of the English-

¹ Günther, *op. cit.* 210, 211; Cantor, *op. cit.* Vol. II, p. 140, 174 et seq.

² Günther, *op. cit.* p. 215; Cf. Hankel, *op. cit.* p. 357. At Leipzig the Algorism, but no other mathematical subject, could be "heard" from a Bachelor. Vide "Tabula pro gradu Baccalauriatus" in Zarncke "Die Urkundlichen Quellen zur Geschichte der Univ. Leipzig, etc.," *Abhandl. der Kön. Sachs. Gesell. der Wiss. Phil. Hist. Class.*, Bd. II, p. 862. This fact re-enforces the argument that the algorismus instruction was only an introductory subject.

³ Vide De Bianco, "Statua Facultatis Artium," *Die Alte Univ. Köln*, Anlage II, pp. 438-443; cf. Hankel, *op. cit.* p. 357; Cantor, *op. cit.* II, p. 442.

man, John Holywood, known as Sacrobosco, whose *Tractatus De Arte Numerandi* or *Algorismus* was reprinted a number of times under various names.¹ It is but an excerpt from Jordanus's *Algorismus Demonstratus*. It contains the rules of arithmetic without the demonstrations or illustrations, and fractions are omitted altogether. The work is in fact barely more than an explanation of the nine operations of arithmetic as laid down by Jordanus. The rules of multiplication are given in verse. The character of the work readily determines its place in the curriculum. It was simply used as a guide—a text from which the elements of arithmetic were taught before the beginning of the more pretentious study of theoretical arithmetic.²

(2) What Sacrobosco did to introduce Jordanus's *Algorismus* into the universities, other writers did for his other two theoretical works. One of these is Thomas Bradwardinus (1290–1349), whose *Arithmetica Speculativa* covered the ground of the advanced arithmetic of Jordanus.³

(3) Sacrobosco and Bradwardinus, then, were the adapters of Jordanus's practical and theoretical arithmetic. In the same way Nicolas Oresmus, a student, and later, in the middle of the fourteenth century, a teacher at the University of Paris, made available Jordanus's arithmetical and algebraic material, especially the portions dealing with fractions and the syncopated algebra. His *Algorismus Proportionum*⁴ was unmistakably

¹ First printed in Paris 1496. Among other titles which this work bore are *Opusculum de praxi numerorum quod algorismus vocant*, Paris 1511; *Algorismus domini Joannes de Sacrobosco*, Venice 1523. Cf. A. De Morgan, *op. cit.* pp. 13, 14; Günther, *op. cit.* p. 176 et seq. Manuscript Copy X510 H74, pp. 211–222, Library of Columbia University, New York.

² This is evident from the existence of commentaries on Sacrobosco's work, one of which by Petrus Philomen de Dacia is described by Cantor, *op. cit.*, II, p. 90 and Günther *op. cit.* p. 167, note 2.

³ Printed at Paris and Vienna in 1495 and 1502 respectively. Cf. Cantor, II, p. 113. A. De Morgan, *op. cit.* p. 11. The treatise on proportion was abridged by Albert of Saxony towards the close of the fourteenth century. It was the text book of the lectures on proportion at most of the Universities, the work of Jordanus being too hard because of its symbolic notation. Cf. Rashdall, *op. cit.* I, p. 442, note 3.

⁴ Text, critically edited and annotated, in *Zeitsch. für Math. u. Phys.*, XIII, Supplementheft pp. 65–73. For a brief summary of Oresmus's didactic and other mathematical works, vide M. Curtze, *Die Mathematischen*

based on the work of Jordanus. That treatise is more than a mere work on fractions and proportions. In fact in the use of fractional exponents Oresmus marks an advance on his source.¹

(4) John De Muris, another French mathematician of the century, labored to simplify Boethius and Jordanus. His *Arithmetica Speculativa* was based on these authors. Its wide use is attested by the large number of editions in existence now. It was a standard work on theoretical arithmetic.²

(5) Johann von Gmünden's *Tractatus De Minutis Phisicis* is the typical text book used in the German universities during the fifteenth century. The author, in his day a famous professor at the University of Vienna, was the first teacher in Europe who lectured on mathematics as a specialty. Before his time, as is well known, it had been the custom to have the professors lecture in turn on the different subjects in their respective faculties. Johann von Gmunden taught at Vienna from 1412 to 1417, both the "Algorismus de Integris" and the "De Minutiis." In his lectures he used the texts popular in his day. On integral arithmetic he read Sacrobosco; on fractions some current commentator on Jordanus, and on astronomical fractions his own "Tractatus de Minutiis Phisicis."³

(6) Johann von Peurbach's *Algorismus* "for the student" was widely used in Germany in the generation following the activity of Johann von Gmünden. The popularity of this book was due to the fact that its author was famous as the successor of Johann von Gmunden at the University

Schriften des Nicolas Oresmus. The numerous mss. still extant show its wide use. Like the work of Bradwardinus, it was undoubtedly the text book on the subject of proportion at the German universities.

¹ The three books of the work are logically arranged. The first treats of the definitions of fractions in which all the rules are given in symbolic terms; the second gives concrete illustrations and problems applying the rules, while the third book deals with geometrical proportions. The essential similarity of this work to that of Bradwardinus *Tractatus de Proportionibus* may be noticed as showing the identical use by these two contemporaries of the same original source—Jordanus. Cf. Cantor, *op. cit.* II, p. 137.

² Günther, *op. cit.* p. 183, note 1; Cf. A. De Morgan, *op. cit.* p. 3, 11.

³ Printed in 1515. Cf. A. De Morgan, *op. cit.* p. 11. Cantor, *Vorlesungne etc.* II, 177; Günther, p. 232 et seq.

of Vienna. As a text book the treatise was an improvement on Sacrobosco, whose work it aimed to supplant.¹

(7) We may close our examination of the text books of this period with the mention of the *Algorismus de Integris* of Prosdocimo de Beldemandi, who, in 1410, lectured on the subject at Padua. This text, in all respects similar to that of Sacrobosco, shows that even by the middle of the fifteenth century the universities of Italy had not been influenced by the work of Leonardo of Pisa. Apparently at this time they still followed the special path of what may be called academic arithmetic.²

With the advance in the knowledge of universal arithmetic—the term applied to algebra—there was a tendency, towards the end of the fifteenth century, to put the elementary arithmetic and proportion outside of the course of requirements for the M. A. degree. This tendency explains the significance of a notice relating to Heidelberg for the year 1443. The study of “Algorism” and “De Proportionibus” is there put in a class of electives “quos non oportet scholares formaliter in scolis ratione alicuius gradus audivisse.”³ These lectures, for which fees were charged, were evidently given as an extra course or to help students to “brush up.” Thus the requirements in arithmetic at the universities had appreciably risen since the days of the fourteenth century.⁴

In the present chapter the aim has been not only to present a brief account of the character and scope of the instruction in arithmetic as one of the studies in the curriculum of the seven liberal arts, but also to give the reader an insight into the nature of the evidence on which the writer has based the views stated in the introductory paragraphs. Although at times the varied and elusive character of the data gathered almost defied analysis, still the inquiry has clearly established

¹ Printed in 1492 as *Opus Algorithmi Jucundissimum*. For the other editions vide Günther, *op. cit.* 237 and A. De Morgan, *op. cit.* p. 11.

² Published in 1483 and 1540 at Padua. Vide Favaro in *Buletino etc.*, Boncompagni, T. XII, p. 60.

³ Quoted in Rashdall, *op. cit.* I, p. 440 note 3.

⁴ In the beginning of the sixteenth century it was customary to publish arithmetical treatises containing all these separate treatises in one. For a description of some of these vide A. De Morgan, *op. cit.* pp. 10, 11.

the historical continuity of the study of arithmetic in the scheme of mediæval higher education. There can be no doubt that at all times mediæval schools taught all that their respective generations knew of arithmetic; that the teachers of arithmetic in the schools were often the famous mathematicians of their day; that this teaching, since it kept pace with the increase in the knowledge of the subject, was progressive in character, and that at no time, not even in the barren generations at the close of the middle ages, when the scholastic education had outlived its usefulness, did arithmetic cease to be a subject of study in the arts faculties of the mediæval universities.

CHAPTER VIII.

Geometry.

In the preceding chapter a detailed analysis was given of the general character of the instruction in the Quadrivium, especially as regards arithmetic. But the same general conclusions there reached apply also to geometry. We may assume that geometry was very generally taught in both the pre-university and the university periods, and that the scope of the instruction expanded *pari passu* with every increase of geometrical knowledge.

In this chapter then it only remains for us to indicate the approximate amount of knowledge of the subject at the various periods, and to give a brief description of the text books used. As in the case of arithmetic, three periods of geometrical teaching in the middle ages may be distinguished: (1) The era before Gerbert; (2) The era between Gerbert and the thirteenth century; (3) The era from the thirteenth century to the rise of Humanism.

FIRST PERIOD.

Up to the end of the tenth century, the age of Gerbert, a knowledge of geometry in our sense of the term hardly existed in Western Europe. In fact the term seems to have been used in its etymological meaning and not in the sense the Greeks understood it. Neglected by the Romans, who cared only for its practical application, surveying, it can hardly be wondered that no geometry worthy of the name of a science was transmitted to the early middle ages.¹ The character of the geometrical text books in use till the time of Gerbert proves this assertion. These were those of Capella, Cassiodorus and Isidore of Seville.

Capella's text is for the most part a brief statement of geography, the location of historical places, and some kindred facts. Only at the end of the work do we find a few definitions

¹ Cantor, *Vorlesungen etc.* Vol. I, p. 522. More in detail Cantor *Die Römischen Agrimensoren und ihre Stellung in der Geschichte der Feldmesskunst*, Leipzig 1875.

of lines, triangles, quadrangles, the circle, the pyramid, the cone. There is nothing in his work of geometry properly so called, or even of surveying.¹ Cassiodorus's chapter is no better.² Isidore of Seville's treatment is similar.³

Though these were apparently the only text books on geometry in use up to the time of Gerbert, it is undoubtedly true that a knowledge of the method of approximately computing the area of a triangle, quadrilateral and circle had come down to the middle ages through the works of the "Gromatici" —the surveyors, of the later Roman empire.⁴

But if the mathematical science of geometry was neglected, geography and cosmography were introduced to supply the deficiency. As available material for these subjects was abundant, they were assiduously cultivated. Most of the twenty books of Isidore's *Etymologiae* were devoted to *Natur-kunde*.⁵ Rabanus Maurus's *De Universo* was another compilation of similar character.⁶ Compendia based on the geographical works of Pliny and others were used in large numbers during this period and references to the study of these works as a part of the quadrivium are very common.⁷

SECOND PERIOD.

Passing to the time of Gerbert we find a small, but comparatively speaking, very important increase in the stock of geometrical knowledge. In this period, through Gerbert's "find" of the copy of Boethius's books on geometry and through his discovery of the "codex arcerius," a morsel of Euclidian geometry and some fragments of the works of eminent surveyors of the later Roman Empire came to be used in the schools of Christendom.⁸ But neither in quantity nor in quality was this newly discovered geometrical knowledge

¹ *De Nuptiis etc.*, Eyssenhardt ed. pp. 194-254.

² Text, Migne, Vol. LXX, c. 1212-1216.

³ Text, Migne, Vol. LXXXII, c. 161-163.

⁴ Cf. Hankel, *op. cit.* 312 et seq. Günther, *op. cit.* 114.

⁵ Günther, *l. c.*

⁶ *De Universo Libri Vigintiduo*, Migne, CXI, c. 9-612 passim esp. Libri VI, VII, VIII, IX, X.

⁷ E. g. Pez, *Thes.* III, 3, 630. For similar references vide Specht, 143-149.

⁸ Cf. Specht, *l. c.*; Günther, *op. cit.* 73 et seq., 115 et seq.

very valuable. The alleged Boethian work which Gerbert found¹ consists of two books. The first is based entirely on Euclid and consists of the "enunciations" of Books I and III, including definitions, axioms and scholiae; of some of the propositions of Books III and IV, and of the complete proofs of the first three propositions of Book I. These, in the words of the author, are given "ut animus lectoris ad enodationis intelligentiae accessum quasi quibusdam gradibus perducatur." The second book consists of calculations of areas of geometrical figures. These, Chasles thinks, are based almost entirely on the works of the Roman gromaticus, Frontinus.²

Comparing this amount of geometrical knowledge with the traditional text of Euclid as transmitted by Theon, we find that Boethius's geometry consists of the Euclidian definitions, the theory of triangles and quadrilaterals, and of some theories of the circles and polygons. In addition to this we find his own original demonstrations of the following problems:—(1) To construct an equilateral triangle on a given length; (2) To draw from a given point a straight line of a given length; (3) To cut off a smaller line from a larger one.

