The University Tutorial Series.

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AN ELEMENTARY TEXT-BOOK OF HYDROSTATICS.

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OF

HYDROSTATICS.

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PREFACE.

The ground covered by this book includes those portions of Hydrostatics and Pneumatics which are usually read by beginners and by candidates for examinations of such a standard as that of the London Matriculation. In the illustrative and other examples, it has been the authors' endeavour to deduce results from first principles, and as far as possible to discourage students from relying on memory for mathematical formulæ. Where new departures have been thought desirable, they have generally been effected in such a way as to allow teachers the opportunity of adhering to older methods of treatment if they so prefer. Thus, according to our arrangement, the student becomes familiar with specific gravity and the very important methods of determining it, including the use of the Hydrostatic Balance, before encountering the difficulties connected with the measurement of pressure and the distinction between *pressure* and *thrust*. But any reader who prefers may pass straight on to Part II., after reading the first three or four chapters of Part I., leaving the remaining chapters of Part I. to be read after Chapter XIII. Again, proofs involving the Principle of Work have been introduced in several cases, but the possibility of omitting them if desired has been pointed out.

We have given considerable attention to the illustrations, notably those of air and water pumps, in which the up and down strokes are figured separately.

Our thanks are due to Mr. F. Rosenberg for his care and attention in revising many of the proofs, and to the Vacuum Brake Company and the Westinghouse Brake Company for their illustrated pamphlets, on which have been based our brief descriptions (page 192) of these two interesting and important exemplifications of the principles of Pneumatics.

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PART III.—*Pneumatics.*

HYDROSTATICS.

INTRODUCTION.

SYSTEMS OF UNITS.

1. **The English System.**—In Hydrostatics we shall have to compare the sizes and weights of different bodies. In measuring these, either the English or the French system of weights and measures may be used.

In the English system, the most usual **unit of length** is the **foot (ft.)**. The foot is one-third of a yard, the **yard** being defined as the distance between two marks on a certain bar of platinum which is now kept in the Tower of London. There is no reason why this particular length should have been chosen as the unit, beyond that of custom. This fact is expressed by saying that the yard is a purely *arbitrary* unit.

Smaller lengths may be measured in inches (1 foot = 12 inches), longer lengths in miles (1 mile = 5280 feet), both units being derived from the foot or yard.

The **unit of area** is to be taken as the area of a square whose side is the unit of length, *i.e.*, a square foot.

The **unit of volume** is to be taken as the capacity of a cube whose length, breadth, and height are each equal to the unit of length. Thus a cubic foot and a cubic inch are the units of volume corresponding to a foot and an inch respectively, and we note that

1 **cubic** foot contains $12 \times 12 \times 12 = 1728$ cubic inches.

Sometimes volumes are measured in **gallons.**

The gallon, like the yard, is an arbitrary unit which is defined by standard.

2. The **weight** of a body is a quantity proportional to the force with which the body is acted on by gravity.*

The usual English **unit of weight** is the **pound (lb.)**.

This is defined as the weight of a certain standard piece of platinum kept in London, and which, like the yard, was chosen arbitrarily.

Smaller weights may be measured in ounces (1 lb. = 16 oz.) or grains (1 lb. = 7000 grs.); larger weights in tons (1 ton = 2240 lbs.).

The following facts are important:—

A cubic foot of pure water weighs about 1000 oz.; and, roughly,

"A pint of clear water
Weighs a pound and a quarter";

and therefore a gallon (8 pints) weighs 10 lbs.

When we say that a body *weighs one pound*, we mean that it would balance the standard pound weight in a pair of scales, and therefore that it tends to fall to the Earth with the same force as a 1-lb. weight at the same place. Hence, in weighing a body in the ordinary way, the force of gravity on it is measured in terms of another force of the same kind, and the *common* measure of the *weight* is a purely *numerical quantity* which does not depend on the intensity of gravity, but merely on the relative *quantity of matter* in the body, as compared with the quantity of matter in the pound or other standard of weight.

The *actual quantity of matter* in a body is called its **mass**; hence *the weight of a body measures its mass*.

The *actual force* with which gravity acts on a body at any particular place may, for convenience, be called the **absolute weight** of the body, to distinguish it from the purely numerical measure of weight obtained with a pair of scales. Where no confusion is likely to arise, the word "absolute" may be omitted.

* If the term "gravity" is taken to include the *universal gravitation* which exists between the Earth, Sun, Moon, and other bodies, it is perfectly correct to speak of the "weight of the Earth." If, however, "weight" is defined *merely* by terrestrial gravity, or by weighing with a pair of scales, the term "weight of the Earth" is meaningless.

3. **The Metric System** of units, originally introduced by the French, is far more convenient for calculations than the English system, and for this reason it is now very generally used in all scientific measurements even in this country.

The metric **unit of length** is the **metre,** and was originally defined as the ten-millionth part of a quadrant of the Earth's circumference, measured from the North Pole to the Equator.*

The submultiples of the metre have been named as follows:—†

1 metre = 10 **deci**_metres_
= 100 **centi**_metres_ (**cm.**)
= 1000 **milli**_metres_ (**mm.**);

and the multiples of the metre are—

1 **deca**_metre_ = 10 metres,
1 **hecto**_metre_ = 100 metres,
1 **kilo**_metre_ = 1000 metres,
1 **myria**_metre_ = 10,000 metres.

In scientific work, the **centimetre** is usually chosen as the unit of length, instead of the metre.

A _metre_ = **39·37 inches.**

A _decimetre_ is nearly 4 inches. _Three centimetres_ are very nearly the diameter of a penny.

760 _millimetres_ (the average height of the mercury in a barometer) = 30 inches.

The unit of area corresponding to the centimetre is the area of a **square centimetre**; and we observe that—

a square decimetre = 10×10 or 100 square centimetres,

and a square metre = 100×100 or 10,000 sq. cm.

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* Since the metre was introduced, the Earth's circumference has been measured with greater accuracy; but it was not considered advisable to alter the standard metre originally adopted, and which is preserved in Paris. The Earth's circumference may be taken as 40,000,000 metres in ordinary calculations.

† The prefixes _deci-_, _centi-_, _milli-_ are derived from the Latin for 10, 100, 1000, and _deca-_, _hecto-_, _kilo-_, _myria-_ from the Greek for 10, 100, 1000, 10,000.

The corresponding **unit of volume** is the capacity of a **cubic centimetre** (**c.c.**) (a cube whose length, breadth, and height are each 1 centimetre). Note that

$$\text{a cubic decimetre} = 10 \times 10 \times 10 \qquad = 1000 \text{ c.c.},$$

$$\text{a cubic metre} \qquad = 100 \times 100 \times 100 = 1{,}000{,}000 \text{ c.c.}$$

The French unit of fluid measure is called a **litre,** and was originally defined as the volume of a *cubic decimetre.*

A litre = 1·76 pints.

4. **The metric unit of weight** is the **gramme (gm.).** It was originally defined as the weight of a cubic centimetre of water, at temperature 4° Centigrade (39° Fahrenheit).*

Since bodies expand with heat and contract on cooling, the *temperature* of the water must be given. If a long-necked flask of boiling water be taken, it will be noticed on cooling down that the bulk of the water diminishes gradually until 4° C. is reached; then, as it continues to cool, water, contrary to the general law, gradually increases in volume, and so becomes lighter. 4° C. is therefore the temperature of maximum density of water, and this temperature will in future be assumed unless otherwise stated.

The submultiples and multiples of the gramme, proceeding by powers of 10, are denoted by the same prefixes to the word gramme as in the case of the metre. Thus a **milli***gramme* = $\frac{1}{1000}$ gramme and a **kilo***gramme* = 1000 grammes.

Reduced to English measure, a kilogramme is nearly represented by **2·2044 lbs.**

A kilogramme is the weight of a **litre** of water at temperature 4° C. *This is the definition of the litre.* Hence, if a cubic centimetre weighed *exactly* a gramme, a litre would be *exactly* a cubic decimetre, and this may be taken to be the case in all ordinary calculations.

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* Like the metre, the gramme is now defined by means of the original standard kilogramme, a piece of platinum preserved at Paris. For all practical purposes however, a cubic centimetre of water may be taken to weigh a gramme.

5. **Units of Force.**—When, as in Hydrostatics, we have to deal chiefly with the forces due to the *weights* of bodies, it is most convenient to measure forces in *gravitation units.* The **gravitation unit of force** is a force equal to the absolute weight of a unit of mass. Hence the English gravitation unit is the weight of a pound, and the Metric gravitation unit is the weight of a gramme. When we speak of a "force of 10 pounds" or a "force of 10 grammes," we mean a "force equal to the absolute weight of 10 pounds" or of "10 grammes," and the force is measured in gravitation units.

The same number which measures the mass of a body also measures its weight in gravitation units. Thus a body of mass 10 lbs. weighs 10 lbs.

Similarly, we may say that "a cubic centimetre of water *weighs* 1 gramme," or "the mass of a cubic centimetre of water is 1 gramme," and both statements are correct.

[In Dynamics it is shown that the absolute weight of a given quantity of matter is not quite the same at different parts of the Earth, and hence that the weights of a pound and a gramme are not constant units of force. For this reason forces are measured in terms of two dynamical units, the poundal and the dyne, both of which are defined without reference to gravity. To reduce pounds' weight to poundals, or grammes to dynes, it is only necessary to multiply by "g" the acceleration of gravity, measured in the foot-pound-second or the centimetre-gramme-second system of units, as the case may be. Taking the usual values of "g," a pound weight = 32 poundals, and a gramme weight = 981 dynes.]

In Hydrostatics, forces should always be calculated in gravitation units unless the contrary is expressly specified.

6. **Work.**—When a force moves its point of application, the **work done** by the force is the product of the force into the distance through which its point of application moves in the direction in which the force acts. When the point of application moves in the opposite direction, the work is negative.

The English gravitation **unit of work** is the **foot-pound,** or the work done by raising a weight of 1 lb. through 1 foot. If W lbs. are raised through a vertical height of h feet, the work done is Wh foot-pounds.

The centimetre-gramme-second gravitation unit is the *gramme-centimetre* or work done in raising a weight of 1 gramme through 1 centimetre. A larger and more convenient unit is the **kilogrammetre** or work done in raising a kilogramme through 1 metre. Hence a kilogrammetre $= 1000 \times 100 = 100{,}000$ gramme-centimetres.

7. **The Principle of Conservation of Energy** asserts that when a body or machine of any kind is acted on by any number of forces (efforts and resistances) which are in equilibrium, the sum of the works done by the several forces in any displacement of the body or machine is zero. In other words, when a machine is acting, no more work will be got out of it than is put into it.

This principle has many important applications to Hydrostatics.

8. **Summary.**—The principal facts connected with the Metric System are shown on page 7. The following statistics are mostly only rough, but may be found convenient for reference.

(1)	Earth's radius		= 4000 miles.	
(2)	Earth's circumference		= 40,000 kilometres.	
(3)	Height of barometer		= 30 inches.	
(4)	,, ,,		= 760 millimetres.	
(5)	Accel. of gravity g		= 32 feet	per sec. per sec.
(6)	,, ,,		= 980 centimetres	per sec. per sec.
(7)	Cubic foot	of water	= 1000 oz.	
(8)	Gallon	of water	= 10 lbs.	
(9)	Cub. centimetre	of water	= 1 gramme.	
(10)	Cub. decimetre or litre	of water	= 1 kilogramme.	
(11)	1000 kilogrammes		= 1 ton (*minus* 36 lbs.).	

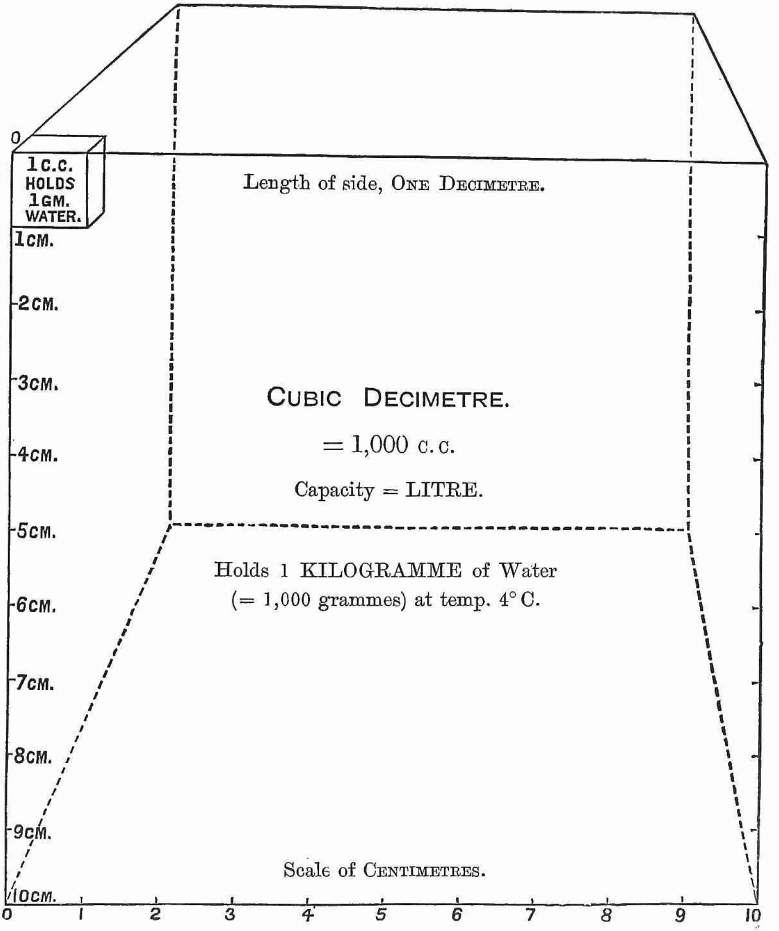
0
1 C.C. HOLDS 1 GM. WATER.
Length of side, ONE DECIMETRE.
1 CM.
2 CM.
3 CM.
CUBIC DECIMETRE.
= 1,000 c.c.
Capacity = LITRE.
4 CM.
5 CM.
Holds 1 KILOGRAMME of Water
(= 1,000 grammes) at temp. 4° C.
6 CM.
7 CM.
8 CM.
9 CM.
10 CM.
Scale of CENTIMETRES.
0 1 2 3 4 5 6 7 8 9 10

Fig 1.

9. **On the use of formulæ in Hydrostatics.**—(1) Although in the following chapters many results will be established in the form of algebraic formulæ, it must be carefully borne in mind that such formulæ are merely mathematical statements of facts, and that the essential feature of Hydrostatics consists in its *principles* and *practical applications* rather than *formulæ*. In order to acquire a sound knowledge of the subject, it is therefore of great importance that numerical calculations should be deduced directly from the principles themselves, and not by substituting numerical values for the symbols in an algebraic formula. For this reason, many of the general algebraic investigations in this book are *preceded* (instead of being followed) by worked-out numerical examples in illustration of them, and this is done in every case where such numerical calculations are of practical importance. In any case where this has not been done, the student is advised to work examples by following (in many cases, word for word) the methods adopted in the bookwork, but substituting at every step the numerical values for the algebraic letters given in the text. By doing so, a far more thorough knowledge of the subject will be acquired.

(2) The elementary student should not attempt to follow an algebraic proof by reading only; he should copy it out, following each line as he sets it down, and then recapitulate.

It is also important that, in writing out calculations, the meaning of each step should be written down; it is of little or no use to obtain the right answer to a question unless the method used has been understood and clearly stated. By adopting this plan, students will be saved from taxing their memory with a number of formulæ which are difficult to remember and are sure to be forgotten when wanted, but which can immediately be deduced from first principles.

(3) *In stating results of numerical calculations, the unit of measurement must always be specified.* Thus, for example—"a force of 100" has no meaning, for it might be taken to mean a force of 100 dynes, or 100 grammes weight, or 100 lbs. weight, or 100 tons weight, or 100 of any other unit whatever; before we can attach any definite meaning to it, we must say which unit is employed. Moreover, it is undesirable to use some units of the metric and others of the English system in the same calculation; one set should be preserved throughout.

PART I.

SPECIFIC GRAVITIES OF SOLIDS AND LIQUIDS.

CHAPTER I.

SOLIDS, LIQUIDS, AND GASES.

1. **Hydromechanics,** as its name implies, comprises all those portions of Mechanics which relate to fluids. It is divided into two branches — Hydrostatics and Hydrodynamics.

Hydrostatics deals with the equilibrium of fluids and with the forces acting on them when at rest.

Hydrodynamics deals with the motion of fluids under the action of forces.

The name *Hydraulics* is generally given to those portions of Hydrodynamics which are useful to the practical engineer; it relates to the flow of water through pipes, mains, and canals, the construction of water-wheels, &c.

2. **The three states of Matter.**—Every one is more or less familiar with matter in its three states of solid, liquid, and gas. In ice, water, and steam we have examples of a single substance which is capable of existing in either of the three states, according to circumstances. When frozen, it takes the form of a solid (ice); at ordinary temperatures it is a liquid (water); and when boiled by

heating, it becomes a vapour or gas (steam). All simple substances (the chemical "elements") are able to exist in each of these states. Thus all the metals can be melted and even turned into vapour by the application of heat.

For some time air and certain other gases were considered to be "permanent gases" which could not be turned into liquids, and a distinction was drawn between these "permanent gases" and "vapours." But in 1878 two physicists, M. Cailletet and M. Pictet, succeeded in liquefying not only air (which is a *mixture* containing oxygen and nitrogen and carbon dioxide), but also oxygen, nitrogen, and other gases previously supposed to be permanent. Moreover, most of these gases (except hydrogen) have been solidified.

3. Solids and Fluids.—From our everyday experience we get a fairly good idea of the general difference between solids, liquids, and gases. In Hydrostatics a general idea is not sufficient; we must give exact definitions, and these we can base on common experience.

We know that a solid body, such as a piece of ice, metal, glass, or wood, always retains the same shape; if put into a bottle, it does not adapt its shape to that of the bottle. We cannot force a piece of stick into it, nor can we stir it up.

On the other hand, liquids and gases, such as water and air, will easily flow from one vessel into another. Thus, if water be poured into a bottle, it adapts itself to the shape of the bottle, and fills the whole of the bottom part. If air be blown into the bottle, it will leave no empty spaces, but will fill the bottle. Again, water is very easily stirred up with a stick, and air is still more easily stirred, so much so, that when we move about we hardly experience any perceptible resistance from the air which we displace.

Hence we may distinguish the two kinds of matter, *solid* and *fluid*, by the property that the former retains a definite shape and cannot be stirred up, while the latter flows easily from one shape to another and can be readily stirred. In exact words—

DEF.—A **solid** is a substance which tends to keep the same shape for an indefinite length of time, and whose various parts cannot move freely among themselves.

DEF.—A **fluid** is a substance which yields to any force, however small, tending to change its shape or to produce movement of its parts among themselves.

It might be remarked that fine sand can be easily stirred, but that thick treacle is much more difficult to stir, and therefore that the sand ought to be considered fluid and the treacle solid. But a sufficiently light piece of stick may be made to stand upright in sand for any length of time, while, if it were stood upright in treacle, it would, in the course of time, fall over. The sand *never* yields to the weight of the stick, and therefore *each of the individual grains of sand* possesses the properties of a solid body. The treacle, on the other hand, yields *in the long run, however light the stick may be*, and this characterizes it as a fluid. Many solids may be moulded from one shape into another by applying considerable forces or pressures to them, but they do not yield to "*the slightest*" force.

*4. **Rigidity.**—The property in virtue of which a body tends permanently to retain the same shape is called *rigidity*. Hence a solid is distinguished from a fluid by being rigid.

5. Liquids and Gases. — Both liquids and gases (*e.g.*, water and air) are fluids according to the above definition. But they differ in one important respect. If a bottle is half full of water, the water cannot be made to occupy either more or less than half of the bottle. If the bottle is full, we cannot get any more water in by squeezing, nor can we squeeze the water into a smaller space by pushing a cork in or otherwise. On the other hand, any amount of air can be compressed into a bottle, or, again, part of the air in a bottle may be sucked out (by means of an air pump, such as will be described in Chap. XVIII.), and then the remainder will still continue to occupy the *whole* of the bottle. An easier experiment is to boil a little water in a corked bottle till it all becomes steam. The whole of the bottle will be filled with compressed steam, and unless the cork be fitted in tightly it will be forced out with considerable violence.

Part of the steam will then escape, but the remainder will still continue to fill the whole bottle. Hence we may distinguish a liquid from a gas by the property that the former cannot, and the latter can, be readily made to occupy a greater or less amount of space, or, in exact language—

DEF.—A **liquid** is a fluid whose volume will not increase beyond a certain limit, and which offers a very great resistance to any decrease of volume.

DEF.—A **gas** is a fluid which always tends to occupy as large a volume as possible, but which may be readily forced to occupy any space, however small.*

6. **Compressibility and Elasticity.** — A liquid is called *incompressible* when it cannot be forced to occupy a smaller volume; a gas is always *compressible*, because it can be easily compressed into any volume. No liquid is *perfectly incompressible;* by means of *great* pressure, water may be forced to occupy a slightly smaller bulk, but in Hydrostatics liquids may be treated as incompressible.

Again, liquids are called *inelastic*, because they have no tendency to expand and increase in bulk, while gases are called *elastic*, because they tend to expand so as to occupy as large a space as possible.

*7. **Perfect and viscous fluids.**—Although all fluids eventually yield to changes of shape or to stirring, different fluids behave differently *while changing their shape or being stirred*. Some seem to yield very readily, others only with apparent reluctance. Water may be stirred up easily and quickly, and little resistance will be experienced. But honey can only be stirred with difficulty, and the faster we try to stir it the more resistance we encounter. If, however, we were to stir it sufficiently slowly, we should feel hardly any resistance, showing that the honey is not *solid*).

* That is, so long as it remains a gas. But, if compressed very much, a gas will become liquid (compare § 2). Conversely, when liquid is introduced into a vacuum, part of it evaporates, and its vapour fills the space unoccupied by the liquid.

But the resistance always tends to *retard* the passage of the spoon through the honey. Hence we have the following definitions:—

A *perfect* fluid is one whose parts can move among themselves without retardation.

A *viscous* fluid is one which continually retards the motion of its parts among themselves.

Strictly speaking, there is no such thing as a perfect fluid. If water were a perfect fluid, a ship when once set in motion would continue to move through it without ever stopping, contrary to experience. Air and some gases much more closely resemble the ideal perfect fluid, but a bullet experiences considerable resistance from the air. Hence air is not a perfect fluid.

At the same time, some fluids are much more viscous than others.

Viscosity of fluids does not affect their equilibrium, but only their motion; and therefore it has not to be considered in Hydrostatics, but only in Hydrodynamics.

8. **The surface of a heavy liquid at rest is horizontal.**—For, if the surface were not perfectly horizontal, some parts would have to be higher than others. We could then draw an inclined plane—such as AB—cutting

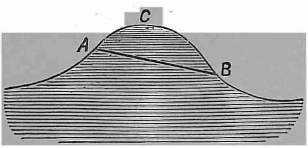

Fig. 2.

off the higher part ACB of the surface. The weight of the liquid above AB would tend to make it slide down the plane towards the lower part. And, by definition, the liquid yields to any force, however small, which tends to make its parts move separately. Hence, even if no other motion were possible, the liquid above AB would slide down the plane towards the places where the surface was lower. Therefore the liquid cannot remain in equilibrium unless the surface is perfectly horizontal.

*9. **Cohesion.**—If we try to break or cut a solid body in two, we experience considerable resistance. The property in virtue of which the different parts of a body resist separation is called *cohesion*. It is very easy to divide a quantity of water in two, showing that but little cohesion exists in most fluids. In some books a fluid is defined as "a substance whose particles yield to the slightest effort tending to separate them"—*i.e.*, a substance devoid of cohesion—but this definition is incorrect.

SUMMARY.

I. SOLIDS.—Permanent shape. Parts cannot move about freely.

II. FLUIDS.—No permanent shape. Yield continually to slightest force tending to move parts. Fluids are sub-divided into—

(i.) LIQUIDS.—*Incompressible*, *i.e.*, definite volume, cannot be reduced;
Inelastic, *i.e.*, volume does not expand (unless they evaporate).

(ii.) GASES.—*Compressible*, *i.e.*, volume can be reduced till they liquefy;
Elastic, *i.e.*, volume tends to expand indefinitely.

Viscosity exists in liquids and gases.

Cohesion exists in solids and liquids.

EXAMPLES I.

1. Distinguish between *solids* and *fluids*, and state what you regard as the essential features of a fluid. What is the special characteristic of a perfect fluid? What are *treacle*, *sand*, *putty*, *india-rubber*, *gold leaf*, *string*, *tar*, *alcohol*, and why?

2. Distinguish between a *liquid* and a *gas*. A bottle is half full of air and half full of water. What will be the effect (i.) of exhausting the air, (ii.) of pumping out the water, (iii.) of pumping in more air, (iv.) of pumping in more water, (v.) of dropping a piece of iron into the bottle?

3. If a gallon of water weighs 10 lbs., and a cubic foot weighs 1000 oz., how many gallons are there in a cubic foot?

4. Taking a ton = 1000 kilog. (roughly), a cubic foot of water = 1000 oz., and a cubic metre of water = 1000 kilog., find how many centimetres there are in a foot.

CHAPTER II.

DENSITY AND SPECIFIC GRAVITY.

10. **Relations between weight and volume of water on the English system.**—From the fact that a cubic foot of water contains 1000 oz., we can find the weight of a quantity of water, having given its volume, expressed in English units.

Examples.—(1) To find the weight of the water contained in a cistern 3 ft. long, 2 ft. broad, 3 ft. deep, filled to a depth of 2 ft.

The volume of the water depends on the depth of the water and not on that of the cistern, and is therefore $= 3 \times 2 \times 2 = 12$ cub. ft. Hence the weight of the water $= 12{,}000$ oz. $= 750$ lbs.

(2) To find the number of gallons of water in the cistern.

Since a gallon of water weighs 10 lbs., and the water in the cistern weighs 750 lbs., therefore its volume is 75 gallons.

11. **Examples on the Metric System.**

We have seen that a gramme is by definition equal in weight to a cubic centimetre of water (at the temperature of greatest density).

Hence, if any vessel is filled with water, the volume of the vessel in cubic centimetres is equal to the weight of the water in grammes.

Examples.—(1) Given a tank of length 25 cm., breadth 20 cm., height 16 cm. The volume $= 25 \times 20 \times 16 = 8000$ cub. cm., and the weight of water filling it $= 8000$ gm. $= 8$ kilog.

(2) To find the capacity of a bottle which weighs 165 gm. when empty, and 915 gm. when full, of water.

Here (weight of water) + (weight of bottle) = 915 gm.
Subtract (weight of bottle) = 165 gm.;
$\therefore$ (weight of water) = 750 gm.;
$\therefore$ volume occupied by water = 750 cub. cm.;

and the capacity is 750 cub. cm., or $\frac{3}{4}$ litre.

(3) To find the length of a tube whose sectional area is 3 sq. cm., and which takes 144 gm. of water to fill it.