Gerbert it appears came into possession of all the available scraps of geometrical knowledge, both theoretical and practical,

¹ The geometrical works of Boethius have for years formed a "Streitfrage" among the historians of mathematics. The fact that between the first and the second book in the oldest Boethian manuscript in existence dating from the tenth century, the use of "apices," the abacus and columnal multiplication and division are explained, has started the controversy as to the origin of the abacus and the introduction of what we call the Hindu-Arabian notation. In this controversy, the foremost historians of mathematics, Kastner, Chasles, Martin, Friedlein, Weissenborn and Cantor, to say nothing of others of less renown, have taken sides, some of them going to the extreme of denying Boethius's authorship of the two books in geometry. The weight of authority (Cantor, *Vorlesungen etc.*, I., 540-551) seems to confirm Boethius as the author of the geometrical portion of the manuscript. On the point, however, with which we are concerned, all are agreed. For whether the original works attributed to Boethius are his own or the works of some anonymous author, certain it is that these two books on geometry were not used till Gerbert's time. Text of the Boethian geometry in Migne, *Pat. Lat.* LXIII, cols. 1307-1364.

² Chasles, *Geschichte der Geometrie*, Sohncke's trans. p. 524. The latest comparison of the sources of Boethius is made by Weissenborn, *Zeit. f. Math. u. Phys.* Vol. 24, 1879, and supports the opinion that Boethius used an excerpt from Euclid and not the original.

and this material formed the basis of his own work on geometry.¹ His text book has not impressed scholars as being of any originality, and may be taken to represent the sum total of geometrical instruction in the schools before the end of the thirteenth century. Gerbert's book and works of a similar character soon replaced in a large measure the geography and cosmography, which for the want of any real knowledge of geometry, had passed under that name up to Gerbert's time.²

THIRD PERIOD.

As in the case of arithmetic, the twelfth and thirteenth centuries form the transition period in the development of the study of geometry. As one would expect, the complete geometry of Euclid was one of the many mathematical works which, in the form of translations from Arabian sources, reached Western Europe during this period. As a matter of course this increase of geometrical knowledge was very soon appropriated by the universities and made a part of the amplified course in geometry.

After the labors of the Englishman, Adelhart of Bath, who in 1120 translated Euclid from the Arabian text, and of Gerhard of Cremona who in 1188 made another translation,³ a knowledge of Euclidian geometry may be said to have been fairly introduced in Western Europe. It was then that the geometrical works of Boethius and Gerbert were entirely discarded in the universities. This left the purely theoretical side of the science as a subject of the curriculum.

We have sufficient evidence that Euclid, as adapted in such works as the *De Triangulis* of Jordanus Nemorarius, was taught throughout the university period till the Renaissance.⁴

¹ Text in Bubnov, *Gerberti opera Mathematica* pp. 48-97.

² Cf. Günther, *op. cit.* p. 115 et seq; Cantor, *op. cit.* I, 809-824; Gow, *op. cit.* pp. 205-206.

³ Cf. Jourdain, *Recherches sur les Traductions Latines D'Aristote*, 1st. ed. Paris, 1819, p. 100. Hankel, *op. cit.* p. 335. Cf. Weissenborn, in *Zeit. f. Math. u. Phys.* vol. XXV, supplementheft pp. 141-166. This work passed throughout the next century as an original translation of Euclid by Giovanni Campani, and was the first of the Latin editions of Euclid to be published, in 1482. For reference to Gerhard, vide Ball, *op. cit.* p. 172.

⁴ For text of Nemorarius vide Curtze's ed., Thorn, 1887. Cf. Cantor, *op. cit.* I, pp. 670, 852 notes 1 and 2.

Nearly all the lists of requirements for the M. A. degree include five or six books of Euclid. Bologna, Prague, Vienna, Liepzig, Padua, Pisa and Cologne invariably demanded that amount.¹ Even the university of Paris, which was rather exceptional in its lack of interest in mathematics, required in the later middle ages the full six books of Euclid, not the first three as is generally assumed.² It is therefore certain that the university candidate for the degree in arts mastered the following minimum of geometrical knowledge of the Euclidian text:

The theory of triangles and quadrilaterals; the various applications of the Pythagorean theory to a large number of constructions; the theorems of the circle; the theorems of the inscribed and circumscribed polygons; proportion explained geometrically; similarity of figures. Add to this the theory of numbers (the contents of the seventh, eighth, ninth and tenth books of Euclid), which was studied as a part of the theoretical arithmetic, and we are forced to the conclusion that a fairly complete knowledge of plane Geometry was imparted as a part of the Quadrivium.

Additional courses on the theory of coördinates were given in the universities in the fourteenth and fifteenth centuries.³ This advanced geometrical teaching prepared the way for the development of the analytical geometry of Descartes in the sixteenth century.⁴ The same statement may be made in regard to the study of perspective, on which at some universities lectures were given as a part of the Quadrivium.⁵

¹ Cf. Rashdall, *op. cit.* I, pp. 250, 442; Hankel, *op. cit.* 356 et seq.; Günther, *op. cit.*, 199, 209 et seq., 215, 217, 281. The statement of Compayre (*Abelard and the Origin and Early History of Universities*, p. 182,) that only the Euclid of Boethius was taught at the universities is erroneous; the statutes of Vienna for 1389, to which the author refers and quotes, expressly say "five books of Euclid." Obviously then this cannot mean the Boethian geometry which consisted of only two books. Vide Kollar *Statua Universitatis, Viennensis* I, 237, as quoted in Mullinger, *The University of Cambridge*, 351.

² Kästner, *Geschichte der Mathematik* I, p. 260.

³ This subject was developed by Nicholas Oresmus in his *Tractatus de Latitudinibus Formarum*, and *Tractatus de Uniformitate et Deformitate Intensionum*.

⁴ Cf. Tropfke, *Geschichte der Elementar-Mathematik* II, p. 407, et seq.; Günther, *op. cit.* pp. 181, 199, 210, 211.

⁵ Günther, *l.c.*

That the old Greek mathematics introduced into Western Europe through the Arabian influences stimulated even original speculation in geometry can be seen from the works of Leonardo of Pisa, *Practica Geometria* (1220); Jordanus Nemorarius's *De Triangulis* (c. 1237); and Thomas Bradwardinus's *Geometria Speculativa* (c. 1327); and of Oresmus' *Tractatus De Latitudine Forarum*. All of these productions are acknowledged to merit high scientific standing even today.¹ It is true, probably, that these last named works, marking as they do a departure from Greek modes, did not find their way into the mediæval curriculum.² But this neglect to assimilate the new ideas and to incorporate them into the curriculum can hardly be cited as a proof of the perfunctory interest of the time in the mathematical teaching.³

Since even in our time, after five centuries of phenomenal development in geometrical studies, the question as to the value of Euclid as a text book is still an open one,⁴ it would seem unreasonable to expect that the mediæval universities—institutions in an age worshipping tradition—should have been in their age more ready to discard Euclid than are our schools today.

¹ Cantor, *Vorlesungen etc.* Bd. II, pp. 35-40, 73-86, 113-118, 128-137. Cf. Curtze in *Zeitsch. f. Math. u. Phys.* XIII, supplementheft pp. 79-104.

² Günther, *op. cit.* p. 162; Cajori, *op. cit.* p. 134.

³ This is charged by Hankel, *op. cit.*, p. 349; and Compayré, *op. cit.* p. 182.

⁴ Cf. D. E. Smith, "*Teaching of Elementary Mathematics*" p. 229, notes 1 and 2; R. W. Ball, "*History of Mathematics*", pp. 56-64.

CHAPTER IX.

Astronomy.

Throughout the middle ages astronomy was perhaps the most popular subject of the Quadrivium. The reasons are not far to seek. In the first place, it was intimately connected with at least two other subjects of the Quadrivium—arithmetic and geometry. Again, the every day life of man was more affected by the practical side of the subject than it is today. Popular interest in celestial motions was obviously greater in the epoch of the sun dial than it is in the age of the watch and the compass. Besides, the peculiar mediæval attachment to astrology also helped to promote the study of astronomy. The potency of this influence can hardly be over-estimated. The great Keppler himself, in the beginning of the seventeenth century, characterized astrology as the “foolish daughter of astronomy,” and he added that “but for the existence of the daughter the mother would have perished of hunger.”¹ The importance of the problem of calculating Easter was likewise a factor stimulating an interest in astronomy. While the computation itself could have been made in a purely mechanical way, it was inevitable that the deeper astronomical problems involved in the operation should also be studied; as a matter of fact material but remotely connected with *computus* proper found its way into these books.²

Then again, “The Philosopher,” had written a work on the heavens. To the schoolmen the fact that Aristotle himself was the author of a work on astronomy must have appeared a cogent reason for the retention of the subject in the curriculum of the arts faculty. It is therefore no wonder that Aristotle’s *De Cælo* should have formed a part of the regular requirements for the M. A. degree in most of the universities. Lastly, the harmony of the geocentric doctrine with

¹ Quoted in Wolf, *Geschichte der Astronomie*, p. 82.

² A striking instance is the *Liber de Computo*, of Rabanus Maurus, where in the ninety-six chapters we find fully forty-two chapters containing purely astronomical material in no way related to the Easter problem. Vide text in Migne, vol. CVII, cols. 669–726.

the theological dogmas of the age¹ must have powerfully stimulated the study of astronomy in the middle ages. In an age when the idea of universality in Church and State dominated man's mind, how enticing must have been the study of a subject so universal in its character. The fascination which the marvels of modern science exercise over our age can hardly be compared in its influence with the appeal which the wonders of astronomy then, even though the science was still in the stage of infancy, made to the mind of the mediæval student.

With these causes at work we are not surprised to find a continued attention to the study of astronomy throughout the middle ages. The general conclusions as to the instruction in arithmetic and geometry apply with greater force to astronomy. There is abundant evidence to show that in the pre-university period the teaching was fully up to the standard of the actual astronomical knowledge of the age; that in the university period both elementary and advanced astronomy were required in nearly all the universities, during the thirteenth, fourteenth and fifteenth centuries;² that during the university period the amount of advanced teaching in astronomy was very great; that the number of eminent astronomical teachers and writers was large; that all the Arabian material was assimilated; that in order to keep up with the advance of the subject improved astronomical tables were continually produced; and that original ideas in this field were not rare. One would have to discard the traditional ideas of the culture of the middle ages before he could fully appreciate the importance of the fact that the manufacture and sale of astronomical instruments became no insignificant a trade towards the end of the fifteenth century.³

¹ E. g. Peter Lombard, *Sent.* II, I, 8, IV, 1, 6, 7.; Thomas Aquinas, *Summa Pars*, I, Qu. 70 quoted in White's *Warfare of Science and Theology*, Vol. I, p. 117.

² H. Suter, *Die Mathematik auf den Universitäten des M. A.* Part III, passim, especially pp. 64, 67, 73, 77, 79, 89; Günther, *op. cit.*, 184-190, 199, 210, 215, 217; Cf. table of lists of lectures in the Faculty of Arts at Prague, Vienna, Erfurt, and Ingolstadt, in Barnard, *Superior Instruction* p. 159, 160.

³ H. Suter, *op. cit.* pp. 65-95, gives an extended account of the teaching activity in elementary astronomy in all the important universities. Cf. Günther, *op. cit.*, 146 et seq; Wolf, *op. cit.*, pp. 197-218. Maedler, *Geschichte der Himmelskunde*, pp. 113-146 treats of the advanced astronomical work at the

In respect to the quantity of astronomical knowledge possessed by the middle ages, we can again divide our period into three divisions: 1st, the period ending with the twelfth century; 2d, the twelfth century; 3rd, the thirteenth to the sixteenth century.

FIRST PERIOD.

In this period, as we might expect, and the text books in use show, that the amount of knowledge of mathematical astronomy was meagre indeed. Among the most noted of these text books, the following may be mentioned:

(1) Capella's section in *De Nuptiis Philologiae et Mercurii*. This chapter contains besides the obvious facts of elementary astronomy, much allegorical material on the constellations. From a study of this text a schoolboy would learn little more than is contained in our school geographies under the pretentious title of "Mathematical Geography."¹

(2) Cassiodorus's compendium of astronomy defines the "lex astrorum" as the "disciplina quae cursus coelestium siderum et figuras omnes contemplatur et altitudines stellarum circa se et circa terram indagabili ratione percurrit." He however does not go beyond the mere definition of a few astronomical terms.²

(3) Isidore of Seville's treatment of the subject is more adequate than that of Cassiodorus. Defining astronomy in substantially the same terms, he widens its scope to include the study of the effect of the heavenly bodies on the earth. His definitions are fuller and clearer. His enunciation of the geocentric theory is significant.³ But he devotes one-third of the space to a mere enumeration of the names of stars; astrology he considers a part of astronomy, having its scientific side (observation and study of the stars), and a superstitious side,

end of the fourteenth and the beginning of the fifteenth centuries. Cf. Hankel, *op. cit.*, p. 349.

¹ Text *De Nuptiis etc.*, Eyssenhardt ed. pp. 296-330.

² Text, Migne, vol. LXX cols. 1216-1218

³ It is supposed that Isidore of Seville was the one whose ideas influenced the mediæval world to adopt the Greek view of the sphericity of the earth, notwithstanding the fact that such an opinion was contrary to the views held by the patristic theologians. Cf. White, *op. cit.* I, pp. 88-99.

namely, the belief that the course of the stars has an influence on the fortunes of man.¹ Isidore's text book on astronomy does not exhaust the material on the subject known to him. Other portions of the *Etymologiae* contain matter of cosmological and cosmographical character. In these he preserved in convenient form abstracts of Roman works on natural history.²

(4) The Venerable Bede's *De Natura Rerum* and *De Temporum Ratione* were the most important astronomical text books of the period. These works are of intrinsic value even from the standpoint of modern criticism, and fully reflect the great erudition of this remarkable mediæval scholar. They contain in ample form all the material required for a complete mastery of the methods of Easter reckoning and of chronology. By adding to this considerable matter from the works of Pliny, the Venerable Bede really prepared a most useful treatise on astronomy and physical geography. The timeliness of the book as well as the fame of the author account for its widespread use during many centuries.³

(5) Alcuin's popular treatise, *De Cursu et Saltu Lunae ac Bissexto* was, as its name shows, limited in scope, and dealt only with the astronomy of the *computus*.⁴

(6) Rabanus Maurus's *Liber de Computo*, partaking of the twofold character of a treatise on the *Computus* and on astronomy, fully sustained the traditional interest in astronomy which the Venerable Bede and Alcuin had fostered.⁵

The effect of the works of Bede, Alcuin and Rabanus was to establish a minimum of astronomical teaching in the schools during this period. This minimum far exceeded the mere technical knowledge required to calculate Easter day. It included among many other kinds of astronomical information, the study of the course of the sun, the moon, the stars

¹ Text, Migne, vol. LXXXII, cols. 169-184.

² *Ibid.* Lib. V., Ch. XXVIII-XXXIX, Lib. VI, Ch. 17; Lib. XIII.

³ Günther, *op. cit.* p. 8, note 1; text in Migne, vol. XC, cols. 187-277, 293-578.

⁴ Migne, vol. CI, cols. 979-1001.

⁵ Rabanus Maurus, *De Cleric. Inst.*, III, p. 25; text in Migne vol. CVII, col. 403. His *De Universo* has much astronomical material, especially Lib. IX, X; Cf. *supra* p. 114.

and the changes of the season. But the more prominent schools went far beyond this minimum. Books on astronomy famous in the days of the later Empire, such as the works of the elder Seneca, the *Phænomena et Prægnostica* of Aratus, the *Astronomicon* of Manlius, the *Pæticon Astronomicon* of Hyginus, and the *Catasterismi* of Eratostenes, were much used in the schools of the pre-university period.¹ To be sure much of this astronomical *Lehrstoff* betrayed the fantastic reasoning of an age in which tradition and the "littera scripta"—not necessarily ecclesiastic—was of more authority than individual observation. It must be granted also that fragments only of the body of knowledge on which the Ptolemaic theory rested were known in Western Europe, and that therefore no adequate theory of the planets was then taught in the schools. Granting the narrow limitations of the age, it is nevertheless true that within these limits there was a wide interest in learning, a spirit of acquisition which in its persistency can be fairly compared with some phases of the modern scientific spirit. The fact that astronomical instruments were not infrequently used, and that even in the beginning of the eleventh century a text book was composed on the use of the astralobe, sufficiently indicate the character of the instruction in this subject. The existence of such a text book at that time seems more significant of the temper of the age than the tradition so often cited to prove the ignorance of the middle ages that Gerbert's use of terrestrial and celestial spheres was in his day taken as a sure proof of his alliance with the devil.²

¹ These text books contain a great deal of the varied Greek astronomical material. Several editions of each of the works mentioned were issued after the invention of printing. Vide Wolf, *op. cit.* pp. 192, 201, 202. For other specific references to cosmological writings of the period vide Peschel, *Geschichte der Erdkunde* pp. 93-100.

² Cf. Günther, *op. cit.* pp. 75-78; Specht, p. 138; and Günther, *Didaktik der Mathematischen Geographie*, p. 6 et seq. The work is: *B. Hermann Contracti monachi Augiensis de utilitatibus astrolobii libri II* in Pez, *Thesaurus Anecdotorum novissimus* III, Pt. II, cols. 110 et seq. The Gerbert tradition is one of the commonplace historical anecdotes. Hermannus Contractus wrote also a few other text books on astronomy, viz: *De Eclipsis* and *De Computo*. Among the minor astronomical text book writers of this period three others may be mentioned: William of Hirschau, the author of *Astronomicarum*

SECOND PERIOD.

In the field of astronomy the twelfth century was one of progress. The gradual assimilation during this period of what for a long time had been the lost knowledge of the world is more marked in the case of astronomy than in the fields of arithmetic and geometry. The elaborate treatise of Ptolemy, summarizing the astronomical knowledge of the Greeks, which under the name of *Almagest* had found its way into Arabian schools of the east and west in the ninth and the succeeding centuries, was one of the first books translated from Arabic into Latin. As early as 1116 Plato of Tivoli made a translation of Al Battani's version of the *Almagest*.¹ From both Italy and Spain volumes upon volumes of astronomical material found their way to other intellectual centres of Europe so that by the middle of the thirteenth century, the Arabic knowledge of astronomy was fully in the possession of Europe.

THIRD PERIOD.

Such an increase in astronomical material from Arabian sources made it possible to take up the study scientifically, supplemented by Aristotle's *De Coelo*, which the universities of Europe possessed by this time.

But this material to be made available for the young university student, had to be worked over, for obviously the immature pupil could not be plunged into the study of either the Arabian or Aristotelian astronomy, nor could he make much use of astronomical tables without some preliminary instruction. The needs of the hour from the standpoint of the mediæval teacher were then, first, a brief treatise introducing the pupil to the elements of astronomy; second, a succinct account for school use of the Ptolemaic theory; third, some treatise on the use of the many astronomical instruments.

The need for such treatises was fully met. The demand for an elementary treatise on astronomy, one that while suitable

Institutiones (1080); Sibertus Gemblacensis, who wrote a *De Computacione Temporum*, and Clemens Langton, the author of the *De Orbibus Coelestibus*. (Vide Maedler, *op. cit.* I, p. 106). On William of Hirschau's work vide Prantl, *Sitzungs. der Königl. Bay. Akad. der Wissen. zu München*, Bd. I, pp. 1-21.

¹ Berry, *History of Astronomy*, 76-85; Ball, *op. cit.* 172; Wolf, *op. cit.*, 193-200, 203-208; Günther, *op. cit.* 146-149.

for school use should also embody the results of the "latest scholarship of the day," as we would say, was filled by the historic *Libellus de Sphaera* of Sacrobosco. The elementary character of its contents is indicated in the author's own introduction.¹ The immense popularity of this elementary book and its wide use for over four centuries (some sixty different editions of the work have been found),² have often been cited to disparage the amount of astronomical teaching at the universities. It must, however, be remembered that its use was only as an elementary introductory volume, and that in all the universities a course on the "sphaera," the subject treated in Sacrobosco's book, was invariably followed by an advanced course on theoretical astronomy.³ As an introductory treatise it well fulfilled its function. Even long after the triumph of Humanism, Melanchthon, the prince of teachers of the Reformation, could not improve upon it, a fact of which his laudatory introduction to an edition of the book in 1531 gives ample proof.⁴

In the thirteenth century the need also for an advanced text book on astronomy embodying the scientific claims of the Ptolemaic theory was fully met. Two Italian writers, Gerardo de Cremona, and Giovanni Compani de Novarro, succeeded in compiling improved adaptations of the large number of existing imperfect Arabian editions of the *Almagest*. Their works entitled *Theorica Planetarum* became standing require-

¹ "Tractatum de sphaera quatuor capitulis distinguimus. Dicturi primo compositionem sphaerae, quid sit sphaera, quid sit eius centrum, quid axis sphaerae, quid sit polis mundi, quot sint sphaerae et quae sit forma mundi. In secundo, de circulis ex quibus sphaera materialis compositur et illa super coelestis (quae peristam imaginatur) componi intelligitur. In tertio, de ortu et occasu signorum et de diversitate dierum et noctium et de divisione climatum. In quarto, de circulis et motibus planetarum et de causis eclipsium." *Libellus de Sphaera* p. 1, Melanchthon's edition of 1531. The book fully illustrated, was composed about 1250 and was also known as *De Sphaera Mundi* and *Sphaera Materialis*. The Library of Columbia University, New York, has also the following printed editions of the work; Wurtemberg, 1545, 1550, 1558, 1578; Venice 1490, 1537, 1564, Paris 1500. It also possesses an interesting ms. edition annotated by John De Muris.

² Wolf, *op. cit.* p. 209.

³ Vide note 2 p. 120.

⁴ Vide Sacrobosco, *op. cit.* Praefatio Melanchtonis.

ments at the universities and represented the advanced astronomical course for the M. A. degree.¹

The third need, that is, the demand for a text book on instruments, was met by a number of writers. Sacrobosco was the author of a popular *De Astrolobio*, another author, Robert Capito, an Oxford teacher of mathematics who died in 1253, also wrote a treatise, *De Astrolobio*, while Pietro D'Abano was the author of an *Astrolobium Planum*. These books became texts of lecturers on the astrolobe or the planosphere in the universities.²

The three different kinds of books on astronomy continued in use in the universities throughout the fourteenth century, and eight decades of the fifteenth. During this time, then, the mediæval graduate in arts was required to have mastered texts treating of elementary and advanced astronomy and of the use of astronomical instruments.

In astronomy, as in the other subjects of the Quadrivium, there existed the same desire to assimilate into the curriculum the new knowledge. When, therefore, towards the end of the fifteenth century Peurbach and Regiomontanus, astronomers famous in their generation, had advanced the science by their perfected translation of the *Almagest* and by their improved astronomical tables, the universities, as we would say, readily raised the requirements in astronomy. The *Theorica Planetarum* and the various epitomes of astronomy, which these writers composed were soon made the standard texts on the subject.³

Summarizing the results of the investigation concerning the scope and character of the teaching of astronomy in the mediæval curriculum, the following general conclusions may be stated: (1) The instruction in the subject continued throughout

¹ Rashdall, *op. cit.* I, pp. 250, 442. Suter, *op. cit.* pp. 76, 77.

² Suter, *op. cit.* p. 67; Wolf, *op. cit.* pp. 160-166; where the history of the use of the astrolobe and planosphere is briefly traced.

³ Berry, *op. cit.* pp. 86, 87, 94; Wolf, *op. cit.* 211. Besides this theoretical treatment of astronomy two practical phases of the subject were taught: Lectures were given on the computus, evidently to meet the practical demand of ecclesiastics who formed such a large number of the student body, and courses on astrology were generally given especially in Italy, where the subject actually formed a part of the curriculum. Vide Suter, *op. cit.* pp. 76, 77, 79; Günther, *op. cit.* pp. 207-264.

the period. (2) During the early middle ages the amount of instruction was not large and was confined chiefly to the teaching of methods of Easter reckoning. (3) With every increase in the knowledge of astronomy the curriculum was broadened, and the newly acquired material was incorporated in the text books in use. (4) The mediæval student at graduation possessed a knowledge not only of the elements of astronomy but also of the scientific basis of the astronomical theories then in vogue. (5) These theories, since the use of the telescope was then unknown, gave a satisfactory explanation of all the astronomical problems of the day, and actually supplied a good working hypothesis to account for all the phenomena with which the men of the time were acquainted.

CHAPTER X.

Music.

The music taught as one of the subjects of the Quadrivium was exclusively theoretical. Studied as a purely speculative science it was entirely mathematical in its character. The mastery of the subject did not require any special aptitude for the art of music. In fact throughout the middle ages the singer or the performer on an instrument was not a musician within the strict meaning of the term.¹ This characteristic mediæval conception of the function of music greatly simplifies the scope of our investigation. As it was only with the Renaissance that the term "musician" began to mean one who possessed a knowledge both of the science and of the art, it is not necessary for the purposes of this inquiry to touch upon the many controversial problems in the general history of the art of music in the middle ages.²

¹ Cf. Ambros, *Geschichte der Musik* II, p. 119 et seq. The tendency of the middle ages to class music as a theoretical science, a part of mathematics, to be studied after arithmetic, has been carefully traced to Roman writers as far back as Cicero. Vide Schmidt, *Quæstiones de Musicis Scriptoribus Romanis imprimis de Cassiodoro et Isidore*. Dissertation 1899. This idea as to what constituted a musician was not merely based on the dicta of the theoreticians like Boethius, who, himself had no knowledge of the art of music; the practical teachers of music, those who had labored to improve the methods of instruction for the needs of church service, took this same view. Thus Aurelian of Rome (IX century) says: "Tantum inter musica distat et cantorum, quantum inter grammaticum et simplicem lectorem . . . Is vero est musicus qui ratione per pensa scientiam canendi non servitio operis sed imperio assumpsit speculationis." Vide Gerbert, *Scriptores de Musica, Sacra*, I, 38, 39. Other practical teachers, such as Huchald, whose work admittedly marks a step in the development of the system of notation, Regino of Prum, Berno, Hermannus Contractus, William of Hirschau, and Guido of Arezzo, accept Aurelian's views as to the scope of music. Vide Brambach, "Verhältniss zwischen Musik theorie und Praxis im Mittelalter" in *Die Reichenaauer Sängerschule etc.* Cf. Ambros, II, p. 40 et seq. The assertion of Williams (*Story of Notation*, pp. 73, 74) that Guido of Arezzo's often quoted stanza on what a musician is, (Gerbert, *op. cit.* I, p. 25) was made in sarcasm, is untenable.