The volume of the tube in cubic centimetres = weight of water in grammes = 144. Let h be its length; then, by mensuration,

$$3 \times h = \text{volume} = 144;$$

$$\therefore \quad h \text{ the required length} = 144 \div 3 = 48 \text{ cm.}$$

12. Density.—In the above examples we have based our calculations on the facts that a cubic foot of water contains 1000 oz., and that a gramme is defined as the weight of a cubic centimetre of water. If, however, we were to use mercury instead of distilled water, we should find that a cubic foot weighs 13,596 oz.; similarly, a cubic centimetre weighs 13·596 gm., and a cubic decimetre or litre weighs 13·596 kilog. Hence mercury is heavier in proportion to its bulk than water.

This shows that the weight of any quantity of matter does not depend only on its volume, but that it also depends on the *kind* of matter.

The same is true of solid, as well as of liquid, matter; thus, a bullet of lead is much heavier than a cork, even though the cork is the larger body of the two.

DEF.—The mass per unit volume of any substance is called the **density** of that substance.

The number which measures the density of a substance depends not only on the substance, but also on the choice of units of length and mass. Thus—

the density of water = **1** in centimetre-gramme system,
= **62½** in foot-pound system,
= **1000** in ounces per cubic foot.

13. **Relation between the volume, mass, and density.**—It is easy to find the mass of any given volume of a substance whose density is given.

Example.—Having given that the density of sea water is 64 lbs. per cub. ft., to find the mass of sea water in a rectangular tank whose base measures 3 ft. by 2 ft., filled to a height of 18 ins.

The volume of water in the tank = $3 \times 2 \times 1\frac{1}{2}$ cub. ft. = 9 cub. ft.
Hence the mass of the water in it is 9 times that of a cubic foot.
But a cub. ft. contains 64 lbs.
Hence the tank contains $64 \times 9 = 576$ lbs.

14. To find the mass of a body whose volume is V and whose density is D.

Let M be the required mass.

From the proportionality of mass to volume, we see that the mass M of the volume V of matter is V times the mass of a unit volume.

But the mass of unit volume is equal to the density D.

Hence $$M = VD,$$

that is, $$\text{mass} = \text{volume} \times \text{density}.$$

From this relation, we have

$$\text{density} = \frac{\text{mass}}{\text{volume}}, \quad \text{or} \quad \text{volume} = \frac{\text{mass}}{\text{density}}.$$

Hence, if we know the mass and volume of a body, its density may be found; or, given any two of them, we can find the third.

Example.—To find the density of lead in the centimetre-gramme system, having given that a bullet of lead, 2 cm. in diameter, weighs 45·7 gm.

The bullet is a sphere whose radius $= 1$ cm.

Hence its volume $= \frac{4}{3} \times \frac{22}{7} \times (1)^3 = \frac{88}{21}$ cub. cm.

Also the mass of the bullet $= 45 \cdot 7$;

$\therefore$ density of lead $= 45 \cdot 7 \times \frac{21}{88} = 11 \cdot 4$ gm. per cub. cm.

15. Def.—The **specific weight** of a substance is the *weight* of a unit volume of the substance.

Since the weight of a unit volume expressed in pounds or grammes is the measure of the **mass** per unit volume (Introduction, § 2), it follows that *the specific weight of a substance is numerically equal to its density, provided that its weight is measured by means of a set of weights*, as is the common practice.

[This relation is no longer true if by "*weights*" are meant "*absolute weights*," measured in poundals or dynes, or other dynamical units of force.]

Thus the specific weight of water

$= 62\frac{1}{2}$ *lbs.* per cub. ft. $= 62\frac{1}{2} \times 32$ or 2000 *poundals* per cub. ft.

$= 1$ *gm.* per cub. cm. $= 981$ *dynes* per cub. cm.

16. Specific Gravity.—In § 12, we saw that the density of a substance depends not only on the kind of matter forming it, but also on the chosen units of length and mass. But, if equal volumes be taken of two different substances, their masses, and therefore also their weights, will always be the same ratio, no matter what be the units of measurement.

Thus the weights of a cubic foot of sea and fresh water are 1024 and 1000 oz., and their ratio = 1·024. The weights of a cubic yard (27 cub. ft.) are, respectively, 1024×27 and 1000×27 oz., but their ratio is still = 1·024, as before. And the ratio is unaltered by reducing both weights to pounds, since this is merely the same as dividing both sides of the ratio by the same number 16.

This ratio will be called the *specific gravity* of sea water.

It is, therefore, more convenient, instead of measuring the actual densities of substances, to compare the masses or weights of equal volumes of different substances, and for this purpose one particular substance is always chosen as the standard substance, with which all others are compared.

The **standard substance** universally adopted (except in comparing certain gases) is **water** at a temperature of 4° C. or 39° Fahr. (its point of maximum density). We have seen that this is the substance chosen in defining the gramme.

Def.—The **specific gravity** of a substance is the ratio of the weight of any volume of that substance to the weight of an *equal* volume of the standard substance.

The abbreviation for specific gravity is **sp. gr.**

Examples.—(1) Thus, from what has been shown in § 16, the specific gravity of sea water is 1·024. This implies that *any* volume of sea water is 1·024 times as heavy as an equal volume of fresh water.

(i.) A cubic foot of sea water weighs $1·024 \times 1000$ oz., or 64 lbs.

(ii.) A gallon of sea water is 1·024 times as heavy as a gallon of fresh water, and therefore weighs 10·24 lbs.

(iii.) A cubic centimetre of sea water is 1·024 times as heavy as a cubic centimetre of fresh water, and therefore weighs 1·024 gm.

(iv.) A litre of sea water is 1·024 times as heavy as a litre of fresh water, and therefore weighs 1 kilog. 24 gm.; and the same proportionality holds for any equal volumes of fresh and sea water.

(2) Again, the specific gravity of **mercury** is about **13·6.** This implies that

(i.) A cubic foot of mercury weighs $13\frac{3}{5}$ times as much as a cubic foot of water, or 13,600 oz.

(ii.) A gallon of mercury weighs $13\frac{3}{5}$ times as much as a gallon of water, or 136 lbs., and so on.

17. Relations between Density and Specific Gravity.

Since weight is proportional to mass, therefore, taking water as the standard,

THE SPECIFIC GRAVITY of a substance is equal to

$$\frac{\textbf{weight}\text{ of any vol. of substance}}{\textbf{weight}\text{ of equal vol. of water}} = \frac{\textbf{mass}\text{ of any vol. of substance}}{\textbf{mass}\text{ of equal vol. of water}}$$

$$= \frac{\text{weight of } unit \text{ vol. of substance}}{\text{weight of } unit \text{ vol. of water}} = \frac{\text{mass of } unit \text{ vol. of substance}}{\text{mass of } unit \text{ vol. of water}}$$

$$= \frac{\textbf{specific weight}\text{ of substance}}{\textbf{specific weight}\text{ of water}} = \frac{\textbf{density}\text{ of substance}}{\textbf{density}\text{ of water}}.$$

Hence the density of any substance

= (specific gravity of substance) × (density of water);

and the specific weight of any substance

= (specific gravity of substance) × (specific weight of water).

The specific gravity of water itself (at temperature 4° C.) is, of course, unity. In fact, the specific gravity of the standard substance is necessarily unity.

Since the gramme is so chosen that the density of water at 4° C. in the centimetre-gramme system is also unity, it readily follows that **the specific gravity of a substance is equal to its density in the centimetre-gramme system.**

This fact constitutes one of the many advantages of the C.G.S. system.

Example.—The density of a piece of crystal is 155·75 in the foot-pound system. What is its specific gravity?

Weight of a cubic foot of the crystal = 155·75 lbs.,

Weight of a cubic foot of water = 62·5 lbs.;

$$\text{Specific gravity} = \frac{\text{weight of substance}}{\text{weight of equal volume of water}}$$

$$= \frac{155{\cdot}75}{62{\cdot}5} = 2{\cdot}492.$$

18. Relations between the volume, weight, and specific gravity.—We can now find the weight of any volume of a substance of given specific gravity.

Examples.—(1) To find in ounces the weight of a cubic inch of lead, taking the specific gravity of lead to be 11·4.

Weight of a cubic foot of water = 1000 oz.;

$\therefore$ weight of a cubic inch of water = $\frac{1}{1728}$ cub. ft. = $\frac{1000}{1728}$ oz.

But a cubic inch of lead weighs 11·4 times as much;

$$\therefore \text{ weight of a cubic inch of lead} = \frac{1000 \times 11{\cdot}4}{1728} = 6{\cdot}55 \text{ oz., approx.}$$

(2) To find the weight of 40 litres of sea water.

40 litres of fresh water contain 40 kilog.

Therefore 40 litres of sea water contain $40 \times 1{\cdot}024$ kilog.

= 40·96 kilog. = 40 kilog. 960 gm.

19. To find the weight of a body whose volume is V and whose specific gravity is S.

Let W be the required weight, w the specific weight of water.

Then weight of unit volume of water = w;

$\therefore$ weight of volume V of water = wV.

But weight of volume V of substance is S times as great;

$\therefore$ W (the required weight) = VSw;

that is, *weight* of body = (volume) × (specific gravity) × (specific *weight* of water).

SUMMARY.

1. The *density* of a body $= \dfrac{\text{mass}}{\text{volume}}$.

2. The *specific weight* of a body $= \dfrac{\text{weight}}{\text{volume}}$.

3. The *specific gravity* of a body $= \dfrac{\text{weight}}{\text{wt. of equal vol. of water}}$.

The density of *water* (temp. 4° C.) = 1 gm. per cub. cm.
= 1000 oz. per cub. ft.
= $62\frac{1}{2}$ lbs. per cub. ft.

The density of *sea water* = 64 lbs. per cub. ft.

The specific gravity of *mercury* = 13·6 roughly [accurately 13·596].

EXAMPLES II.

[The following specific gravities are given : — Air, 0·0012 ; alcohol, 0·835 ; copper, 8·9 ; gold, 19·25 ; ice, 0·92 ; lead, 11·35 ; mercury, 13·6 ; sea water, 1·024.]

1. Find the weights of fresh and sea water, respectively, required to fill the following vessels, and their capacities in gallons :—
   (i.) a rectangular trough 5 ft. long, 1 ft. broad, 1 ft. deep ;
   (ii.) a tank 5 ft. long, 4 ft. broad, 6 ft. deep ;
   (iii.) a piece of hose 30 ft. long and $\frac{1}{2}$ in. internal diameter.

2. Find the weights of water and mercury, respectively, required to fill the following vessels, and their capacities in litres :—
   (i.) a trough 5 cm. long, 4 cm. broad, filled to a depth of 1 cm. ;
   (ii.) a barometer tube 760 mm. long, 1 cm. in diameter ;
   (iii.) a hemispherical bowl 20 cm. in diameter.

3. If the rainfall is 1 in., how many tons of water fall on an acre?

4. If the rainfall is 1 cm., how many tonnes fall on a hectare?
   [An *are* = 100 sq. metres ; a *hectare* = 100 ares.]

5. If 5 cub. ins. of mercury weigh 2·45 lbs. and 2 cub. ins. of cast iron weigh 0·52 lb., what ratio does the density of mercury bear to that of cast iron?

6. The density of cast iron in the C.G.S. system of units is 7·2. What is its density in the foot-pound system of units?

7. Explain what is meant by the statement that the specific gravity of mercury is 13·596.

8. Write down the weights of
   (i.) a cubic foot of copper ; (ii.) a cubic inch of lead ;
   (iii.) a cubic yard of air ; (iv.) a gallon of alcohol ;
   (v.) a cubic centimetre of gold ; (vi.) a cubic metre of ice ;
   (vii.) a litre of mercury ; (viii.) a hectolitre of sea water.

9. What are the specific gravities of substances of which
   (i.) 1 cub. in. weighs 1 oz. ; (ii.) 1 cub. yd. weighs 1 ton ;
   (iii.) 1 pint weighs 1 lb. ; (iv.) a ball, 10 cm. in diameter, weighs 1 kilog. ;
   (v.) 1 kilog. fills 240 cub. cm. ; (vi.) 1000 kilog. fill 625 litres.

10. If 5 cub. ins. of silver weigh as much as 21 cub. ins. of plate glass, and the specific gravity of silver be 10·5, find that of plate glass.

11. A body A has a volume of 2 cub. yds. and specific gravity of 1·1. A second body B has a volume of $\frac{1}{8}$ cub. ft. and specific gravity 4·95. What ratio does the quantity of matter in A bear to that in B?

12. Show that, if the specific gravity of a substance be multiplied by $\frac{3}{4}$, the product will be the weight of a cubic yard of the substance in tons, very nearly, water being the standard substance.

13. Show that, if 35·84 be divided by the specific gravity of a substance, the quotient will be the number of cubic feet contained in a ton of the substance very nearly, water being the standard substance.

14. Show that the specific gravity of any substance

$$= \frac{\text{volume of an equal weight of water}}{\text{volume of substance}}.$$

15. The outer radius of a hollow leaden bullet containing a spherical cavity is R, and its weight is W. If w is the weight of a unit volume of lead, show that the radius of the cavity

$$= \sqrt[3]{\left\{R^3 - \frac{3}{4\pi}\frac{W}{w}\right\}}.$$

16. Show that the units may be chosen so that the specific gravity and the density of a substance are identical. What is the relation between the unit of volume and the unit of weight when the weight of a body is numerically equal to 1000 times the product of the volume and specific gravity?

17. Show that the volume of a body varies directly as the weight and inversely as the specific gravity.

18. The specific gravity of any substance is the weight of any volume of that substance divided by the weight of an equal volume of water. Is it correct to substitute the word "mass" for weight in the above statement?

CHAPTER III.

SPECIFIC GRAVITIES OF MIXTURES.

20. **Mixtures by Volume and by Weight.**—It is often necessary to find the specific gravity of the mixture formed by taking given quantities of different substances and mixing them together. When the volumes of the several substances are given, the mixture is said to be a **mixture by volume.** When their weights are given, it is said to be a **mixture by weight.**

The total weight of a mixture is invariably equal to the sum of the weights of its component parts.