² Among the mooted questions in this field are the following: the extent of the influence of Pope Gregory the Great on the development of the so-called traditional Gregorian music; the extent of the original services of

But although the present investigation is unrelated to the general history of the art of music in the middle ages, facts as to the extent of the practical musical instruction during this period are pertinent, and throw much light on the scientific side of the subject. While it is true that those who had been taught the art of singing were not thereby made musicians in the mediæval sense, and that a knowledge of the theory of numbers was considered of more value to the student of music than a good voice, or the ability to play an instrument, it is nevertheless equally true that in the pre-university period the science of music as a part of the quadrivium was taught best in those schools which had a reputation for good musical teaching in the elementary grades.

In the succeeding paragraphs, therefore, the facts relating to the history of the teaching of the art of singing will be considered for the light they throw on the general position which the subject occupied in the curriculum of the seven liberal arts.¹

GENERAL CHARACTER.

In tracing the history of the first three subjects of the Quadrivium—arithmetic, geometry and astronomy—we found that

Hucbald and of Guido of Arezzo; the development of musical notation; whether there was any harmony in our modern sense in the middle ages. The treatises embodying the latest researches on these subjects are Hugo Riemann's *Geschichte der Musiktheorie* and *Studien zur Geschichte der Notenschrift*. Vide also C. F. A. Williams, *The Story of Notation*, 1903. The work of Franchinus Gaforius, *Thericum opus armonicae disciplinae* seems to be the first text book to represent the new tendency of the Renaissance. This fact possibly accounts for its being the next oldest printed work on music. Cf. J. B. Mathews, *Literature of Music*, p. 21.

¹ On the other hand the influence which the theoretical study of music had on the art cannot be overlooked. A mere comparison of the names that are considered of epochal significance in the development of the art of music with the names of those who are famous as theorizers shows the presence of the same names in both categories. This may be taken as an index of the influence which the theorist had on the art. It would seem, then, that there were in the middle ages a science and an art of music, the former forming a component part of the Quadrivium, and that many of the practical teachers of music like Hucbald, Berno of Reichenau, Oddo, Hermannus Contractus, Guido of Arezzo and Johannes Cotton, were famous theorists at the same time. Cf. W. Brambach, *Das Tonsystem und die Tonarten des Christlichen Abendlandes im Mittelalter* passim. Cf. Riemann, *Geschichte der Musiktheorie*, pp. 50-96; Ambros, *op. cit.*, II, pp. 92-216.

three characteristic periods were common to all of them. Each had an early period in which the knowledge of the subject, though small in amount and intrinsically almost insignificant, was nevertheless sacredly guarded and transmitted to the succeeding generations by means of the instruction in the schools; each had a period in which the chief concern of the age was to assimilate the new knowledge of the subject which was coming into Western Europe from beyond the Pyrenees; each had a period during which the assimilated knowledge was made a part of the curriculum and assiduously cultivated. But in the case of music the situation was different. There was no period of assimilation, which should make progress possible in the succeeding period. Nay, the advance made in the subject was conditioned only upon the abandonment of the theories of the previous age. In this field the work of the middle ages was entirely original and was of value only to the extent to which traditions of the classical period were ignored.¹

Two periods may be distinguished in the history of the subject: one ending roughly with the eleventh century; the other with the end of the fifteenth. In the earlier period music was cultivated essentially to serve the purposes of Christian worship. The inherent conditions and necessities of the church made for improvements in both the theory and practice of the subject. Music, all agree, was at that time, studied only in the schools of the church—monastic and cathedral. The age of popular secular music had not yet come.²

The importance which the study of music had for the church made a knowledge of its elements essential for the education of a priest.³ The patronage which Charlemagne gave to the art was not without its influence. It is therefore not surprising to find that in the ninth, tenth and eleventh centuries an excep-

¹ Advance was only possible after the Greek theories which had been partially and in some instances incorrectly transmitted through Boethius were gradually discarded in the eleventh century. The historians of musical theory seem to agree that the incubus of Boethian music greatly hampered its development throughout the middle ages. Cf. W. Brambach, *Die Musik Literatur des Mittelalters etc.* pp. 13-24; Ambros, *op. cit.* II, p. 41.

² Naumann, *History of Music*, Vol. 1, p. 168 et seq.; Langhans, *Die Musik Geschichte* pp. 11-17.

³ Cf. Rabanus Maurus *De Clericorum Inst.* III, c. 24; Migne, vol. CVII, cols. 401 et seq.

tionally great interest in the subject should have shown itself in many of the monasteries in Western Europe. While the schools of Metz and St. Gall shine out as the most famous for the extent of their teaching of music, those of Soissons, Fulda, Mainz, Treves, Reichenau, St. Amand and many of the other schools mentioned in connection with the teaching of the Quadrivium were likewise famous for their instruction in theoretical and practical music.¹

The evidence of the activity in the teaching of music in this period can be seen in many ways. The numerous incidental references in the scattered accounts of the study of the Quadrivium cited before all show that music was studied along with the other subjects of the course in the seven liberal arts. The number of intensive studies relating to musical instruction in particular schools of the early middle ages shows abundantly an ever increasing amount of teaching in the subject and the great influence of these schools on the theoretical writers.² The vast number of mediæval theoretical text books,—books, good, bad, and indifferent,—which Gerbert and Coussemaker have gathered in their monumental collections covering the ground up to the fifteenth century are eloquent proof of the extent of theoretical musical teaching, if not of its quality.³

In the second epoch, which roughly corresponds to the university period, the speculative side of the study of music was still more emphasized. In all the references to the requirements for the licentiate and the master's degree music is always mentioned. The facts more than warrant the cautious generalization of Rashdall that "Oxford and the German universities seem to have agreed also in requiring Boethius' *De Re Musica* or some other musical work so as to keep up

¹ Cf. Specht, *op. cit.* p. 140; Ambros, *op. cit.* II, 96 et seq. Schubiger, *St. Gallische Sängerschule* passim, esp. p. 86 et seq.

² Brambach, *Die Sängerschule zu Reichenau im M. A.* passim; Schubiger, *op. cit.*, passim.

³ Vide, Gerbert, *Scriptores ecclesiastici de musica sacra etc.* St. Blais, 1784, III, vols.; E. De Coussemaker, *Scriptorum de musica mediævi etc.*, Paris, 1864-1867 IV vols. These two collections by no means exhaust the field. A number of theoretical treatises of the early middle ages have since been found. Cf. W. Brambach, *Die Musik Literatur des M. A. (500-1050)*, 1883 p. 5 et seq.

the theory that the arts course consisted of the complete Trivium and Quadrivium."¹

TEXT BOOKS.

Because of the unique development of the study of music in the Quadrivium, the text books in use cannot be classified in the way the text books on arithmetic, geometry and astronomy were grouped. It will be more to our purpose to divide them into two classes. In the first may be put all those written by authors who followed implicitly the standard of Greek musical theory and the distinctive tetrachordal scale. The second will include all books which to a greater or lesser extent are emancipated from the Greek influence, in which are included the consideration of the elements of "organon" (harmony in our modern sense) and in which the scales are used from which our modern system of notation has gradually developed.

The text books on which a considerable number of the treatises of this first period² are based comprise the works of Capella, Boethius, Cassiodorus, and Isidore, since these combine all the vestiges of Greek musical theory to which the middle ages became heir. Although very brief, the compilations of Capella, Cassiodorus, and Isidore of Seville³ must have been of great influence, since traces of their works have been found in nearly all the subsequent writers on music during this period.⁴ But the all pervading influence in this period was the elaborate compendium of Boethius.⁵ His five books *De Musica* formed the absolute basis of all theoretical music. Indeed the work remained a standard authority throughout the middle ages—

¹ Rashdall, *op. cit.* I, p. 443. Vide also Suter, *op. cit.* pp. 76, 77, 79, 80, 91 and Günther, *op. cit.* pp. 199, 210, 211, 215, for detailed references to university requirements in music.

² Partially collected in Gerbert, *op. cit.* I and Coussemaker, *op. cit.* II.

³ Texts: Capella, in Eyssenhardt ed. pp. 331-375; Cassiodorus, in Migne vol. LXX cols. 1208-1212; Isidore, in Migne, vol. LXXXII, cols. 163-

⁴ See the tables of Brambach, *Die Musik Literatur etc.* pp. 7, 9. Schmidt in *op. cit.* pp. 17-20 records the existence of 27 manuscripts of Isidore.

⁵ Text in Migne, vol. LXIII cols. 1166-1300. I have used the German translation by O. Paul, Leipzig, 1872, which enabled me to utilize the elaborate commentary, larger in volume than the text, without which even Günther pronounced the text unintelligible. Cf. Günther, *op. cit.* p. 112.

the great representative text of the "ars musica antiqua." The importance of the book warrants an analysis of its contents.

The text occupies one hundred and thirty-two closely printed pages, with numerous illustrations, tables, diagrams, and schematic arrangements. It is divided into five books: Book I comprises about one-fifth of the work; it deals with the history, character and the divisions of the science. The theory of consonance is set forth and the numerical proportions which form consonance and dissonance are given at great length with their appropriate elaborate Greek names. Following many classifications and definitions in which analogies to the harmony of the planetary motions are emphasized, the book concludes with a chapter on "what a musician is," and the definition which was long to remain classical is enunciated.¹ Book II treats exclusively of arithmetical quantities. The mystical Pythagorean theories of numbers are set forth. Proportion is amplified to include the special subject of "musical proportion," which in the opinion of Boethius forms one of the different kinds of arithmetical proportion. Book III is of a controversial character wherein the theories of Aristoxenus are criticized at length. Book IV is devoted to an exposition of the division and the use of the monochord in the theory of music. It also treats of the system of notation in which letters of the alphabet are used, though not in any regular systematic order. Book V is again given over to an examination of those points of musical theory on which writers on music have differed. This brief analysis shows that the only portions of the treatise fit for school purposes are the second and the fourth books. They were probably the only portions of the text used. The supposition is supported by the fact that it is mainly the subject matter of these two books which appears to have been the basis for the many derivative treatises.²

¹ "Is vero est musicus qui ratione perpensa, canendi scientiam, non "servitio operis, sed imperio speculationis assumit" "Isque "musicus est cui ad est facultas secundum speculationem rationemque pro- "positam ac musicae convenientem de modis ac rythmis de que generibus "cantilenarum ac de permixtionibus ac de omnibus de quibus posterius explicandum est ac de poetarum carminibus iudicandi." Boethius, *De Musica etc.* Lib. I, Ch. XXXIV, Migne, *Pat. Lat.* LXIII col. 1596.

² Vide tables of Brambach on the Boethian origin of the early mediæva

Among these derivative texts the following may be noted:

(1) Aurelian of Reome's *Musica Disciplina*, which was composed in the ninth century.¹

(2) Remigius of Auxerre's Commentary on Capella's *De Musica* which was written in the next century.²

(3) Hucbald's (880-930) *Liber de Harmonica Institutione*, which represents an attempt to build up a musical system out of the church music based on the theories of Boethius.³

The advance caused by the emancipation from Boethius was so gradual that it is most difficult to name the first text book in which this change is clearly seen. The *Cita et Vera Divisio Monochordi* of Bernelinus, composed in the tenth century, may however be considered the epoch making production. Most competent critics believe that the book approaches the mathematical aspects of musical theory in an essentially original spirit.⁴

From the days of Bernelinus the advance along the lines of distinctive mediæval musical theory was steady but sure. The increased knowledge was gradually incorporated in the text books in use. By the fourteenth century this improved material was epitomized by John De Muris, a professor of mathematics at the University of Paris. His book was the standard text in use in the universities during the fourteenth, and fifteenth centuries.⁵

text book writers on music, showing from what book the portions are taken, in *Die Musik Litteratur etc.*, passim.

¹ Text Gerbert's *Scriptoris etc.* I, pp. 27-63.