The total volume of the mixture is in most cases equal to the sum of the volumes of its parts, but not invariably so.

When sulphuric acid and water are mixed together, the mixture contracts and occupies a smaller volume than its separate parts together occupied before mixing, and, generally, where chemical action takes place, there is a change in the total volume. *Where no data are given* by which the amount of the contraction could be determined, it is always to be assumed that no contraction takes place, and, therefore, that the general principle holds good.

21. **Determination of the specific gravities of mixtures by volume.**—If then the volumes of each of the ingredients forming a mixture are given, and the specific gravity of each is also known, the weight of each can be found from the formula of § 19,

$$\text{(weight)} = \text{(volume)} \times \text{(specific gravity)} \times \text{(specific weight of water)}.$$

Hence, by applying the principle of addition stated above, the weight and volume of the mixture are obtained, and from these its specific gravity may be determined.

Examples.—(1) To find the specific gravity of a mixture of 2 cub. ft. of fresh water and 3 cub. ft. of sea water, having given that the specific gravity of sea water is 1·026.

Here 2 cub. ft. of fresh water weigh 2000 oz., and 3 cub. ft. of sea water weigh $3 \times 1{\cdot}026 \times 1000$ oz. $= 3078$ oz.

Hence the weight of the mixture $= 5078$ oz.

Also the volume of the mixture $= 5$ cub. ft.

$\therefore$ the weight of an equal volume of water $= 5 \times 1000$ oz.

$\therefore$ the specific gravity of the mixture $= \frac{5078}{5000} = 1{\cdot}0156$.

It is not really necessary to know the actual volumes of the components, provided that their relative proportions are known. In this case, we may proceed as in the following examples, which may be taken as types.

(2) To find the specific gravity of a mixture of 3 parts (by volume) alcohol, 2 parts water, and 1 part glycerine; given that the specific gravity of alcohol is 0·794, and that of glycerine is 1·26.

Let w be the weight of 1 part of water.

Then the weight of 3 parts of alcohol $= 3 \times {\cdot}794w$
$= 2{\cdot}382w$.

Also the weight of 2 parts of water $= 2w$.

Also the weight of 1 part of glycerine $= 1{\cdot}26w$.

$\therefore$ the weight of 6 parts of the mixture $= 5{\cdot}742w$.

But the weight of an equal volume of water $= 6w$.

$$\therefore \text{ specific gravity of mixture} = \frac{5{\cdot}742}{6} = {\cdot}957.$$

(3) An amalgam is formed by mixing 3 volumes of potassium with 7 of mercury, the volume of the amalgam being four-fifths of that of its constituents. Find its specific gravity, being given that specific gravities of mercury and potassium are 13·596 and 0·860, respectively.

Let w be the weight of 1 volume of water.

Then weight of potassium $= 3w \times 0{\cdot}860$,
weight of mercury $= 7w \times 13{\cdot}596$.

Volume of mercury and potassium $= 3$ vols. $+ 7$ vols. $= 10$ vols.,

and volume of amalgam $=$ four-fifths of this $= 8$ vols.

$\therefore$ weight of equal volume of water $= 8w$.

$$\therefore \text{ specific gravity} = \frac{\text{weight of amalgam}}{\text{weight of equal volume of water}}$$
$$= \frac{3w \times 0{\cdot}860 + 7w \times 13{\cdot}596}{8w} = 12{\cdot}219.$$

The following is the general theorem of which these examples are illustrations :—

22. To find the specific gravity of a mixture in terms of the volumes and specific gravities of the several components.

Let V_1, V_2, V_3, ... V_n be the volumes of the different components, S_1, S_2, S_3, ... S_n their respective specific gravities, and let w be the weight of a unit volume of water.

Then the weights of the different components are

$$V_1S_1w, \quad V_2S_2w, \quad V_3S_3w, \ \dots \ V_nS_nw;$$

hence the weight of the mixture is

$$= w\,(V_1S_1 + V_2S_2 + V_3S_3 + \dots + V_nS_n).$$

Again, if no contraction takes place on mixing, the volume of the mixture

$$= V_1 + V_2 + V_3 + \dots + V_n.$$

Hence the weight of an equal volume of water

$$= w\,(V_1 + V_2 + V_3 + \dots + V_n);$$

$\therefore$ **specific gravity of mixture**

$$= \frac{\text{weight of mixture}}{\text{weight of equal vol. of water}}$$

$$= \frac{V_1S_1 + V_2S_2 + V_3S_3 + \dots + V_nS_n}{V_1 + V_2 + V_3 + \dots + V_n} \quad \dots\dots \ \mathbf{(1)}.$$

If the volume contracts on mixing, we must know the new volume of the mixture. Let this be V; then the weight of an equal volume of water is wV. Hence

specific gravity of mixture

$$= \frac{V_1S_1 + V_2S_2 + V_3S_3 + \dots + V_nS_n}{V} \quad \dots\dots \ \mathbf{(1a)}.$$

The above formulæ might be applied to numerical examples on the determination of specific gravities of mixtures, but it is better to work out each case from first principles, as in Examples 1, 2, 3, § 21. The student is, however, recommended to *verify* the above examples now by substitution in the formulæ as an instructive exercise.

23. **Determination of specific gravities of mixtures by weight.**—When the weights of the ingredients forming a mixture are given, and their specific gravities are known, their volumes may be readily found by § 19. Hence, by § 20, we know both the total weight and volume of the mixture, whence its specific gravity can be found.

Examples.—(1) To find the specific gravity of a mixture of 2000 oz. of fresh water and 3000 oz. of sea water, having given that the specific gravity of sea water is 1·026.

Here 2000 oz. of fresh water occupy 2 cub. ft., and 3000 oz. of sea water occupy $\frac{3000}{1026}$ cub. ft. $= 2{\cdot}924$ cub. ft.

$\therefore$ the volume of the mixture $= 2{\cdot}924 + 2 = 4{\cdot}924$ cub. ft.,

and the weight of an equal volume of water $= 4924$ oz.

But the weight of the mixture $= 5000$ oz.;

$\therefore$ the specific gravity of the mixture $= \frac{5000}{4924} = 1{\cdot}0154$.

Note.—The specific gravity is slightly *less* than in Ex. 1, § 21. This is because 3000 oz. of the heavier sea water occupy *less* than 3 cub. ft., and therefore the proportion *by volume* of the heavier component is now less than before. Cf. § 26.

(2) To find the specific gravity of a mixture of 3 parts (*by weight*) of alcohol, 2 parts of water, and 1 part of glycerine; given that the specific gravity of alcohol is 0·794, and that of glycerine is 1·26.

Let W be the weight of each part, w the weight of a unit volume of water.

Then a unit volume of alcohol weighs $\cdot 794w$, and a unit volume of glycerine weighs $1{\cdot}26w$.

But the weights of alcohol, water, and glycerine are

$$3W, \quad 2W, \quad W.$$

Hence their volumes are

$$\frac{3W}{\cdot 794w}, \quad \frac{2W}{w}, \quad \frac{W}{1{\cdot}26w}.$$

$\therefore$ the whole volume

$$= \left(\frac{3}{\cdot 794} + 2 + \frac{1}{1{\cdot}26}\right)\frac{W}{w} = (3{\cdot}778 + 2 + 794)\frac{W}{w} = 6{\cdot}572\frac{W}{w};$$

and the weight of an equal volume volume of water $= 6{\cdot}572W$.

But the weight of the mixture $= 6W$.

$$\therefore \text{ specific gravity of mixture} = \frac{6{\cdot}000}{6{\cdot}572} = \cdot 913.$$

The student should carefully compare this example with Ex. 2, § 21.

The general theorem is as follows:—

24. To find the specific gravity of a mixture in terms of the weights and specific gravities of its several components.

Let W_1, W_2, ... W_n be the weights of the different components, S_1, S_2, ... S_n their respective specific gravities, and let w be the weight of unit volume of water.

Then the weight of the mixture

$$= W_1 + W_2 + \dots + W_n.$$

If V_1, V_2, ... V_n are the volumes of the components,

then $\quad W_1 = wS_1V_1$, &c.; $\quad \therefore \quad V_1 = \dfrac{W_1}{wS_1}$.

Hence the total volume of the mixture

$$= \frac{1}{w}\left(\frac{W_1}{S_1} + \frac{W_2}{S_2} + \dots + \frac{W_n}{S_n}\right),$$

and the weight of an equal volume of water

$$= \frac{W_1}{S_1} + \frac{W_2}{S_2} + \dots + \frac{W_n}{S_n};$$

therefore **specific gravity of mixture**

$$= \frac{W_1 + W_2 + \dots + W_n}{\dfrac{W_1}{S_1} + \dfrac{W_2}{S_2} + \dots + \dfrac{W_n}{S_n}} \quad \dots\dots\dots\dots\dots\dots (\mathbf{2}).$$

*25. **The specific gravity of a mixture is increased by increasing the proportion of its heaviest constituent, and decreased by increasing the proportion of its lightest constituent.** For if we replace any volume of any substance in the mixture by an equal volume of a heavier substance, we increase the weight of the mixture without altering the volume. Hence we increase its specific gravity. And conversely.

*26. *The specific gravity of a mixture in given proportions by volume is greater than that of a mixture in the same proportions by weight.*— For, if m *volumes* of a heavier substance are mixed with n volumes of a lighter substance, the former are heavier in proportion to their bulk than the latter, and therefore their proportion by weight is greater than m to n. Hence, by § 25, the specific gravity is greater than if the proportions *by weight* were as m to n.

27. Determination of the composition of a mixture from its specific gravity.

When a mixture is known to consist of *two substances only*, whose specific gravities are known, we may always find the relative proportions of these components if we know the specific gravity of the mixture.* Such determinations are of the greatest practical value. The specific gravity of a mixture of spirit and water enables us to find the proportion of spirit in it; the specific gravity of an alloy of two metals enables us to find the relative amounts of the metals, and the specific gravity of a nugget enables us to find the amount of gold which the nugget contains.

The following examples illustrate the method of solving problems of this kind:—

Examples.—(1) Having given that the specific gravities of gold and quartz are 19·35 and 2·15, respectively, to find the proportions of gold and quartz in a nugget of specific gravity 5·69.

Let x be the volume of gold per unit volume of the nugget.

Then $1-x$ is the corresponding volume of the quartz.

Taking the unit of weight such that the specific weight of water is unity, the weights of the gold and quartz in a unit volume are, respectively, $19{\cdot}35x$ and $2{\cdot}15\,(1-x)$.

But the weight of a unit volume of the nugget is 9·49.

$$\therefore\quad 19{\cdot}35x+2{\cdot}15\,(1-x) = 5{\cdot}69\,;$$

$$\therefore\quad 17{\cdot}20x = 3{\cdot}44\,;$$

$$\therefore\quad x = \tfrac{1}{5},\ \text{and}\ 1-x = \tfrac{4}{5}.$$

Therefore the volumes of the gold and quartz are, respectively, $\frac{1}{5}$ and $\frac{4}{5}$ of the whole volume. The weights of the gold and quartz are therefore in the proportion of

$$\tfrac{1}{5}\times 19{\cdot}35 : \tfrac{4}{5}\times 2{\cdot}15,$$

i.e., $$9 : 4.$$

Hence the weights of gold and quartz occurring in the nugget are $\frac{9}{13}$ and $\frac{4}{13}$ of the whole weight.

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* The specific gravity of the mixture must be found *by actual experiment*. We shall see how to do this in the next few chapters, and then the process of determining the composition of the mixture will be complete.

(2) To find the weights of copper (sp. gr. = 8·8) and zinc (sp. gr. = 7) in 1 lb. of brass (sp. gr. = 8).

Let the weight of copper be x lbs. Then the weight of zinc is $(1-x)$ lbs.

Hence we may substitute $W_1 = x$, $W_2 = 1-x$, $S_1 = 8·8$, $S_2 = 7$, in formula (2) of § 24, and since the specific gravity of the mixture is 8, therefore

$$8 = \frac{x + (1-x)}{\frac{x}{8·8} + \frac{1-x}{7}}$$

$$= \frac{8·8 \times 7}{7x + 8·8\,(1-x)} = \frac{88 \times 7}{88 - 18x};$$

$$\therefore \quad 88 - 18x = 77;$$

whence

$$x = \tfrac{11}{18},$$

$$1 - x = \tfrac{7}{18}.$$

Therefore 1 lb. of brass contains $\frac{11}{18}$ lb. of copper and $\frac{7}{18}$ lb. of zinc.

Summary.

1. If S is the specific gravity of a mixture, and if no contraction takes place,

$$S = \frac{V_1 S_1 + V_2 S_2 + \ldots + V_n S_n}{V_1 + V_2 + \ldots + V_n} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1).$$

If the volume of the mixture contracts to V,

$$S = \frac{V_1 S_1 + V_2 S_2 + \ldots + V_n S_n}{V} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1a).$$

2. If the weights of the ingredients are given,

$$S = \frac{W_1 + W_2 + \ldots + W_n}{\frac{W_1}{S_1} + \frac{W_2}{S_2} + \ldots + \frac{W_n}{S_n}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2).$$

Here $S_1, S_2, \ldots, S_n$ are the specific gravities of the components

$V_1, V_2, \ldots, V_n$ their volumes,

$W_1, W_2, \ldots, W_n$ their weights.

EXAMPLES III.

1. Define *specific gravity* and *density*. A certain mass of liquid, whose specific gravity is 0·5, is mixed, without suffering contraction, with four times that mass of a second liquid, whose specific gravity is 1·25. Find the specific gravity of the mixture.