² Text, *ibid.* pp. 63-94.

³ Text in Gerbert, *Scriptores etc.* I, pp. 104-152. Cf. Ambros, *op. cit.* II, pp. 122 et seq. For the treatises of Notker and of other music teachers of the period, vide, Gerbert, *Scriptores etc.*, I, II, passim.

⁴ Vide Brambach, *Die Musik Litteratur etc.* p. 15 et seq. Text of Bernelinus in Gerbert, *Scriptoris etc.* I, pp. 313-330.

⁵ Rieman, *Geschichte der Musik-theorie* p. 235 et seq. The references cited in p. 132, note 1 show the requirements to have been either "aliquis musica" or "musica de Muris." Text of DeMuris's work on music, ms. X510 H74, Library of Columbia University, New York.

CHAPTER XI.

Conclusion.

The results of this study may be summarized as follows: The curriculum of the seven liberal arts was fully developed in the schools of the Roman Empire in the fourth century. Christianity triumphant over paganism found it necessary to appropriate to its own use the content of this curriculum and adopted it in its entirety, since secular studies would have a value for the Christian if the liberal education obtained through them could be made only a means towards an end, namely, a preparation for the higher study of theology.

Although tradition, the characteristic spirit of the age, maintained this curriculum in its entirety throughout the middle ages, the quantity and quality of instruction in the various subjects did not remain the same throughout the period. In accordance with the needs of the times certain subjects were emphasized at certain times at the expense, though not to the entire neglect, of the other subjects. Thus, in the period before the twelfth century, the study of Latin literature was assiduously cultivated, partly because a knowledge of the Latin language and literature was essential to this time, the formative period of the middle ages, and partly because the other subjects of the curriculum—logic and the mathematical studies—had not yet sufficiently developed to make the instruction in them of any cultural value. Later, when correct thinking was the pre-requisite for a consideration of questions which occupied the minds of the thoughtful people—questions of theology and metaphysics—the study of logic came gradually to hold a commanding position in the scheme of education. Still later, towards the close of the middle ages, when with the advance of mathematical knowledge this subject began to occupy the minds of thoughtful men, the study of arithmetic, geometry and astronomy came to be emphasized in the curriculum.

The actual amount of instruction in the subjects given throughout the period invariably approximated closely to the

amount of knowledge of the particular subjects possessed in the different periods of the middle ages.

The attitude of the mediæval church and of the leaders of thought was not hostile to secular culture. Eminent churchmen, even those who took the distinctly ascetic view on the question of the study of the liberal arts, were all trained in the traditional way. Naturally not all of them retained an interest during their busy lives in these secular studies, any more than the average man of affairs today always retains an abiding enthusiasm in the subjects which form the staples of education in the modern college.

There was a general interest in classical studies in the schools of the middle ages. This was never stronger than at the time when the church controlled all the educational institutions. The abatement of the interest in classical studies of the later university period was a reaction from, and not a continuation of, pre-university conditions.

The text books on the different subjects of the seven liberal arts, in an age worshipping tradition, an age without the printing press, an age of deference to authority and the "*littera scripta*," form unmistakable evidence of the quantity and quality of instruction afforded. The detailed examination of these text books has shown an amazing activity and desire to adapt the material of instruction to the requirements of the time. They show rare pedagogical skill, a general understanding of underlying conditions, marked individuality, and an eager desire to incorporate every advance in the knowledge of the subject.

In at least two subjects of the curriculum—rhetoric and music—the mediæval schoolmasters forsook the traditional paths and developed these studies along original lines in harmony with the spirit of the age.

In spite of the dialectical tendencies of the later middle ages, the instruction in logic as a part of the curriculum of the seven liberal arts was surprisingly free from what we generally assume to have been the dialectical puerilities in the study of philosophy. The evidence shows that whatever niceties of metaphysical distinctions occupied the minds of mediæval philosophers, the instruction given in the liberal arts was largely free from them. These subtleties formed part of the intellectual interests of the specialist and not of the man seeking liberal culture.

CRITICAL BIBLIOGRAPHY.¹

I. ORIGINAL SOURCES.

- Aelfric.** Colloquium ad pueros linguae Latinae locutione exercendos. Thorpe, editor. *Analecta Anglo-Saxonica*. London, 1846.
- Abelard, P.** *Ouvrages inédites* (edited by V. Cousin). Introduction. (in *Collection de Documents Inédits sur l'Histoire de France*.) Paris, 1836. Contains elaborate critical introduction.
- Albertus Magnus.** *Opera*. Edition of 1651. Tom. I.
- Alcuin, H. F.** *De Cursu et Saltu lunae ac Bissexto*. Migne, *Patrologia Latina*, Tom. CI.
- Alcuin, H. F.** *De Dialectica*. Migne, *Patrologia Latina*, Tom. CI.
- Alcuin, H. F.** *De Fide Sanctae et Individualis Trinitatis libri tres*. Migne, *Patrologia Latina*, Tom. CI.
- Alcuin, H. F.** *De Rhetorica et de Virtutibus*. Migne, *Patrologia Latina*, Tom. CI.
- Alcuin, H. F.** *Liber de Ratione Computi*. Migne, *Patrologia Latina*, Tom. CI.
- Alexander de Villa-Dei.** *Doctrinale*. Th. Reichling, editor. (*Monumenta Germaniae Paedagogica*, XII). Berlin, 1893.
- Anselm, St.** *Dialogus de Grammatico*. Migne, *Patrologia Latina*, Tom. CLVIII.
- Anselm, St.** *Monologium*. Migne, *Patrologia Latina*, Tom. CLVIII.
- Anselm, St.** *Prosologium*. Migne, *Patrologia Latina*, Tom. CLVIII.
- Arnulf, Deliciae Cleris.** J. Huemer, editor. *Romanische Forschungen*. Bd. II.
- Aristotle.** *Politics*. Translation by B. Jowett. Oxford, 1885.
- Augustine, St.** *Ars (Grammatica) Breviata*. Keil, *Grammatici Latini*, Tom. V.
- Augustine, St.** *De Doctrina Christiana*. Migne, *Patrologia Latina*, Tom. XXXIV.
- Augustine, St.** *De Rhetorica*. Halm, *Rhetores Latini Minores*.
- Augustine, St. (Pseudo).** *Principia Dialecticae*. Migne, *Patrologia Latina*, Tom. XXXII.
- Bede, The Venerable.** *De Natura Rerum*. Migne, *Patrologia Latina*, Tom. XC.

¹ The following named works were consulted in the preparation of this monograph. Some of these have already been criticized briefly, others more fully, both in the text and in the foot-notes. For this reason critical notes will only be given on such of the books quoted as require farther notice.

- Bede**, The Venerable. *De Temporum Ratione*. Migne, *Patrologia Latina*, Tom. XC.
- Bede**, The Venerable. *Liber de Schematibus et Tropis*. Halm, *Rhetores Latini Minores*.
- Bede**, The Venerable. (Pseudo.) *De Numerorum Divisione Libellus; De Loquella Per Gestum Digitorum et Temporum Ratione Libellus; De Unciarum Ratione*. Migne, *Patrologia Latina* Tom. XC.
- Boethius**, A. M. S. *De Arithmetica Libri Duo*. Migne, *Patrologia Latina*, Tom. LXIII.
- Boethius**, A. M. S. *De Musica Libri V*. Translated into German, and edited by O. Paul. Leipzig, 1872.
- Boethius**, A. M. S. *Interpretatio Euclidis Geometriae Libri Duo*. Migne, *Patrologia Latina*, Tom. LXIII.
- Boethius**, A. M. S. *Opera Philosophica*. Migne, *Patrologia Latina*, Tom. LXIV.
- Capella**, M. M. F. *De Nuptiis Philologiae et Mercurii*. F. Eyssenhardt, editor. Leipzig, 1866.
- Cassiodorus**, M. A. *De Artibus et de Disciplinis Liberalium Literarum*. Migne, *Patrologia Latina*, Tom. LXX.
- Catholicon**, Extracts. Edited by A. Scheler (in *Mémoires Couronnées de L'Académie Royale de Belgique*, T. XXXVII.) Bruxelles, 1886.
- Catonis Distichia De Moribus**. English and Latin Texts (in James Wright's *Sales Epigrammatum*). London, 1663.
- Cicero**, M. T. *De Oratore*, J. S. Watson, translator. London, 1896.
- Cicero**, M. T. *Opera*. (Edited by I. C. Orellus). Turin, 1845.
- Corpus Glossariorum Latinorum**. Edited by G. Lowe, vols. 3, 4, 5. Leipzig, 1892.
A monumental collection of Latin dictionaries. Has superseded all previous attempts in that direction. Still incomplete.
- D Andeli**, Henri. *La Bataille des Sept Arts, Fiction Critique et Satirique*. (Notices et Extraits des Manuscrits de la Bibliothèque Nationale, V., pp. 496-510). Paris.
- Donatus**, A. *Ars Grammatica Major*. Keil, *Grammatici Latini*, T. IV.
- Donatus**, A. *Ars Grammatica Minor*. Keil, *Grammatici Latini*, T. IV.
- Du Meril**, M. E. P. *Poésies Inédites du Moyen Age*. Paris, 1854.
An essay on the history of the Aesopic fables and a collection of fables and similar poetic popular literature.
- Eberard of Bethune**. *Laborinthus*. Polycarp Leyser, editor. In *Historia Poetarum et Poematum Medii Aevi*. Halle, 1721. Partly also in Mari Giovanni's *I Trattati Mediævali de Ritmica Latina*. (Memorie del Reale Istituto Lombardo, T. XX, Milan.)
- Egbert of Liege**. *Fecundia Ratis*. E. Voigt, editor. Halle, 1889.
- Erigena**, J. Scotus. *De Divisione Naturae*. Migne, *Patrologia Latina*, Tom. CXXII.
- Gerbert**, (Pope Sylvester II). *Opera Mathematica*. B. Bubnov, editor. Berlin, 1899.
- Halm**, C. F. *Rhetores Latini Minores*. Leipzig, 1863.
The only full collection of Latin rhetorical text books. In scholarship it is fully as good as Keil's collection of grammars. Contains most of the texts on technical rhetoric quoted in the work.
- Hermannus Contractus**. *De Utilitatibus Astrolobii Liber*. (Pez, *Thesaurus Anecdotorum Novissimus*, III, Pt. II.)

Hervieux, L. *Les Fabulistes Latins depuis le Siècle d'Auguste jusqu' à la fin du Moyen Age.* 5 vols. 1893-1899.

The most comprehensive investigation, superseding previous works on the subject. Contains, besides the critical investigation, the best texts in full.

Honorius of Autun. *De Animae Exsilio et Patria alias De Artibus.* Migne, *Patrologia Latina*, T. CLXXII.

One of the few authentic sources giving a detailed account of the scope of the mediæval curriculum by an eminent churchman of the twelfth century.

Hugo of Trimberg. *Registrum Multorum Auctorum.* J. Huemer, editor. (In *Sitzungsberichte der Philologisch-Historischen Classe der Kaiserlichen Akademie der Wissenschaft, Wien*, Bd. CXVI).

Interpretes Veteres Virgilii Maronis. (Mai, *Classici Auctores e Vaticanibus Codicibus editi*, T. VII) Romae, 1835.

Isidore of Seville. *Etymologiae Libri XX.* Migne, *Patrologia Latina*, T. LXXXII.

John of Salisbury. *Polycraticus.* Migne, *Patrologia Latina*, T. CXCIX.

John of Salisbury. *Metalogicus.* Migne, *Patrologia Latina*, T. CXCIX.

Johannes De Garlandia. *Poetria de Arte Prosaica Metrica et Rithmica.* G. Mari, editor. (*Romanische Forschungen*, Bd. XIII).

Jordanus Nemorarius. *Geometria vel de Triangulis Libri IV.* (Mittheilungen des Copernicus-Vereins für Wissenschaft und Kunst) Thorn, 1887.

First annotated text of the work published. "Einleitung" gives an extended account of all Nemorarius's works. Diagrams in Appendix. 71 propositions demonstrated.

Jordanus Nemorarius. *De Numeris Datis.* P. Treutlein, editor. (*Zeitschrift für Mathematik und Physik*, XXXVI, Supplementheft.)

Karolus Magnus. *Capitula De Doctrina Clericorum.* (Monumenta Germaniae Historica, Leges, Tomus I. pp. 107, 108.)

Keil, H. *Grammatici Latini.* 7 vols. Leipzig, 1857-1880.

The only complete collection of standard Latin grammars. The work of an eminent philologist, containing most of the grammatical texts described in the study.

Kleine Lateinische Denkmäler der Thiersage. E. Voigt, editor. (In *Quellen und Forschungen zur Sprach und Kultur Geschichte der Germanischen Völker.*)

A collection of a few but important sources.

Langlois, Ch. V. *Formulaires des Lettres du xii, du xiii et du xiv siècle.* (Notices et Extraits des Manuscrits de la Bibliothèque Nationale. Paris T. 34, 35.)

A collection of sources of Formulae chiefly, critically edited.

Monroe, P. *Source book of the History of Education for the Greek and Roman Periods.* New York, 1902.

A collection of sources with full introductions and running commentary. Arranged to show evolution of educational theories.

Nagl, A. *Eine Algorismusschrift des XII Jahrhunderts.* (*Zeitschrift für Mathematik und Physik*, Bd. XXXIV, pp. 129-146, 161-170).

Nicole Oresme. *Algorismus Proportionum.* Edited by M. Curtze (in *Zeitschrift für Mathematik und Physik*, XIII, Supplementheft.)

Nicomachi Geraseni Pythagorei Introductionis Arithmeticae Libri II. R. Hoche, editor. Leipzig, 1866.

The only available text (Greek) of this epoch-making book.

- Nicolai De Bibera.** *Carmen Satiricum.* Th. Fisher, editor. (Geschichtsquellen der Provinz Sachsen). Halle, 1870.
- Notices et Extraits de divers Manuscrits Latins Pour Servir A' L'Histoire des Doctrines Grammaticales au Moyen Age.** edited by M. Thurot. (in *Notices et Extraits des Manuscrits de la Bibliothèque Nationale.* Paris. T. XXII, Pt. 2)
Valuable annotated original material.
- Notker.** *Opera* (in *Denkmäler des Mittelalters.* H. Hattemer, editor.) St. Gallen, 1844-1849.
- Othlo of Emmeran.** *De Doctrina Spirituali Liber Metricus* (Pez, *Thesaurus Anecdotorum Novissimus*, III, Pt. 2).
- Othlo of Emmeran.** *Liber Proverborum.* (Pez, *Thesaurus Anecdotorum Novissimus*, III, Pt. 2.)
- Paulus Diaconus.** *Sexti Pompei Festi de verborum significatu quae supersant cum Pauli epitome.* A. D. Thewrewk, editor. Budapest, 1889.
- Plato.** *Republic.* B. Jowett, translator. *Dialogues of Plato*, vol. III. Oxford, 1875.
- Priscianus.** *Institutio de Arte Grammatica.* Keil, *Grammatici Latini*, vols. II, III.
- Priscianus.** *Partitiones duodecim versuum Aeneidos principalium.* Keil *Grammatici Latini*, T. III.
- Prudentius.** *Cathemerinon.* English metrical translation, E. G. Smith. London and New York, 1898.
- Prudentius.** *Psychomachia.* J. Bergman, editor. Upsala, 1897.
- Quintilian.** *Institutes of Oratory.* J. S. Watson, translator. 2 vols. London, 1882.
- Rabanus Maurus.** *De Clericorum Institutione.* Migne, *Patrologia Latina*, Tom. CVII.
- Rabanus Maurus.** *Excerptio de Arte Grammatici Prisciani.* Migne, *Patrologia Latina*, Tom. CXI.
- Rabanus Maurus.** *De Universo Libri Vigintiduo.* Migne, *Patrologia Latina*, Tom. CXI.
- Rabanus Maurus.** *Liber de Computo.* Migne, *patrologia Latina*, Tom. CVII.
- Richer.** *Historiarum Libri IV.* (Monumenta Germaniae Historica, *Scriptores*, Tom. III.)
- Röckinger, L.** *Briefsteller und Formelbücher des Elften bis Vierzehnten Jahrhunderts.* (Quellen zur Bayerischen und Deutschen Geschichte, Bd. IX, I, II.) München, 1863.
The most complete collection of its kind.
- Röckinger, L.** *Drei Formelsammlungen aus der Zeit der Karolinger* (Quellen zur Bayerischen und Deutschen Geschichte, Bd. VII) München, 1858.
In this collection some of the earliest sources on the subject are re-edited and others are published for the first time.
- Rozière, E. D.** *Recueil Général des Formules Usitées dans L'Empire de France du Vme au Xme siècle.* 3 vols. 1859-1871.
- Sacrobosco, (J. Holywood).** *Algorismus.* Ms. Columbia University, New York.
- Sacrobosco, (J. Holywood).** *Libellus de Sphaera cum Praefatione Melanctonis.* Württemberg, 1531.
- Schefflarer Proverbia.** *Anzeiger für Kunde der Deutschen Vorzeit Neue Folge*, Bd. XX.
- Scriptores Ecclesiastici De Musica ex Variis Italiae, Galliae et Germaniae codicibus manuscriptis collecti.** Martin Gerbert, editor. St. Blaise, 1784.

- Scriptorum De Musica Medii Aevi Nova Series.** 4 vols. E. De Coussenaker, editor. Paris, 1864-1867.
- Seneca, L. A.** Opera Omnia. 5 vols. Leipzig, 1882.
- Servius.** Commentarii in Virgilii Carmina. G. Thilo and H. Hagen, editors. Leipzig, 1878. 3 vols.
- Steinmeyer, E.** Die Deutschen Virgilglossen (Zeitschrift für Deutsches Alterthum, Neue Folge. Bd. III Berlin, 1872.)
- Steinmeyer, E. and Sievers, E.** Die Althochdeutschen Glossen, 3 vols. 1879-1895.
The only large collection of its kind in existence.
- Theodulfus.** Carmina. (In Poetae Latini Medii Aevi. E. Duemmler, editor.) Berlin, 1881-1884. (Monumenta Germaniae Historica, Scriptores).
- Trattati D'Aritmetica.** Boncompagno, editor. Roma, 1857.
A critically edited collection of important mediæval arithmetical texts.
- Walther. Spirensis.** Libellus de Studio Poetae, Acta Santi Christophori. (Pez, Thesaurus Anecdotorum Novissimus, II, Pt. III.)
- Wright, Thomas.** A Volume of Vocabularies. (10th to 12th centuries). 1857.
- Zarncke, F.** Die Urkundlichen Quellen zur Geschichte der Universität Leipzig in den ersten 150 Jahren ihres Bestehens. (Abhandlungen der Königlich-Sächsischen Gesellschaft der Wissenschaften, Philologisch Historische Classe, Bd. II.) 1857.

2. SECONDARY WORKS.

- Ambros, A. W.** Geschichte der Musik. 5 vols. Breslau, 1862-1881.
One of the best books on the subject in any language. Covers the period through the Middle Ages to Palestrina's times. The fifth volume consists of musical compositions to illustrate the text of the third volume. Continued for the 17th, 18th and 19th centuries by W. Langhans. Final general authority.
- Aschbach, Joseph.** Geschichte der Wiener Universität im ersten Jahrhundert ihres Bestehens. 2 vols. Wien, 1865.
A "Festschrift" written for the five hundredth anniversary of the University. Very full; based on original sources. Has half of the space devoted to the life and works of the famous teachers at the University. One of the best detailed histories of mediæval universities. Chapter III, on "Studien," is especially lucid on the subject.
- Bäbler, J. J.** Beiträge zur einer Geschichte der Lateinischen Grammatik im Mittelalter. Halle, 1885.
A series of disconnected studies treating in general of the subject chosen as the title of the book.
- Ball, W. R. R.** Mathematical Recreations and Problems of Past and Present Times. 3rd edition. London, 1896.
A collection of famous mathematical puzzles and quaint problems. Full of information for one interested in this phase of mathematics.
- Ball, W. R. R.** A Short Account of the History of Mathematics. 3rd edition. London, 1901.
The best short general account in English for the advanced student. Accurate. Useful bibliographical references.
- Barach, C. S.** Bernard von Chartres, sein Leben, seine Lehrwirksamkeit, seine Schriften. (Barach and Wrobel's edition of Bernardi Sylvestris de Mundi Universitate).
Brief sketch of the educational activity and influence of Bernard. Based on original sources.

- Barnard, H.** An account of Universities and other Institutions of Superior Instruction in Different Countries. (Comprising Part III of National Education.)
Mainly reprints of articles and chapters from standard European works on the history of higher education.
- Beaurepaire, Charles Marie de Robillard de.** Recherches sur l'Instruction publique dans le Diocèse de Rouen avant 1789. 3 vols. Evreux, 1872.
An intensive study. The only one in its field.
- Bergmann, J.** Geschichte der Philosophie. 2 vols. Berlin, 1892.
A general history of philosophy with emphasis on the development of philosophical systems. Readable.
- Berry, A.** A Short History of Astronomy. New York, 1899.
A brief general history of astronomy. Popular and accurate. Illustrated. Probably the most useful general treatise in English.
- Bianco, Fr. J.** Versuch einer Geschichte der ehemaligen Universität und der Gymnasien der Stadt Köln. Cologne, 1833.
A general account. The value of the book at present lies chiefly in its collection of documents printed as "Beilagen," which comprises nearly half of the book.
- Blakey, R.** Historical Sketch of Logic. London, 1851.
Of little value.
- Boissier, G.** Etude sur la Vie et les Ouvrages de M. T. Varron. Paris, 1861.
The most authoritative treatise on the whole life and work of Varro. The chapter on Varro's "Disciplinarum Libri" (the one quoted in this study) is very brief.
- Boissier, G.** La Fin du Paganisme. 2 vols. Paris, 1891.
General account of the obvious causes of the decline of paganism. The second part of the first book contains some suggestive chapters on the interaction of Roman education and Christian ideals.
- Bosanquet, B.** The Education of the Young in the Republic of Plato. Cambridge, 1900.
Introduction and selections from the first four books of the Republic.
- Brambach, W.** Das Tonsystem und die Tonarten des Christliche Abendlandes im Mittelalter. Leipzig, 1881.
A valuable investigation on the subject.
- Brambach, W.** Die Musik Literatur des Mittelalters bis zur Blüthe der Reichenauer Sängerschule. Karlsruhe, 1883.
A comparative study of standard books of the period to determine the extent of Boethius's influence on these text books.
- Brambach, W.** Die Reichenauer Sängerschule, Beiträge zur Geschichte des Gelehrsamkeit und zur Kenntniss Mittelalterlicher Musikhandschriften. (In Beihefte zum Centralblatt für Bibliothekenwesen I.)
Excellent monograph. Treats admirably of the difficult subject of the relation of mediæval musical theory to practice.
- Braniss, J.** Die Logik in Ihrem Verhältniss zur Philosophie geschichtlich betrachtet. Berlin, 1823.
An old brief "Preisschrift." Logical and clear; the only treatise of its kind.
- Brockman, F. J.** System der Chronologie. Stuttgart, 1883.
Convenient handbook. Contains in small volume the essential material of Ideler's older and more voluminous work on the subject.
- Burnet, John.** Aristotle on Education. Cambridge, 1903.
An excellent discussion of Aristotle's views on education, based on abstracts from Aristotle's "Ethics" and "Politics."

- Bursian, C.** *Geschichte der Classischen Philologie in Deutschland.* 2 vols. München, 1883.
A general history of classical scholarship in Germany. The chapter on the period before Humanism is brief and inadequate.
- Cajori, F.** *A History of Elementary Mathematics.* New York, 1896.
A brief survey of the history of elementary mathematics. Accurate. Very readable; based on results of modern researches on the subject.
- Cantor, M.** *Mathematische Beiträge zum Kulturleben der Völker.* Halle, 1863.
Chapters on the history of mathematics, readable and scholarly. One of the early works of this famous historian of mathematics.
- Cantor, M.** *Die Römischen Agrimensoren und ihre Stellung in der Geschichte der Feldmessenkunst.* Leipzig, 1875.
The latest investigation on the subject. The book covers a broader field than its title indicates. It treats of the Greek forerunners of the Roman Gromatici and of the latter's influence on the Middle Ages.
- Cantor, M.** *Vorlesungen über Geschichte der Mathematik.* 3 vols. 2 Auflage. Leipzig, 1894-1900.
A final authority on subjects relating to the history of mathematics. Volume I to the year 1200; Volume II to 1668; Volume III from 1668 to 1758. An inexhaustible mine of information.
- Chasles, M.** *Geschichte der Geometrie.* Translated by L. A. Sohnke. Halle, 1839.
A general survey of the history of the science before the nineteenth century by the famous French mathematician. Two-thirds of the book consists of appended notes. Erudite.
- Comparetti, D.** *Virgil in the Middle Ages.* Translated by E. F. M. Benecke. London, 1895.
The best book on the subject. Comprehensive.
- Compayré, G.** *History of Pedagogy.*
A general popular account. The author's antipathy to the Middle Ages is a marked characteristic of the work. Partisan.
- Compayré, G.** *Abelard and the Origin and Early History of Universities.* New York, 1902.
A brief popular sketch of the subject, based entirely on secondary sources.
- Curtze, M.** *Die Mathematischen Schriften des Nicole Oresme.* Berlin, 1870.
A brief pamphlet giving a bibliographical account of the author's complete mathematical works.
- Davidson, Thomas.** *Aristotle and Ancient Educational Ideals.* New York, 1892.
A scholarly and suggestive treatment of the subject.
- De Morgan, A.** *Arithmetical Books. From the Invention of Printing to the Present Time.* London, 1847.
A unique compilation. Each book is fully described.
- Denk, V. M. Otto.** *Geschichte des Gallo-Frankischen Unterrichts—und Bildungswesens von den ältesten Zeiten bis auf Karl den Grossen.* Mainz, 1892.
The only complete scientific work on the early period of Frankish education. Very scholarly; based entirely on original sources.
- Dill, S.** *Roman Society in the Last Century of the Western Empire.* London, 1898.
A general account of the period treated. Accurate; judicious; scholarly.
- Ebert, Ad.** *Allgemeine Geschichte der Litteratur des Mittelalters im Abendlande.* 3 vols. Leipzig, 1874-1887.