2. A nugget of gold mixed with quartz weighs 12 oz., and has a specific gravity 6·4; given that the specific gravity of gold is 19·35, and of quartz is 2·15, find (to one place of decimals) the quantity of gold in the nugget.

3. Four pints of alcohol, having a specific gravity of ·75, are mixed with one pint of water (specific gravity 1). Find the specific gravity of the mixture, no change of volume being supposed to take place.

4. Two vessels each contain 3 pints of fluid, the specific gravity of the one fluid being twice that of the other. Two pint tumblers are filled, one out of each vessel, and then each tumbler is emptied into the vessel from which it was not drawn. Prove that, after the process has been three times gone through, the specific gravities of the fluids are to each other as 41 : 40.

5. To a salt solution, whose specific gravity is 1·08 and weight 27 oz., 4 oz. of water are added. Find the specific gravity of the mixture.

6. A Prussian dollar, made of an alloy of silver and copper, has the specific gravity 10·05. Determine the relative amount of silver and copper in it, the specific gravity of silver being 10·5, that of copper 8·7.

7. Three equal vessels A, B, C are half full of liquids, densities d_1, d_2, d_3, respectively. If now B is filled up from A and then C from B, find the density of the liquid now contained in C, the liquids being supposed to mix completely.

8. A mixture is made of 7 cub. cm. of sulphuric acid (specific gravity $= 1·843$) and 3 cub. cm. of distilled water, and its specific gravity when cold is found to be 1·615. Determine the contraction which has taken place.

9. How many gallons of water must be mixed with 20 gallons of

milk of specific gravity 1·032 in order to give a mixture of specific gravity 1·02?

10. The specific gravity of gold is 19·3, that of silver is 10·4. What is the composition of an alloy of gold and silver whose specific gravity is 17·6, no change of volume being supposed to accompany the admixture of the metals.

11. If the specific gravity of zinc be 6·88, and that of copper be 8·92, how much of each must be taken in order to obtain 100 gm. of an alloy of the two metals whose specific gravity is 8·41?

12. Determine the volumes of two liquids, the densities of which are 1·2 and ·8 respectively, which must be mixed in order to obtain a mixture of 8 volumes whose density is ·95.

13. If equal volumes of two liquids be mixed, a mixture is obtained the specific gravity of which is 1·12. If, however, two volumes of one liquid are added to one volume of the other, the specific gravity of the mixture is 1·16. Find the specific gravities of the two liquids.

14. If a volume v_1 of a liquid whose specific gravity is s_1 be mixed with a volume v_2 of a liquid whose specific gravity is s_2, and the specific gravity of the mixture is s, find the change of volume.

15. When equal volumes of two substances are mixed together, the specific gravity of the mixture is 4; when equal weights of the same substances are mixed together, the specific gravity of the mixture is 3. Find the specific gravities of the two substances.

16. A mixture has to be made by taking m parts by weight of one substance and n parts by weight of another. Instead of this, m parts by volume of the first and n parts by volume of the second are taken. Show that the specific gravity of the mixture is greater than if the proper proportions were taken.

17. The specific gravity of a mixture of two different liquids being supposed to be an arithmetic mean between those of the component liquids, required the ratio of the volumes of the latter contained in the mixture.

18. If equal weights of two different substances be mixed, show that the specific gravity of the mixture is the harmonic mean of the specific gravities of the component substances.*

* x is said to be the harmonic mean between a and b if $2/x = 1/a + 1/b$.

EXAMINATION PAPER I.

1. Define a *fluid*. What are the distinguishing features of *liquids* and *gases?* What is a *powder*, a *soft solid*, a *viscous fluid?*

2. Define *density* and *specific gravity*. How are they measured?

3. Find the density of the standard substance, water, when 1 metre and 1 kilog. are the units of length and mass, respectively.

4. What is the weight of 10 cub. ft. of a substance whose specific gravity is 6·4?

5. Show how to find the specific gravity of a mixture when the volumes and specific gravities of the components are given.

6. 500 cub. cm. of a gas whose density is 14 are mixed with 200 cub. cm. of a gas whose density is 16, and the mixture occupies 510 cub. cm. Find its density.

7. Equal volumes of alcohol (specific gravity = ·796) and water are mixed, and the specific gravity of the mixture is found to be ·938. Find the percentage diminution of volume.

8. Weights W_1, W_2 of two substances whose specific gravities are, respectively, s_1, s_2 are mixed together, and the volume of the mixture is found to be less than the combined volumes in the ratio of $r : 1$. Find the specific gravity of the mixture.

9. If zinc (specific gravity = 7) and copper (specific gravity = 8·8) are mixed in the proportion of 2 : 5 by weight, find the specific gravity of the mixture.

10. How much tin (specific gravity = 7·3) must be mixed with 5 oz. of antimony (specific gravity = 6·7) so that the specific gravity of the mixture may be 7·2?

CHAPTER IV.

DIRECT DETERMINATION OF DENSITIES. THE SPECIFIC GRAVITY BOTTLE.

28. **To find the density and specific gravity of a solid or liquid by direct measurements.**—If the shape of a solid body is any one of the solid figures treated in Mensuration, its volume can be found by direct measurements of its size.

The mass of the solid can be found by weighing with a common balance. By dividing the mass by the volume, the mass per unit volume is found, and this is the required density of the solid. If the C.G.S. system is used in measuring and weighing the solid, its specific gravity is equal to its density (§ 17). If not, the calculated density must be divided by the density of water (in terms of the chosen units of length and mass) in order to obtain the specific gravity.

In order to find the density of a liquid by this method, a vessel must be taken whose capacity must be calculated from direct measurements of its interior. The vessel must then be placed in the scale-pan of a balance and weighed empty. If it be now filled with liquid and again weighed, the difference of the weights when empty and when full determines the mass of the liquid filling the vessel.

Dividing this by the calculated capacity, the density of the liquid is found as before.

29. **The specific gravity bottle** is much used for finding the specific gravities of solids and liquids. It is constructed for the purpose of weighing exactly equal volumes of different liquids, and it consists of a glass flask having a tightly fitting stopper through which a

very fine hole (ab) is bored. In using the bottle, it is completely filled with the liquid to be weighed, and the stopper is then pushed in till it reaches a certain mark (P) on the neck of the bottle. The superfluous liquid overflows through the hole ab, and is wiped off; so the bottle, when filled in this way, always contains the same volume of liquid.

To obviate the necessity of allowing for the weight of the bottle in every observation, a counterpoise is provided, whose weight is exactly equal to that of the bottle. This counterpoise is usually a little metal case containing small shot, and its weight is adjustable by adding or subtracting shot.

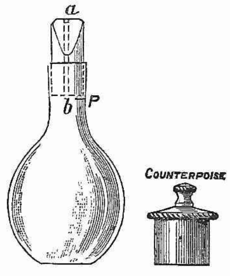

Fig. 3.

When the bottle, filled with liquid, is placed in one of the scale-pans of a balance, the counterpoise is placed in the other pan in addition to the weights used in weighing. Since the counterpoise balances the weight of the bottle, the additional weights give the weight of the contained liquid alone.

30. To find the specific gravity of a liquid by means of the specific gravity bottle. — The process is as follows:—

(i.) Adjust the weight of the counterpoise (if necessary) till it balances the bottle when empty.

(ii.) Fill the bottle with water, carefully insert the stopper, and weigh, placing the counterpoise in the scale-pan containing the weights.

(iii.) Fill the bottle with the liquid whose specific gravity is required, carefully insert the stopper, and again weigh, as before.

The second process gives the weight of the water contained in the bottle. The third process gives the weight of an equal volume of the given liquid. Dividing the latter by the former, the specific gravity of the liquid is found.

Example.—A flask weighs 7·2 gm. when empty, 53·45 gm. when filled with sulphuric acid, and 32·2 gm. when filled with water. To find the specific gravity of sulphuric acid.

Weight of sulphuric acid = 53·45 − 7·2 gm. = 46·25 gm.;
weight of equal volume of water = 32·2 − 7·2 gm. = 25·0 gm.
∴ specific gravity of sulphuric acid = 46·25 ÷ 25 = 1·85.

When the counterpoise has been made equal in weight to the empty bottle its weight is never altered.

Moreover, specific gravity bottles are usually constructed to hold 10, 20, 25, 50, or 100 gm., or 250, 500, or 1000 grains of water, and when this is the case there is no need to weigh the bottle when filled with water.

31. To find the specific gravity of a solid by the original method of Archimedes.

In order to find the specific gravity of a solid, it is necessary—

(i.) To weigh the solid.

(ii.) To find the weight of an equal volume of water.

Now let any vessel be filled to the brim with water, and let the solid be then immersed in it. A quantity of liquid equal in volume to the solid will overflow. Let this liquid be weighed. Then the weight of the solid divided by this weight gives the specific gravity required.

This property was discovered by **Archimedes**, a mathematician of Syracuse, in Sicily, where he flourished about 250 B.C. Hiero, the king of Syracuse, had given to a goldsmith a certain weight of gold to be made into a crown. Suspecting that a portion of the gold had been replaced by an equal weight of alloy, the king applied to Archimedes for a test. While thinking the matter over, Archimedes chanced to enter his bath, where it occurred to him that he displaced a quantity of water equal to the volume of his body. This suggested that, if the crown contained an alloy of less specific gravity than the gold, it would, when immersed in water, displace a greater quantity of water than a crown of pure gold and of the same weight. When the experiment was made, the king's suspicions were justified.

32. To find the specific gravity of a solid substance insoluble in water, by means of the specific gravity bottle.

(i.) Weigh the solid.

(ii.) Fill the specific gravity bottle with water, and place it, together with the solid, in one of the scale-pans of a balance, and weigh.

(iii.) Take the solid and insert it in the bottle. A quantity of water will overflow whose volume is equal to that of the solid, and the volume of water in the bottle will be less than before by the volume of the solid. If, therefore, the bottle containing the solid and water be again weighed, their total weight will be less than before by the weight of the displaced water. Dividing the weight of the solid by the latter weight, the required specific gravity of the solid is found.

The specific gravity bottle can only be used to find the specific gravity of a solid substance when broken up into fragments sufficiently small to go into the bottle. It is, therefore, particularly useful in finding the specific gravities of powders—*e.g.*, sand.

Examples.—(1) The weight of a solid is 13 gm. When the specific gravity bottle is filled with water, its weight, together with that of the solid, is 63 gm. When the solid is put into the bottle, the combined weight is 53 gm. To find the specific gravity of the solid.

After the solid is dropped into the bottle, the volume of water in the bottle is less than it was before by an amount equal to the volume of the solid.

Hence the difference of weights, $63 - 53$ or 10 gm., equals the weight of a quantity of water equal in volume to the solid.

But the weight of the solid is 13 gm.;

$$\therefore \quad \text{specific gravity of solid} = \tfrac{13}{10} = 1{\cdot}3.$$

(2) The weight of a quantity of powder (insoluble in water) is p. The weight of a specific gravity bottle filled with water is A, and when the bottle contains the powder and is filled with water its total weight is B. To find the specific gravity of the powder.

Let w be the weight of water whose volume is equal to that of the powder.

Then, before the powder is placed in the bottle, the total weight of the powder, bottle, and water $= p + A$.

When the powder is placed in the flask it displaces a quantity of water equal in volume to the powder, whose weight is w.

Therefore the total weight B is less than before by w, that is,

$$B = p + A - w;$$
$$w = p + A - B.$$

$$\therefore \quad \text{sp. gr. of powder} = \frac{\text{weight of powder}}{\text{weight of water displaced}} = \frac{p}{p + A - B}.$$

Hence, if p, A, and B are known, the specific gravity can be found. Notice that it is not necessary to know the weight of the specific gravity bottle itself.

Summary.

1. With *the specific gravity bottle*, the specific gravity of a *liquid*

$$= \frac{\text{weight of liquid which fills the bottle}}{\text{weight of water which fills the bottle}}.$$

2. *Archimedes* discovered that when a solid is immersed in liquid it displaces an equal volume of liquid.

3. To find the specific gravity of a *solid*,

(i.) Weigh the solid;

(ii.) Weigh the specific gravity bottle full of water;

(iii.) Drop in the solid, and weigh again.

Hence calculate weight of water which overflows at third observation. Then

$$\text{specific gravity of solid} = \frac{\text{weight of solid}}{\text{weight of water displaced}}.$$

EXAMPLES IV.

1. A rectangular block of marble whose length is 75 cm., width 50 cm., and depth 25 cm., weighs $266\frac{1}{4}$ kilog. Find its density.

2. Describe the process of determining the specific gravity of a liquid by means of a specific gravity bottle, and show how the capacity of the bottle may be found by filling it with liquid of known density.

3. Define *density*. A specific gravity bottle, completely full of water, weighs 38·4 gm.; and, when 22·3 gm. of a certain solid have been introduced, it weighs 49·8 gm. Calculate the density of the solid, explaining clearly why the result is density in accordance with your definition, and also why the second weighing differs from the first by less than the weight of the introduced solid.

4. If water be taken as the standard substance, a cubic foot of which weighs 1000 oz., what is the specific gravity of a substance of which 16 cub. yds. weigh $84\frac{3}{4}$ tons?

5. If water be taken as the standard substance, a cubic foot of which weighs 1000 oz., what is the specific gravity of a substance of which 27 cub. ins. weigh 10 drs.?

6. The weight of a specific gravity bottle when empty is 42 gm., and when full of water and glycerine, respectively, its weight is 222 gm. and 292 gm. Find the specific gravity of glycerine.

7. A specific gravity bottle full of water weighs 44 gm.; and when some pieces of iron weighing 10 gm. in air are introduced into the bottle, and the bottle again filled up with water, the combined weight is 52·7 gm. What is the specific gravity of the iron?

8. A specific gravity bottle weighs 500 gm. when full of water; 50 gm. of a given powder is put into the bottle, which is filled up with water, and the whole weighs 530 gm. What is the specific gravity of the powder?

9. How much mercury of density 13·6 will be required to fill a tube whose length is 20 cm. and mean section ·015 sq. cm.?

10. Find the mean section of a tube 28 cm. long which holds 1 gm. of glycerine of density 1·26.

11. A pound of iron is to be drawn into wire, having a diameter of ·05 in. What length will it yield, the specific gravity of iron being 7·6?

12. The weight of a flask when empty is w, when filled with water its weight is A, and when filled with a certain liquid its weight is B. What is the specific gravity of the liquid?

CHAPTER V.

FLOTATION—THE PRINCIPLE OF ARCHIMEDES. THE HYDROSTATIC BALANCE.

We now come to certain methods of finding specific gravities which depend on measuring the forces that a fluid exerts on an immersed solid.

33. **Buoyancy.**—When a body lighter than water is dropped into water, it floats at the top of the water. If, however, the body is heavier than water, it sinks to the bottom. This is a fact which we know from everyday experience. Thus a cork floats on water while a stone sinks to the bottom. If we push a cork down under water, it will again rise to the surface, though the force of gravity on it acts *downwards*.

Therefore we can infer that a fluid is capable of exerting an upward force or thrust tending to lift any immersed body to the surface.

We commonly speak of this action as due to the **buoyancy** of the fluid. The upward force is really produced by the pressure which the fluid exerts against the surface of the solid. We shall now investigate the amount of this force by a simple application of the Principle of Work, leaving a fuller discussion of the pressures of fluids on immersed or floating bodies till Chap. XIII.*

34. **To find the upward force which a heavy fluid exerts on an immersed body.**

When a body of volume V sinks in a fluid, an equal volume V of the fluid is displaced to make room for it (§ 31). As the body sinks, this fluid is raised, and hence work must be done against its weight. And in sinking

* The student who has not read the Principle of Work in Dynamics may omit what follows, and pass on to the statement of the Principle of Archimedes (§ 35), relying on the experiments of § 39 to establish it.

from A (Fig. 4) to B (Fig. 5) the solid displaces the fluid at B, and fluid rises and fills the space V at A.

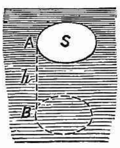

Fig. 4.

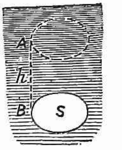

Fig. 5.

Hence the solid in sinking through a vertical height h must do work against the reaction of the fluid sufficient to raise an equal volume V of fluid through an equal height h.

Hence, if R be the reaction of the fluid, we have, by equating the works done,

$$R \times h = (\text{weight of volume } V \text{ of fluid}) \times h\,;$$

$$\therefore \quad R = \text{weight of volume } V \text{ of fluid},$$

or **upward thrust of fluid on solid**

= weight of fluid equal in volume to the solid

= weight of fluid displaced by the solid.

35. The Principle of Archimedes.—The principle proved above is known as the **Principle of Archimedes,*** and is generally stated thus:—

A solid immersed in fluid loses as much of its weight as is equal to the weight of the fluid which it displaces.

Example.—The upward thrust on a body of volume 8 cub. ft. totally immersed in water

= wt. of 8 cub. ft. of water = 8000 oz. = 500 lbs.

In the case of sea water (p. 20) the thrust = 8 × 64 lbs. = 512 lbs. wt.

36. **Case of a floating body.**—If the weight of a solid is less than the upward thrust due to the weight of the fluid displaced, the solid rises till it *floats*.

When the solid is only partially immersed (Fig. 6), the space which it occupies is divided into two parts U, V by AB, the plane of the surface of the fluid.

It is clear that no fluid is displaced by the upper portion U, nor does the fluid exert any pressure on this

* Whether Archimedes used this principle in experimenting with the crown of Hiero (§ 31), or discovered it later, is uncertain.

portion. Hence the *fluid displaced* is the fluid which would fill the submerged portion V, and is less than the whole volume of the solid. The solid therefore comes into a position of equilibrium in which *the weight of the fluid displaced* (V) *is equal to the weight of the solid.*

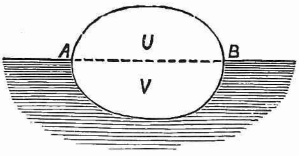

Fig. 6.

Example.—To find the weight and specific gravity of a body of volume $\frac{1}{2}$ cub. ft., which, when attached to 5 cub. ft. of cork (sp. gr. $\cdot 24$), floats with 1 cub. ft. of the whole projecting.

Total wt. supported = wt. of $4\frac{1}{2}$ cub. ft. of water displaced = 4500 oz.

But wt. of 5 cub. ft. of cork $= 5 \times \cdot 24 \times 1000$ oz. = 1200 oz.;

$\therefore$ wt. of body $= 4500 - 1200$ oz. = 3300 oz. = $206\frac{1}{4}$ lbs.;

and sp. gr. of body $= 3300/500 = 6 \cdot 6$.

37. **Equilibrium of immersed bodies.**—An immersed body is always acted on by two forces:

1st. The weight of the body acting downwards.

2nd. The thrust of the fluid acting vertically upwards and equal to the weight of the fluid displaced.

If the weight of the solid exceeds that of the fluid displaced, the body will *sink*. To support it, we must suspend the body by a string, whose tension

= weight of solid − weight of fluid displaced.

Cor. Hence a solid placed in fluid will sink or float according as the solid or the fluid has the greater density.

[For illustrative examples, see p. 127.]

38. **The Hydrostatic Balance.** — When a common balance is adapted for weighing bodies suspended in fluid, it is called a *Hydrostatic Balance* (Fig. 7). The only difference between a hydrostatic balance and an ordinary pair of scales is that one of the scale-pans in the former is at a sufficient height to allow a vessel of fluid to be placed under it, and has a hook on its under side from which any small solid may be suspended by means of a fine wire, and weighed when immersed in the fluid.

39. To verify the Principle of Archimedes by experiments with a Hydrostatic Balance.

EXPERIMENT I. — (i.) Take two brass cylinders, one hollow, the other solid and of such a size as just to fit into the first. Suspend them from the scale pan of the hydrostatic balance, the hollow one uppermost, and weigh.

(ii.) Now let the solid cylinder be immersed in water and the two again weighed; they will be found to be considerably lighter than before.

(iii.) Lastly, let the upper cylinder be filled with water and the two weighed, the lower cylinder still being immersed. It will be found that their combined weight is exactly the same as at the first observation.

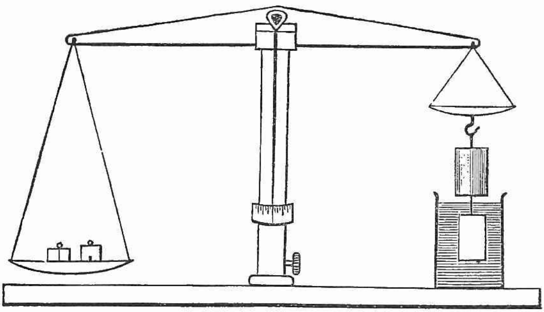

Fig. 7.

Since the lower cylinder exactly fits the upper, the volume of the water in the upper cylinder is exactly equal to the volume of the lower cylinder.

Hence the apparent loss of weight when the lower cylinder is immersed in water is exactly equal to the weight of an equal volume of water, that is, to the weight of the fluid displaced.

EXPERIMENT II.—(i.) Take a vessel or flask (such as a specific gravity bottle). Place in it any body, fill it up with water, and weigh it in the scale-pan of the hydrostatic balance.

(ii.) Take the body out of the flask and hang it from below the scale-pan by means of a fine thread, so that it is immersed in a vessel of water. Fill the flask up with water again, place it in the scale-pan, and weigh. It will be found that exactly the same weights have to be placed in the opposite scale-pan as before.

Now, when the solid is removed from the flask, an additional volume of water equal to that displaced by the solid has to be poured in to fill the flask up. Hence the weight of the flask and of the water in it is greater than before by the weight of the water displaced by the solid. Therefore part of the weight of the solid is supported by the reaction of the water, and this reaction is equal to the weight of the fluid displaced.

40. To verify the Principle of Archimedes by experiments with floating bodies.

Take any open vessel filled to the brim with water. Take any body which is lighter than an equal volume of water, and gently lower it into the water until it floats. A quantity of water will overflow out of the vessel, whose volume is equal to that of the immersed portion of the solid. Let this water be weighed; then its weight will be found to be equal to that of the solid.

Hence, when a solid floats in equilibrium, the weights of the solid and of the fluid displaced are equal.

41. **Effect of density of fluid.**—In order to prove the principle perfectly, generally it would be necessary to repeat the above experiments, using different liquids. When this is done, it is found that in every case the loss of weight on immersion is equal to the weight of the fluid displaced. Since this is proportional to its density, it follows that the reaction of a fluid on a given immersed body is proportional to the density of the fluid.

[Examples illustrative of the principles contained in this chapter will be found on p. 127.]

Summary.

The Principle of Archimedes asserts that a heavy fluid exerts an upward reaction on an immersed or floating solid equal to the weight of fluid displaced by the solid.

If the solid floats, the weight of the displaced fluid equals that of the solid, but its volume is only equal to that of the submerged portion.

EXAMPLES V.

1. It is required to determine with great accuracy the weight of a cubic centimetre of water, and it would be very difficult to construct a vessel whose internal capacity is exactly 1 cub. cm. Can you suggest any alternative plan?

2. A ship is said to draw more water in a river than at sea. If this be so, what is the reason?

3. A man weighing 160 lbs. floats with 4 cub. ins. of his body above the surface. What is his volume in cubic feet?

4. A cube of wood, whose edge is 10 ins. and specific gravity ·8, floats in water. What weight must be placed on it in order to just totally immerse it?

5. A cube of wood floating in water descends 1 in. when a weight of 270 oz. is placed upon it. Find the size of the cube.

6. A cylinder weighing 1 lb., floating in water with its axis vertical and each of its ends horizontal, requires a weight of 4 oz. to be placed on its upper surface to depress it to the level of the water. Find the specific gravity of the cylinder.

7. What is the specific gravity of a substance a cubic foot of which will just float in water when attached to a cubic foot of cork of specific gravity ·2?

8. What is the specific gravity of a metal a cubic foot of which will just float in glycerine of specific gravity 1·25 when attached to 6 cub. ft. of cork of specific gravity ·24?

9. How many cubic feet of cork of specific gravity ·24 must be attached to 1 cub. ft. of glass of specific gravity 2·9, in order that the whole may float in water just immersed?

10. A solid of which the volume is 1·6 cub. cm. weighs 3·4 gm. in a fluid of specific gravity ·85. Find the specific gravity and weight of the substance.

11. A piece of cork whose weight is 19 oz. is attached to a bar of silver weighing 63 oz., and the two together just float in water. The specific gravity of silver is 10·5 times that of water. Find the specific gravity of the cork.

12. A body, whose mass is 10 lbs. and specific gravity ·75, dips into water, and is supported partly by the buoyancy of the water and partly by the tension of an attached string which passes over a smooth pulley, and carries at its other end a mass of 2 lbs. hanging freely in the air. Find what fraction of the volume of the first body is immersed.

13. A piece of iron weighing 275 gm. floats in mercury of density 13·59 with five-ninths of its volume immersed. Determine the volume and the density of the iron.

14. An earthenware box and its lid form a hollow cube which floats just immersed in water. The thickness of the material is on all sides one-eighth of an edge of the cube. Find the specific gravity of the earthenware.

15. Describe experiments to prove that the upward force which a fluid exerts on an incompressible solid immersed in it depends only on the bulk of the body, the density of the fluid, and the intensity of gravity, and is independent of the depth of immersion and of the shape of the body. What difference would it make if the body were readily compressible?

16. A stone of specific gravity 4 is dropped from a height of 16 ft. above the surface of a lake 36 ft. deep. Supposing no sudden change of velocity takes place at the surface of the water, in what time will the stone reach the bottom of the lake?

EXAMINATION PAPER II.

1. 15 cub. ft. of a substance weigh $2\frac{11}{32}$ tons; find its specific gravity.

2. A cylinder, whose height is 21 ins. and the radius of whose base is 8 ins., weighs $412\frac{1}{2}$ lbs.; find its density. $(\pi = \frac{22}{7}.)$

3. A cone of lead (specific gravity = 11·34), whose height is 5 cm. and the radius of whose base is 4 cm., balances a sphere of brass (specific gravity = 8·4). Find the radius of the sphere.

4. Describe the method of using the specific gravity bottle in finding the specific gravities of liquids and powders.

5. Enunciate the Principle of Archimedes.

6. A specific gravity bottle holds 154·5 gm. of a liquid whose specific gravity is 1·03, and 108 gm. of ether; find the specific gravity of ether.

7. A specific gravity bottle filled with water weighs 36·8 gm. A piece of spar weighing 1·68 gm. is placed in it, and the whole now weighs 37·85 gm. Find the specific gravity of the spar.

8. 100 gm. of a certain powder are placed in a specific gravity bottle weighing 50 gm. and capable of holding 500 gm. of water. The bottle is filled with ether of specific gravity ·72, and the whole is then found to weigh 474 gm. Find the specific gravity of the substance forming the powder.

9. A glass globe, weighing 100 gm. when exhausted of air, holds 2 litres of water at standard temperature. Full of air it weighs 102·586 gm., and full of hydrogen it weighs 100·1788 gm. Find the specific gravity of air with respect to water and hydrogen.

10. Give a short account of the Hydrostatic Balance. How would you use it to verify experimentally the Principle of Archimedes?

CHAPTER VI.

DETERMINATION OF SPECIFIC GRAVITIES BY THE HYDROSTATIC BALANCE.

42. To find the specific gravity of a solid which is heavier than an equal volume of water, using the hydrostatic balance.