A classic on the subject. A remarkably complete and thorough book covering so large a field.

Eckstein, F. A. Lateinischer Unterricht (in Schmid's Encyklopädie des gesamten Erziehungs und Unterrichtswesens, Bd. IV, pp. 204-405).

Full of information. It has been re-printed in book form.

Friedlein, G. Das Rechnen mit Columnen vor dem 10 Jahrhundert, (in Zeitschrift für Mathematik und Physik, Bd. IX.)

An article relating to the columnal computation controversy about Boethius.

Friedlein, G. Die Zahlzeichen und das Elementare Rechnen der Griechen und Römer und des Christlichen Abendlandes vom 7 ten bis 13 ten Jahrhundert. Erlangen, 1869.

A general treatise full of information on the history of notation and computation.

Gaskoin, C. J. C. Alcuin, His Life and His Work. London, 1904.

The latest and best biography of Alcuin. Thorough in its treatment of the many-sided activity of Alcuin. Based on all available sources.

Gottlieb, Th. Ueber Mittelalterliche Bibliotheken. Leipzig, 1890.

An excellent book on the subject. The book contains a large number of titles of mediæval library catalogues. Under each heading the most important books mentioned in those catalogues are given.

Glover, T. R. Life and Letters in the Fourth Century. Cambridge, 1901.

Emphasizes Prudentius's importance in the development of Christianity in the West.

Gow, J. A Short History of Greek Mathematics. Cambridge, 1884.

The title is misleading. The book is really a history of elementary mathematics; scholarly; bibliographical notes very full. One of the few thorough, original works on mathematics in English.

Grasberger, Lorenz. Erziehung und Unterricht im Klassischen Alterthum. 3 vols. Würtemberg, 1864-1881.

A very full treatment of the subject.

Grote, G. Aristotle. Bain & Robertson, editors. 2 vols. London, 1872.

An unfinished work on Aristotle, chiefly "The Organon" by the famous historian of Greece. Keen; erudite.

Güdeman, M. Geschichte des Erziehungswesens und der Kultur der Juden in Italien während des Mittelalters. Wien, 1884.

One of a work of three volumes on Jewish education and culture in the Middle Ages in Spain, Italy and France. Final authority in its special field.

Guizot, F. History of Civilization. W. Hazlitt, translator. New York, 1850.

The 16th, 17th and 18th lectures of this book give an interesting account of the general character of literary activity in the 6th to the 8th centuries. Suggestive. In sympathy with the spirit of the Middle Ages.

Günther, S. Geschichte des Mathematischen Unterrichts im Deutschen Mittelalter bis zum Jahre 1525. Berlin, 1887. (In Monumenta Germaniae Paedagogica III.)

A remarkably thorough work on the subject. Includes material strictly foreign to its title. The author is a master of all the aspects of his theme.

Günther, S. u. Kirchhoff Alfred. Didaktik und Methodik des Geographieunterrichts. 2 vols. München, 1895.

Günther's contribution (the portion quoted in the text) is a brief history of geography as an introduction to the subject.

Haase, Fr. Vorlesungen über Lateinische Sprachwissenschaft. Leipzig, 1874.

The published notes of this eminent philologist's lectures. Fragmentary.

- Hallam, H.** *Introduction to the Literature of Europe.* 1 vol. 5th edition. London, 1873.
The chapter quoted shows the inaccurate character of the work for modern students of mediæval culture.
- Hankel, H.** *Zur Geschichte der Mathematik im Alterthum und Mittelalter.* Leipzig, 1874.
A scholarly, well written book. Full of bibliographical references. Emphasis on classical period. Very brief on the later Middle Ages (the 12th to the 15th centuries). Belittles the mathematical knowledge of the middle ages.
- Hauréau, B.** *De La Philosophie Scolastique.* 2 vols. Paris, 1850.
When first published, a pioneer research work in a neglected field of the history of philosophy. Still authoritative.
- Hazlitt, W. C.** *Contributions toward a History of Earlier Education in Great Britain.* (In *Antiquary*. Vols. 35, 36.)
An annotated bibliography of books of all kinds bearing on the subject, printed in England.
- Heerdegen, F.** *Lateinische Lexicographie.* I. Müller's *Handbuch der Klassischen Alterthums-Wissenschaft*, II, Pt. 2.
A monograph embodying the latest researches. The series of which this monograph forms a part is the latest comprehensive collection of works on classical antiquity collaborated by most eminent classical scholars, and covers the whole range of the subject—Language, Literature, Culture, Religion, Art, Statecraft.
- Histoire Littéraire de la France.** 1733–1895.
Fullest literary history of France. Contains analysis of most important French works.
- Huemer, J.** *Ueber ein Glossenwerk zum Dichter Sedulius.* (In *Sitzungsberichte der Philologisch-Historischen Classe der Kaiserlichen Akademie der Wissenschaften* Bd. CXVIII. Wien, 1888.)
- Hurter, Ammann, F. E.** *Tableau des Institutions et des Moeurs de L'Eglise au Moyen Age, Particulièrement au treizième siècle, sous le Règne du Pope Innocent III.* (Translated from the German.) 3 vols. 1843.
A general survey emphasizing the brighter side of the church's manifold activities and relations to mediæval life.
- Jourdain, A.** *Recherches Critiques sur l'âge et l'origine des Traductions latines d'Aristote.* Nouvelle edition. Paris, 1843.
The first scholar to cover this all important subject. His work was epoch making. Still standard.
- Kästner, A. G.** *Geschichte der Mathematik.* Göttingen, 1296–1800.
This work of the famous mathematician of the eighteenth century is old and superseded, but full of information.
- Kaufman, G.** *Rhetorenschulen und Klosterschulen oder Heidnische und Christliche Cultur in Gallien während des 5 und 6 Jahrhunderts.* (Historisches Taschenbuch, 4te Folge 1869.)
An interesting study based on original sources.
- Kirchen Geschichtliche Studien.** Leipzig, 1890.
The article quoted from this volume is characterized in the text.
- Langhans, W.** *Die Musik-Geschichte in Zwölf Vorlesungen.* Leipzig, 1878.
A brief survey by an authority.
- Laurie, S. S.** *Historical Survey of Pre-Christian Education.* London, 1895.
A popular treatise. General account. Based on secondary sources.
- Laurie, S. S.** *Rise and Early Constitution of Universities.* New York, 1903.
Of the same character as this author's above mentioned work.
- Lowe, G.** *Prodromus Glossariorum Latinorum.* Leipzig, 1876.
A pioneer investigation in the field of Latin lexicography. The book

treats fully of the whole subject of sources of the numerous dictionaries. The chapter on mediæval dictionaries describes the best known manuscripts.

Mabillon, J. *Traité des Etudes Monastiques*. Paris, 1691.

The first scholarly defense in modern times of the culture of the middle ages. The author's name is associated with the work of the modern Benedictines.

Maitland, S. R. *The Dark Ages*. 3rd edition. London, 1853.

A work written especially to combat the narrow-minded view of the Middle Ages, so common in the early part of the nineteenth century. Apologetic in tone, but well written and eminently fair. The author finds no difficulty in disproving the glaring exaggerations and misstatements as to the "state of learning" in the 10th, 11th and 12th centuries.

Mädler, J. H. *Geschichte der Himmelskunde von der ältesten bis auf die neueste Zeit*. 2 vols. Braunschweig, 1873.

1st volume to Herschel. A student's text book; readable; very brief on the period before Copernicus.

Manitius, M. *Beiträge zur Geschichte Römischer Dichter im Mittelalter*. (Philologus-Zeitschrift für das Classische Alterthum, Bde. XLVII, XLVIII, XLIX, L, LI, LII, LVI, Supplement Bd. VII.)

Marty, Martin (anonymous author). *Wie man vor tausend Jahren lehrte und lernte*. Beilage-Jahresbericht über die Erziehungsanstalt des Benedictinerstifts Maria Einsiedelen, 1856-1857.

An imaginative diary of Walafrid Strabo in which the poet's schooldays are vividly described. The author certainly shows historic imagination. The facts of Strabo's life are skillfully woven into the narrative.

Mattew, J. E. *The Literature of Music*. London, 1896.

An excellent annotated bibliography of the subject; well classified; accurate; full of information.

Meier, G. *Die Sieben Freien Künste im Mittelalter*. Chapters on Grammar, Rhetoric and Dialectic. (Jahresbericht der Benedictinerstifts Maria Einsiedelen.) 1885-1886.

A eulogistic account of the methods of mediæval education on Grammar, Rhetoric and Dialectic, by a Catholic scholar. The sketch is not an original contribution to the subject, though it contains interesting facts.

Montalembert, C. F. *The Monks of the West*. Casquet, translator. Vol. 5. New York, 1896.

Scholarly defense of monasticism and of its services to Western civilization; apologetic in tone.

Mullany, P. F. (Brother Azarias). *Educational Essays*. Chicago, 1896.

A series of collected essays, mainly controversial in character.

Mullinger, J. B. *Schools of Charles the Great and the restoration of education in the ninth century*. London, 1877.

An authority in its field; sound in its conclusions.

Mullinger, J. B. *University of Cambridge from the Earliest Times*. Cambridge, 1873.

Covering the earliest period to 1535. Based on sources. The first two chapters deal with general educational conditions in the Middle Ages.

Nagl, A. *Gerbert und die Rechenkunst des 10 ten Jahrhundert* (Sitzungsberichte der Philologisch-Historischen Classe der Kaiserlichen Akademie der Wissenschaften. Wien, Bd. CXVI.)

An investigation into the history of mathematics. The author argues for original mathematical achievements of Gerbert.

- Nauman, Emil.** *The History of Music*, 2 vols.
A general popular account; fully illustrated. The period of the middle ages treated fully.
- Norden, E.** *Die Antike Kunst-prosa vom 6 ten Jahrhundert vor Christus bis in die Zeit der Renaissance*. 2 vols. Leipzig, 1898.
A comprehensive work by an eminent philologist. It is authoritative and full of bibliographical references.
- Oznam, A. F.** *Des Ecoles et de l'Instruction Publique en Italie aux Temps Barbares* (In Documents Inédites pour Servir à l'Histoire Littéraire de l'Italie.) Leipzig and Paris, 1897.
A collection of sources to illustrate the state of literary culture in Italy from the 8th to the 13th centuries. To the collection is prefixed a scholarly essay on public education in Italy in the Dark Ages, in which the author has made good use of the original sources to defend his thesis.
- Parker, H.** *The Seven Liberal Arts*. *English Historical Review*. Vol. 5, July 1890.
The best study on the evolution of the mediaeval curriculum. The author establishes his contentions with much force.
- Paulsen, Friedrich.** *Geschichte des Gelehrten Unterrichts auf den Deutschen Schulen und Universitäten*. 2 vols. Leipzig, 1885.
Treats of the history of higher education in modern times. The first chapter is a remarkable statement of the significance of the mediæval university.
- Peiper, R.** *Beiträge zur Lateinischen Cato-Literatur*. (Zeitschrift für Deutsche Philologie, Bd. V.) Halle, 1874. A critical study of a Berlin manuscript of Ludolf von Lüchow's version of the *Distichia Catonis*.
- Peschel, O.** *Geschichte der Erdkunde bis auf Alexander von Humboldt und Carl Ritter*. 2 Auflage. München, 1877.
A standard work in an official series covering the History of Sciences in Germany. Numerous references.
- Poole, R. L.** *Illustrations of the History of Mediæval Thought*. London, 1894.
A series of suggestive studies. Very full bibliographical notes. The topics taken up are treated in a remarkably thorough fashion.
- Prantl, C.** *Abt Wilhelm von Hirschau's Philosophicae et Astronomicae Institutiones*. (Sitzungsberichte der Königlichen Bayerischen Akademie der Wissenschaften zu München,) 1861.
A discussion of the contents and importance of Hirschau's work.
- Prantl, C.** *Geschichte der Logik im Abendlande*. Leipzig, 1855-1870.
A brilliant monumental work. Still unsurpassed in its comprehensive treatment of the logico-metaphysical problems of the Middle Ages; full of erudition, but the author is too polemical and at times even abusive of his opponents.
- Putnam, G.H.** *Books and their Makers during the Middle Ages*. New York, 1896. 2 vols.
Volume I contains an abundance of interesting material on libraries and manuscripts in the Middle Ages.
- Rashdall, Hastings.** *Universities in Europe in the Middle Ages*. 2 vols. Oxford, 1895.
An excellent book. Perhaps the best general account of the history of mediæval universities. Based on full investigations. Strong on the early development of universities.
- Reichling, Th.** *Das Doctrinale des Alexander de Villa-Dei*. (Introduction to Volume XII, *Monumenta Germaniae Paedagogica*) Berlin, 1893.
An admirable example of "German thoroughness." The work is the final authority on the subject.

- Rémusat, F. M. Ch. De.** *Abelard.* 2 vols. Paris, 1845.
The first extended investigation into the life and influence of Abelard. Authoritative in its field.
- Rieman, Hugo.** *Geschichte der Musiktheorie; IX-XIX Jahrhundert.* Leipzig, 1898.
The authority on the subject.
- Rieman, Hugo.** *Studien zur Geschichte der Notenschrift.* Leipzig, 1878.
A thorough investigation of the subject.
- Ritschl, F.** *De M. Terentii Varronis Disciplinarum Libris Commentarius* (*Opuscula Philologica, III*) 1877.
Originally a dissertation printed in 1845. Argument based on keenest textual criticism.
- Robertson, W.** *History of the Reign of the Emperor Charles V. with a view of the Progress of Society in Europe.* Harpers, 1829.
The work, so popular in its day, the end of the eighteenth century, strikingly illustrates the advance made in historical investigations in recent times. The author makes sweeping generalizations and in two pages sums up the "fatal effects" of that "state of society on sciences, letters and religion." The "proofs and illustrations" cover 2 more pages. On the subject relating to this investigation, the work of the Rev. W. Robertson may represent the typical biased view of the Middle Ages so commonly accepted before the advent of the modern historical school.
- Röckinger, L.** *Die Ars Dictandi in Italien.* (Sitzungsberichte der Königlich Bayerischen Akademie der Wissenschaften zu München, 1861).
A critical investigation of the topic based on general sources.
- Sandys, J. E.** *History of Classical Scholarship.* Cambridge, 1903.
The only book of its kind in English. Fragmentary in treatment and not accurate. It is inferior to the foreign treatises on classical literature.
- Saintsbury, G.** *A History of Criticism and Literary Taste in Europe.* 3 vols. New York, 1900.
The first volume devoted to classical and mediæval criticism. Scope—very general; delightfully readable but not very accurate.
- Savigny, F. C.** *Geschichte des Römischen Rechts.* 7 vols. Heidelberg, 1834.
An epoch making work in its day. Replete with information and much original material. It is still an authority in its field.
- Schmidt, C.** *Quaestiones de Musicis Scriptoribus Romanis, imprimis de Cassiodoro et Isidoro.* Darmstadt, 1899.
A Doctor's dissertation. A critical investigation into the sources determining the position of music in the scheme of liberal studies. The theses are well proven.
- Schmid, K. A.** *Geschichte der Erziehung vom Anfang an bis auf unsere Zeit, bearbeitet in Gemeinschaft mit einer Anzahl von Gelehrten und Schulmännern.* 5 vols. Stuttgart, 1884-1902.
A comprehensive history of education containing a connected series of monographs by eminent specialists; scholarly; full; authoritative. Even in its thoroughness.
- Schmidt, Karl.** *Die Geschichte der Paedagogik.* Lange's edition. Popular, but not scholarly nor accurate. General survey.
- Smith, D. E.** *Teaching of Elementary Mathematics.* New York, 1903.
Contains much up-to-date accurate material on the history of the different mathematical subjects.
- Schubiger, P. Anselm.** *Die Sängerschule St. Gallens vom Achten bis Zwölften Jahrhundert.* Einsiedeln. New York, 1858.

- An exhaustive treatment of its limited field; based on sources. The notes of important songs associated with the names of teachers of the school are reproduced. Altogether an important contribution.
- Specht, F. A.** *Geschichte des Unterrichtswesens in Deutschland von den ältesten Zeiten bis zur Mitte des 13 Jahrhunderts.* Stuttgart, 1885.
A remarkable book; based entirely on original sources. The best in its field. References very complete.
- Stallaret, Ch. et Vanderhaegen.** *De l'Instruction Publique au Moyen Age (viime au xvime Siècle).* In (*Memoires Couronnées de l'Académie Royale de Belgique*). Vol. 23. Bruxelles, 1853.
Deals especially with conditions in the Netherlands. Authoritative.
- Stolz, F.** *Lateinische Grammatik* (in I. Müller's *Handbuch der Klassischen Alterthumswissenschaft*, Bd. II, Pt. 2.) München, 1900.
- Suter, H.** *Die Mathematik auf den Universitäten des Mittelalter.* (Festschrift der Kantonsschule zu Zürich) Zürich, 1887.
A valuable contribution based entirely on original sources. Full bibliographical notes.
- Taylor, H. O.** *The Classical Heritage of the Middle Ages.* New York, 1901.
A charming work; up-to-date in its scholarship.
- Teuffel, W. S.** *Geschichte der Römischen Literatur* (bearbeitet von L. Schwabe, 5th Auflage). Leipzig, 1890.
Very full in its notes and bibliography. The work is a standard.
- Thesaurus Novus Latinitatis.** (*Classici Auctores e Vaticanis Codicibus editi.* T. VIII. A. Mai, editor). Romae, 1836.
- Trendelenberg, A.** *Geschichte der Kategorienlehre.* Berlin, 1846.
Authoritative, the only work on the subject; thorough.
- Tropfke, J.** *Geschichte der Elementar-Mathematik.* Bd. 2. Leipzig, 1903.
Arranged to show separately the historical development of each subject—arithmetic, algebra etc. A good book, based on the best secondary sources.
- Ueberweg, F.** *History of Philosophy.* G. S. Morris, translator. 4th German edition. New York, 1885.
A standard book for the student. Unexcelled for its bibliography. The middle ages are treated in full.
- Ueberweg, F.** *System of Logic and History of Logical Doctrines.* Translated by T. M. Lindsay. London, 1871.
The historical chapters are the best on the subject.
- Unger, F.** *Die Methodik der Praktischen Arithmetik in Historischer Entwicklung.* Leipzig, 1888.
Good account of the development of the subject in modern times. Chapter on the period of the middle ages is very brief and inadequate.
- Wattenbach, W.** *Das Schriftwesen im Mittelalter.* 2 Auflage. Leipzig, 1875.
A scholarly general account of the manners and methods of book writing, making and preservation in the middle ages.
- Wattenbach, W.** *Deutschlands Geschichtsquellen im Mittelalter bis zur Mitte des 13 Jahrhunderts.* 2 vols. 7th Auflage. Bd. I. Stuttgart und Berlin, 1904.
Invaluable as a reference work for the student of mediæval history. Full of information and biographical material, and criticism of most important sources.
- Weddingen, A. Van.** *Essai Critique sur la Philosophie de S. Anselme* (In *Memoires Couronnées de l'Académie Royale de Belgique*). Bruxelles, 1875.
A good account of Anselm's life and work forms part of the study.
- Weissenborn, H.** *Gerbert—Beiträge zur Kenntniss der Mathematik der Mittelalters.* Berlin, 1888.

An original investigation on the controverted questions relating to Gerbert's arithmetical and astronomical works.

West, A. F. *Alcuin and the Rise of the Christian Schools.* New York, 1892.
An interesting book; popular yet scholarly.

White, A. D. *History of the Warfare of Science with Theology, in Christendom.* 2 vols. New York, 1901.
Polemical in tone; very valuable for bibliography; full notes and references.

White, C. L. *Aelfric.* (Yale Studies in English.) Boston, 1898.
The final authority on the subject.

Williams, C. F. A. *The Story of Notation.* London and New York, 1903.
A very readable brief account; up to date.

Wolf, R. *Geschichte der Astronomie.* München, 1877.
Best on the subject. An authoritative work. Bibliographical notes and references very full. Comprehensive.

Wustenfeld, F. *Die Übersetzungen Arabischer Werke in das Lateinische seit dem 11 ten Jahrhundert* (In *Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen*, Bd. XXII.)
The work is a descriptive catalogue of all Arabian books translated into Latin and includes lists of manuscripts and printed works extant.

Zarncke, F. *Beiträge zur Mittel-Lateinischen Spruchpoesie* (In *Berichte über die Verhandlungen der Königlichen Sächsischen Gesellschaft der Wissenschaften, Philologisch-Historische Classe*, Bde, XV, XVII, XXII,) Leipzig, 1862-1868.

Contains four similar critically edited versions of Cato; a Cato Novus; a Cato Rhythmicus; a Cato Interpolatus; and a Cato Leoninus.

Ziegelbauer, R. P. M. *Historia Rei Literariae Ordinis S. Benedicti.* 4 vols. 1754.

A unique contribution; Volume I on Benedictine schools and libraries; Volume II on different writers in all lines; Volume III, Biographical; Volume IV, Bibliographical.

ERRATA.

- P. VI, line 31; read with the spirit of tradition *of the middle* ages, etc.
P. 4, line 18; read *first* Punic War.
P. 15, line 22; after *appear* there should be a comma, not dash. Line 25; read
understood.
P. 73, line 8; read *throughout the middle ages*.
P. 91, line 25; read still greater *interest* etc.
P. 92, line 21; after *age* insert 3
P. 103, line 1; read period *of*.
P. 133, last line of notes; read *mediaeval*
P. 134, note 4; read *scriptores*.
P. 138, line 25; read *Orelli*.

VITA.

Paul Abelson—Brown School, Hartford, Conn., 1894; B. A. College of the City of New York, 1899; graduate student, School of Political Science, Columbia University, 1899-1901—courses in history, economics, political philosophy and education under Professors Burgess, Castle, Dunning, Mayo-Smith, Monroe, Osgood, Robinson, Seligman and Sloane—Secondary Diploma in History, Teachers College, Columbia University, 1901, teacher Evening Elementary Schools, New York City, 1898-1902; teacher of History and Political Science, New York Evening High School, 1904-1905; instructor in History and Civics, De Witt Clinton High School, New York City, 1902—; chairman of Committee on Course of Study and Syllabus in History and Political Science for the Evening Public High Schools, New York City, 1904; lecturer, New York City Board of Education lecture corps 1903—; director Department of Civics, Educational Alliance, New York, 1905; social worker, 1899—; age 28.

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filled their mission and transmitted all the mathematical knowledge they possessed to future generations, and that the student was obliged to master this knowledge before he took up the advanced study of philosophy.

(2) The standard of mathematical teaching in the middle ages in the well known schools was very high. Although there is no evidence of creative mathematical ability in the early periods, the later centuries show advance in assimilation of new material.¹

(3) The quantity and character of the mathematical instruction throughout the age improved *pari passu* with the advance of mathematical knowledge in the several subjects.

(4) Even after the thirteenth century, when in the university period the Quadrivium was merged in the general course of philosophy in the school of arts, mathematical studies were far from being neglected. Even then when scholasticism held sway, the amount of instruction in mathematics kept pace with the gradual advance of the sciences.²

B. EXTENT OF KNOWLEDGE.

The arithmetical knowledge of the middle ages can be classified into three periods. During the first, ending with the tenth century, Europe possessed a very small amount of knowledge of the kind of arithmetic which was cultivated so much by the Greeks in the so-called Alexandrian age. What was known comprised chiefly the contents of the text books on arithmetic of the Neo-Pythagorean Nichomachus, composed about the close of the first century.³ During this time the study

¹ Günther, *op. cit.* pp. 81-121, 146-207.

² The traditional view as to the neglect of these studies before the Renaissance is based on an erroneous assumption. Paris, being the mother of the universities, it is supposed that its *special* neglect of the mathematical studies was characteristic of the entire period. In opposition to this belief it might be urged that the University of Vienna made much of the teaching of these very subjects. As a matter of fact there was a mean position between the two extremes—Paris and Vienna—taken by most of the universities of the middle ages. These afforded a reasonable and ample amount of instruction in mathematics. Cf. Rashdall, *op. cit.* I, pp. 440-443; Günther, *op. cit.* p. 207 et seq.

³ It was through Boethius who translated and adapted this text that this peculiar form of arithmetic came to be known as, Boethian arithmetic. For text of Nicomachus, vide edition of R. Hoche Leipzig, 1866. For analysis of Nicomachus, vide Gow, *Short History of Greek Mathematics*, pp. 89-95.

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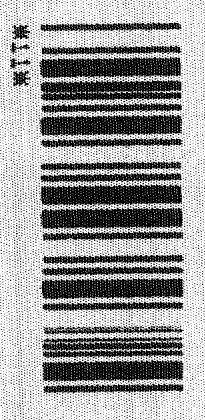
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