(i.) Let the solid be first placed in the scale-pan and weighed in air.

(ii.) Let the solid be suspended in water by a very fine thread attached to the scale-pan of the balance and again weighed.

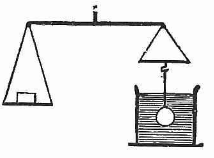

Fig. 8.

By the Principle of Archimedes, the difference of the observed weights in air and water is the weight of a quantity of water equal in volume to the solid. Dividing the weight of the solid by this, the specific gravity of the solid is found.

Examples.—(1) A solid weighs 15 gm. in air and 5 gm. in water; to find its specific gravity.

The weight in water is less than in air by the weight of the water displaced.

$$\text{weight of water displaced} = 15-5 = 10 \text{ gm.}$$

Also $$\text{weight of solid} = 15 \text{ gm.};$$

$$\therefore \quad \text{specific gravity of solid} = \tfrac{15}{10} = 1{\cdot}5.$$

(2) A piece of gold weighs 598·3 gm. in air, and 567·3 gm. in water; to find its volume and specific gravity.

$$\text{Weight of water displaced} = 598{\cdot}3-567{\cdot}3 \text{ gm.} = 31 \text{ gm.};$$

$$\text{volume of gold} = 31 \text{ cub. cm.},$$

and $$\text{specific gravity of gold} = 598{\cdot}3 \div 31 = 19{\cdot}3.$$

43. To find the specific gravity of a solid whose weight in air is W and whose weight in water is P.

The weight of the water displaced

$$= (\text{weight of solid in air}) - (\text{weight in water})$$

$$= W - P;$$

$$\therefore \text{ specific gravity of solid} = \frac{\text{weight of solid}}{\text{weight of water displaced}}$$

$$= \frac{W}{W-P} \quad \ldots\ldots\ldots\ldots\ldots\ldots \textbf{(1)}.$$

When the weights of a solid in air and in water are given, it would, of course, be possible to write down the specific gravity at once from this formula, but it is far better to proceed as in the preceding examples.

44. To find the specific gravity of a solid which is lighter than an equal volume of water.

If the solid were suspended by itself, it would float in water, and we could not find the weight of a quantity of water equal in volume to the whole solid. To remedy this, a heavy piece of metal, called a *sinker*, is attached to the thread which supports the body to be weighed, and this keeps the body under water.

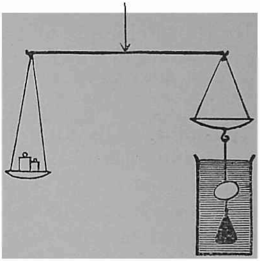

Fig. 9.

The operations are best performed in the following manner :—

(i.) Weigh the solid in air.

(ii.) Suspend the solid and sinker together from the scale-pan of the balance, and weigh them in water (Fig. 9).

(iii.) Weigh the sinker in water.

The weight in water of the solid and sinker combined is less than the weight in water of the sinker alone by the amount of the total resultant upward force on the solid. This force is the excess of the weight of water displaced by the solid over the weight of the solid itself. This being known and the weight of the solid being also known from the first observation, the weight of the water displaced is found, and the specific gravity of the solid can be found, as before.

Examples.—(1) A solid weighs 16 gm. in air. When attached to a sinker and immersed in water, the two together weigh 6 gm. The weight of the sinker in water alone is 10 gm. To find the specific gravity of the solid.

Here the weight of the solid and sinker in water together

$$= (\text{weight of sinker in water}) + (\text{weight of solid in air}) - (\text{weight of water displaced by solid}) = 6 \text{ gm.}$$

But weight of sinker in water = 10 gm.,

and weight of solid in air = 16 gm.;

$\therefore$ weight of water displaced by solid = $16 + 10 - 6$ gm.
= 20 gm.;

$$\therefore \text{ specific gravity of solid} = \frac{\text{weight of solid}}{\text{weight of water displaced}} = \tfrac{16}{20} = \cdot 8.$$

45. **The weight of a solid in air is W, the weight of a sinker in water is A, and the weight of the solid and sinker together in water is B. To find the specific gravity of the solid.**

Let w be the weight of the water displaced by the solid. Then we have, evidently,

weight of solid and sinker in water together

$$= (\text{weight of sinker in water}) + (\text{weight of solid in air}) - (\text{weight of water displaced by solid});$$

or
$$B = A + W - w.$$

Therefore, by transposition,

$$w = A + W - B.$$

Hence the weight of the water displaced by the solid is known, and we then have

$$\text{specific gravity of solid} = \frac{\text{weight of solid}}{\text{weight of water displaced}} = \frac{W}{w}$$

$$= \frac{W}{W + A - B} \quad \ldots\ldots\ldots\ldots \quad \textbf{(2)}.$$

46. To find the specific gravity of a *liquid* by means of the hydrostatic balance.

Take any solid body of greater specific gravity than either the liquid or water (the sinker used in the experiments of the last paragraph would do).

(i.) Weigh the solid in air.

(ii.) Weigh it in the given liquid whose specific gravity is required.

(iii.) Weigh it in water.

The difference between the weights in air and in the given liquid is the weight of the liquid displaced by the solid. The difference between the weights in air and in water is the weight of water displaced. And, since the volumes displaced in both cases are equal, the ratio of their weights is the specific gravity of the liquid.

Example.—To find the specific gravities of glass and glycerine, from the following data:—

Weight of a piece of glass in air				= 10 grs.,
,,	,,	,,	water	= 6 grs.,
,,	,,	,,	glycerine	= 5 grs.

Here weight of water displaced by glass = 10 − 6 = 4 grs.,
,, glycerine ,, ,, = 10 − 5 = 5 grs.,
and weight of glass = 10 grs.

$$\therefore \text{ specific gravity of glycerine} = \frac{\text{weight of glycerine displaced}}{\text{weight of water displaced}}$$
$$= \tfrac{5}{4} = 1 \cdot 25;$$

and specific gravity of glass $= \frac{10}{4} = 2 \cdot 5$.

47. **Having given that the weight of a body in air is W, its weight in water is P, and its weight in a given liquid is Q, to find the specific gravity of the liquid.**

Here weight of water displaced by solid $= W - P$,
,, liquid ,, ,, $= W - Q$.

$$\therefore \text{ specific gravity of liquid} = \frac{\text{weight of liquid}}{\text{weight of equal volume of water}}$$

$$= \frac{W - Q}{W - P} \quad \text{..............................} \quad (3).$$

We have hitherto considered only solids which do not dissolve in water; we now proceed—

48. To find the specific gravity of a solid which will dissolve in water.

If a solid is soluble in water, its specific gravity may be found by taking some liquid in which it is not soluble, and weighing the solid first in air, then in that liquid. The liquid taken must be of known specific gravity.

In this case, the apparent loss of weight in the liquid is equal to the weight of the liquid displaced by the solid.

Dividing this by the specific gravity of the liquid, the weight of an equal volume of water is found, and hence the specific gravity of the solid.

Example.—To find the specific gravity of a substance soluble in water, but not in turpentine, from the following data:—

$$\begin{aligned}\text{Weight of solid in air} &= 32 \text{ grs.},\\ \text{,, \quad ,, \quad turpentine} &= 3 \text{ grs.};\\ \text{specific gravity of turpentine} &= \cdot 87.\end{aligned}$$

The weight of turpentine displaced $= 32 - 3 = 29$ grs.

But weight of turpentine $= \cdot 87$ (weight of equal volume of water);

$$\therefore \text{ weight of equal volume of water} = \frac{29}{\cdot 87} = \frac{29 \times 100}{87} = \frac{100}{3} \text{ grs.};$$

$$\text{specific gravity of solid} = \frac{32 \times 3}{100} = \cdot 96.$$

NOTE.—In this example, the solid was specifically lighter than water, but heavier than turpentine. Had the solid been insoluble in water, it could not have been weighed in water without attaching it to a sinker.

49. **To find the specific gravity of a solid, having given that its weight in air is W, and that its weight in a liquid of specific gravity s is Q.**

The weight of liquid displaced by solid $= W - Q$,

and weight of equal volume of water $= \dfrac{W-Q}{s}$.

$$\text{Hence specific gravity of solid} = W \div \frac{W-Q}{s} = \frac{Ws}{W-Q} \quad \ldots\ldots\ldots \textbf{(4)}.$$

The above method may also be used to find the specific gravity of a solid which is lighter than water by weighing it in a liquid of still smaller specific gravity, thus dispensing with the use of a "sinker."

*50. *The above methods may be used to find the sectional diameter of a fine wire*, as in the following example :—

Example.—To find the diameter of a wire 1 metre long, and weighing 20 gm. in air and 18 gm. in water.

The weight of water displaced by the wire = 20 − 18 = 2 gm.

$\therefore$ the volume of the wire = 2 cub. cm.,

and its length = 100 cm.

$\therefore$ the area of its cross section $= \frac{2}{100} = \frac{1}{50}$ sq. cm.

Hence, if d be the diameter of the wire in centimetres, we have

$$\frac{\pi d^2}{4} = \frac{1}{50};$$

whence, taking $\pi = \frac{22}{7}$, we get

$$d^2 = \frac{4 \times 7}{22 \times 50} = \frac{7}{11 \times 25} = \frac{77}{11^2 \times 5^2}.$$

$$d = \frac{\sqrt{77}}{55} = \frac{8{\cdot}775}{55} = {\cdot}1595 \text{ cm.}$$

$$= 1{\cdot}595 \text{ mm.}$$

*51. Effect of displaced air on the weight of solids.

In finding specific gravities of solids, we supposed their weights found by weighing them in air with a common balance. If great accuracy is required, it will be necessary either to weigh the bodies *in vacuo*, or to allow for the fact that the bodies, as well as the set of weights employed, all displace more or less air, and therefore the apparent weight of a body in air is less than its true weight by the weight of this displaced air. But the density of air is very small compared with that of most solids and liquids, being $\frac{1}{1000}$ of that of water. Hence the weight of the displaced air is in most cases so small a fraction of the weight of the body that no serious error is introduced by neglecting it altogether.

It is easy, however, to make allowance for the displaced air, if necessary. For when a body is placed in one pan of a pair of scales, and balanced by weights in the other, the apparent weights or resultant forces tending to draw the body and weights towards the ground are equal. Hence

true weight of body − weight of air displaced by body

= weight of weights − weight of air displaced by weights.

Consider, for example, the well-known case of a pound (mass) of lead and a pound (mass) of feathers. The weights of the two *in vacuo* are equal. But the feathers displace more air than the lead, and therefore their apparent weight in air is less.

In investigations requiring very delicate weighings, it is, therefore, necessary to specify the metal of which the weights are made. Brass weights are commonly used, but platinum weights are employed in the most expensive chemical balances.

Summary.

Let W be the weight of a solid in air,
S its specific gravity,
P its weight in water, if heavier than water,
Q its weight in a given liquid,
s the specific gravity of the liquid,
A the weight of a sinker in water,
B the weight in water of a sinker and a solid lighter than water.

1. *To find the specific gravity of an insoluble solid heavier than water.*

Observe W and P. Then

$$S = \frac{W}{W-P} \quad \text{.................................} \quad (1).$$

2. *To find the specific gravity of a solid lighter than water.*

Observe W, A, B. Then

$$S = \frac{W}{W+A-B} \quad \text{.........................} \quad (2).$$

3. *To find the specific gravity of the liquid.*

Observe W, P, Q. Then

$$s = \frac{W-Q}{W-P} \quad \text{.................................} \quad (3).$$

4. *To find the specific gravity of a solid soluble in water but not in the liquid.*

Observe W, Q, s. Then

$$S = \frac{Ws}{W-Q} \quad \text{.................................} \quad (4).$$

EXAMPLES VI.

1. Distinguish between *actual* and *apparent* weight of a body. The apparent weight of a piece of platinum in water is 60 gm.; the actual weight of another piece of platinum twice as big as the former is 126 gm. Determine the specific gravity of platinum.

2. A piece of lead weighs 125 oz. in air and 114 in water. Find its specific gravity.

3. A piece of silver, whose specific gravity is 10·5, weighs 120 oz. in air. How much does it weigh in water?

4. A piece of chalk weighs 48 gm. in air and 28 gm. in water. Find its specific gravity.

5. A piece of copper weighs 10 lbs. in air and $8\frac{3}{4}$ lbs. in water. Find its specific gravity and its volume in cubic inches.

6. A piece of lead, whose weight in air is 285 gm. and specific gravity 11·4, is weighed in water. What will be its apparent weight?

7. Calculate the mass of 1 cub. cm. of a certain solid from the following data:—a mass of 720 gm. hanging from one pan of a balance is totally immersed in water, and found to be counterpoised by a weight of 645 gm. in the other pan.

8. A piece of iron (specific gravity = 7·21) weighing 216·3 gm. is attached to a piece of cork weighing 36 gm., and the weight of both in water is 36·3 gm.; find the specific gravity of the cork.

9. A piece of cork weighing 12 gm. is joined to a piece of iron (specific gravity = 7·21) weighing 72·1 gm. The loss of weight of the two in water is 72 gm. What is the specific gravity of cork?

10. What weight of cork of specific gravity ·24 must be attached to 57 cub. cm. of zinc of specific gravity 7·2 in order that the two may just float immersed in water?

11. A block of wood, the volume of which is 26 cub. ins., floats in water with two-thirds of its volume immersed. Find the volume of

a piece of metal, the specific gravity of which is 8 times that of the wood, which, when suspended from the lower part of the wood, will cause it to be just totally immersed.

When this is the case, find the upward force which will hold the wood half immersed.

12. A piece of paraffin weighs 4·273 gm. in air, and, when attached to a piece of lead which weighs 7·596 gm. in water, the two together weigh 6·423 gm. in water; determine the specific gravity of the paraffin.

13. A piece of glass weighed 8·602 gm. *in vacuo*, 5·854 gm. in water, and 6·395 in alcohol. Calculate the specific gravity of alcohol.

14. A solid body weighs 117 gm. in air, 98 in water, and 101 in another liquid. Calculate the specific gravities of the solid and the liquid.

15. A ball of glass weighing 665·8 grs. in air is found to weigh 465·8 grs. in water and 297·6 grs. in sulphuric acid. What is the specific gravity of the latter?

16. A bar of metal weighs 1275 grs. in air, 1147·5 grs. in spirit, and 1125 grs. in water. Find the specific gravities of the metal and the spirit compared to that of water.

17. A ball of metal weighs 9 lbs. in air and 8 lbs. when suspended in water. What would be the specific gravity of a liquid in which it would weigh $7\frac{1}{2}$ lbs.?

18. A piece of glass, specific gravity = 2·5, weighs 25 gm. in air and 15·6 gm in oil; find the specific gravity of the oil.

19. A piece of gold weighs 96 gm. in air, 91 gm. in water, and 92·4 gm. in ether. Find the specific gravities of the gold and ether.

20. A piece of glass weighs 47 gm. in air, 22 gm. in water, 25·8 gm. in alcohol. Find the specific gravity of alcohol.

21. Describe some method of finding the specific gravity of a fluid. A certain body just floats in water. On placing it in sulphuric acid of specific gravity 1·85, it requires an addition of 42·5 gm. to immerse it; find its volume.

22. A ball of metal is attached to a spring balance, and the index shows that it weighs 5 lbs. It is then allowed to dip below the surface of the water, and the weight appears to be 4·375 lbs. If it be immersed in a liquid of specific gravity ·75, what will then be the apparent weight?

23. A glass ball weighs 3000 grs., and has a specific gravity 3; what will be its apparent weight in a liquid whose specific gravity is ·92?

24. It is required to find the specific gravity of potassium, which decomposes water. A lump weighing 432·5 grs. in air is suspended in naphtha, the specific gravity of which is ·847, and is found to weigh 9 grs. What is the specific gravity?

25. A piece of iron weighs 260 gm. in water and 250 gm. in glycerine of specific gravity 1·25. Find its specific gravity.

26. A piece of silver, whose weight in water is 19 lbs. and specific gravity 10·5, is weighed in oil of specific gravity ·9. What will be its apparent weight?

27. An iron shell is found to lose half its weight when weighed in water. What portion of its volume is hollow? (Specific gravity of iron = 7·2.)

28. How would you determine the specific gravity of a gold medal by means of a hydrostatic balance furnished with brass weights? Explain how each weighing and the final result will be affected by the presence of air, if no correction is made for the air displaced.

29. A piece of silver and a piece of gold are suspended from the arms of an equal-armed balance beam, which is in equilibrium when the silver is immersed in alcohol (density ·85) and the gold in nitric acid (density 1·5). The densities of the silver and gold being 10·5 and 19·3, respectively, what are their relative masses?

30. Find the volume of a solid which weighs 500 gms. in air and 375 gms. in glycerine of specific gravity 1·25.

CHAPTER VII.

THE HYDROMETERS.

The instruments now to be described are chiefly used for finding the specific gravities of liquids.* They all depend on the principle of flotation, namely, that the weight of a floating body is equal to the weight of liquid which it displaces.

52. **Dr. Wilson's Glass Beads** are a series of hollow balls of glass, the diameters of the hollows in them being so adjusted that the average specific gravities of successive beads form a series of numbers increasing by ·002. Each bead is numbered according to its specific gravity, and the specific gravity of any liquid may be found by throwing them all into it.

All the beads of greater specific gravity than the liquid sink, and all those of lesser specific gravity float. By these means the specific gravity of the liquid is found to within ·002, and this degree of accuracy is sufficient for most purposes.

53. **Nicholson's Hydrometer** (Fig. 10) consists essentially of a hollow globe or cylinder of metal *B*, from the top of which projects a stem of hardened steel wire carrying a small cup or scale-pan *A*. To the bottom of *B* is fixed another cup or scale-pan *G*, which is of sufficient weight to keep the hydrometer from becoming top-heavy without sinking the whole of the bulb even in the lightest liquids. A set of weights is provided with the hydrometer, and these are to be placed in the upper scale-pan so as

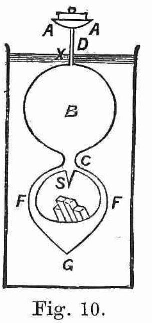

Fig. 10.

* Commercially, this is a very important operation, the specific gravity often being a rough but ready test of the purity of a liquid or degree of concentration of a solution. Thus the strongest ammonia has a specific gravity of ·880; it is a concentrated solution of ammonia gas in water.

to sink the hydrometer till the liquid in which it floats reaches a fixed mark X on the upright wire. For convenience, a cylindrical jar is usually provided, to contain the liquid whose specific gravity is required.

54. To find the specific gravity of any liquid by means of Nicholson's Hydrometer, the observations are made as follows :—

(i.) Find the weight of the hydrometer (in air).

(ii.) Lower the hydrometer into a jar of water, and add weights to the upper scale-pan A until the instrument sinks to the fixed mark on the stem.

(iii.) Repeat the last operation, replacing the water by the liquid whose specific gravity is required.

The weight of the hydrometer, together with the weights in the scale-pan at the second operation, is equal to the weight of the water displaced by the part of the hydrometer below the fixed mark.

The weight of the hydrometer and weights in the scale at the third operation are equal to the weight of the given liquid displaced. And the volume displaced is the same as before.

Hence, by dividing the latter weight by the former, the specific gravity of the liquid is at once found.

Example.—To find the specific gravity of brandy by means of a Nicholson's Hydrometer weighing 60 gm., having given that 23·7 gm. are required in the upper scale-pan to sink the hydrometer to the fixed mark when placed in brandy, and that 40 gm. are required to sink it to the same mark in water.

At the first observation, the total weight supported by the brandy

$$= 60 + 23{\cdot}7 = 83{\cdot}7 \text{ gm.}$$

Hence weight of brandy displaced $= 83{\cdot}7$ gm.

At the second observation, we have, in like manner,

$$\text{weight of water displaced} = 60 + 40 = 100 \text{ gm.}$$

But the volumes of the brandy and water displaced are equal.

$$\therefore \quad \text{specific gravity of brandy} = \frac{83{\cdot}7}{100} = {\cdot}837.$$

55. **The weight of a hydrometer is W, the weight required to sink the bulb in water is P, and the weight required to sink it in another liquid is Q. To find the specific gravity of the liquid.**

Since the hydrometer floats in equilibrium in each case,

$\therefore$ weight of water displaced by hydrometer $= P + W$,

and weight of liquid displaced $= Q + W$.

$$\therefore \text{ specific gravity of liquid} = \frac{\text{weight of liquid}}{\text{wt. of equal vol. of water}} = \frac{Q+W}{P+W}.$$

56. To find the specific gravity of a solid by means of Nicholson's Hydrometer.—We have to find—

(1) The weight of the solid in air.

(2) The weight of the water displaced by the solid when immersed.

To do this, we proceed as as follows :—

(i.) Plunge the hydrometer in water, and place weights in the upper scale-pan till the stem sinks to the fixed mark.

(ii.) Place the solid in the upper scale-pan, taking off weights to make the stem again sink to the fixed mark.

The total weight supported by the hydrometer is the same as before. Hence the weights taken off must be equal to the added weight of the solid, which is therefore known.

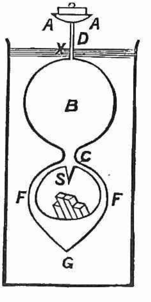

Fig. 10.

(iii.) Place the solid in the *lower* cup and again plunge the hydrometer in water. The water displaced will now exert an upward force on the solid. Hence extra weights must be placed in the upper scale-pan to sink the hydrometer to the fixed mark, and these added weights are equal to the weight of the water displaced by the solid.

Hence the specific gravity of the solid is at once found.

57. *If the solid is specifically lighter than water*, it must be fastened down to the lower cup of the hydrometer during the third process of weighing, so as to prevent its rising to the surface. For this purpose the cup is sometimes provided with a cap of wire gauze. In Atkin's form of the instrument, there is a small spike projecting downwards over the cup (S, Fig. 10), on the point of which any small bodies lighter than water can be impaled.

In such cases the cup acts as a sinker.

In other respects, the method of finding specific gravity is exactly the same.

Example.—A Nicholson's Hydrometer when placed in water required a weight of 40 gm. in the upper scale to sink it to the fixed mark. When a piece of silver was placed in the upper scale-pan, 8·5 gm. were required to sink it; and when the silver was placed in the lower scale-pan, 11·5 gm. were required in the upper. To find the specific gravity of silver.

Here, when the silver was placed in the upper scale-pan, $40-8{\cdot}5$ or 31·5 gm. had to be taken out in order to make the total weight the same as before.

Therefore the weight of the silver was 31·5 gm.

When the silver was transferred to the lower pan and immersed, we had to add $11{\cdot}5-8{\cdot}5$ or 3 gm. to the upper pan to counteract the upward thrust of the water on the silver.

$$\therefore \text{ weight of water displaced by silver} = 3 \text{ gm.}$$

$$\therefore \text{ specific gravity of silver} = \frac{31{\cdot}5}{3} = 10{\cdot}5.$$

58. **The weight required to sink the bulb of a hydrometer is P. When a body is placed in the upper scale-pan, the weight required to sink the bulb is Q; and when the body is placed in the lower pan, the weight required is R. To find the specific gravity of the solid.**

Let W be the weight of the body, w the weight of the water it displaces. Then, since the resultant force required to sink the bulb is the same in each case,

$$\therefore \quad P = Q + W,$$

and
$$Q + W = R + W - w;$$

$$\therefore \text{ weight of solid } W = P - Q,$$

and
$$\text{weight of water displaced } w = R - Q;$$

$$\therefore \text{ specific gravity of solid} = \frac{W}{w} = \frac{P-Q}{R-Q}.$$

59. The Common Hydrometer is adapted for finding the specific gravities of liquids only. It consists of a glass tube or stem AEG blown out into two bulbs B, G at its lower end, and closed at its upper end. The stem and the upper bulb B are filled with air, the lower bulb G being loaded with mercury or small shot, so that when the hydrometer is in liquid it floats upright with the whole of the bulb and part of the stem submerged.

No moveable weights are used, but the stem is provided with a graduated scale. The height to which the liquid rises on the stem is indicated by the scale, and serves to determine the specific gravity of the liquid.

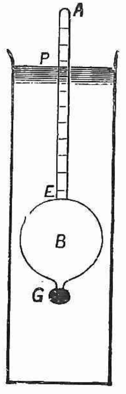

Fig. 11.

60. **To find the specific gravity of a liquid by means of the Common Hydrometer.**

In order to find the specific gravity of a liquid, it is sufficient to know—

(1) The weight of the hydrometer.

(2) The volume of liquid displaced when the hydrometer floats in it.

Now the stem of the hydrometer is cylindrical; hence, if its thickness be known, the volume of any length of it can be found. Hence, if the volume of the bulbs or of the whole hydrometer be known, we can find the required volume immersed when the liquid reaches a given height on the stem.

The weight of the hydrometer is equal to the weight of the displaced liquid, and, dividing this by the volume, the weight of a unit volume is found, and hence the specific gravity can be determined.

The general formula being somewhat complicated, it is usual to deduce the specific gravity of liquids by first principles in the manner illustrated in the following examples.

Examples.—(1) To find the specific gravity of a liquid, having given that a hydrometer weighing $1\frac{1}{2}$ oz. sinks in it until 2·4 cub. ins. are immersed.

Here weight of 2·4 cub. ins. of liquid $= 1{\cdot}5$ oz.;

$\therefore$ weight of 1 cub. in. of liquid $= \dfrac{1{\cdot}5}{2{\cdot}4}$ oz. $= \frac{5}{8}$ oz.,

and weight of 1 cub. ft. of liquid $= 1728 \times \frac{5}{8}$ oz. $= 1080$ oz.

ut weight of 1 cub. ft. of water $= 1000$ oz.;

. . specific gravity of liquid $= \frac{1080}{1000} = 1{\cdot}08$.

(2) To find the density of a liquid in which a common hydrometer floats with $3\frac{1}{2}$ ins. of its stem immersed, having given that the diameter of the stem is ·2 in., the volume of the two bulbs is ·754 cub. in., and the weight of the hydrometer $1\frac{1}{2}$ oz.

Here the portion of the stem immersed is a cylinder of height $3\frac{1}{2}$ ins., the radius of whose base is

$$= \tfrac{1}{2} \times {\cdot}2 = {\cdot}1 \text{ in.}$$

Hence the volume of the immersed portion of the stem

$$= \tfrac{22}{7} \times ({\cdot}1)^2 \times \tfrac{7}{2} = {\cdot}11 \text{ cub. in.}$$

Moreover, the volume of the bulbs = ·754 cub. in.

Hence the whole volume of the displaced liquid

$$= {\cdot}754 + {\cdot}11 = {\cdot}864 \text{ cub. in.}$$

But weight of displaced liquid = weight of hydrometer $= \frac{1}{2}$ oz.;

. . ·864 cub. in. of liquid weighs $\frac{1}{2}$ oz.;

$\therefore$ 1 cub. in. of liquid weighs $\dfrac{1}{2 \times {\cdot}864} = \dfrac{1}{1{\cdot}728}$ oz.;

· 1 cub. ft. of liquid weighs $\dfrac{1728}{1{\cdot}728}$ oz. $= 1000$ oz.

Hence the liquid is of the same density as water, and its specific gravity is unity.

(3) The stem of a hydrometer is divided into 100 equal parts. It reads 0 in water and 100 in liquid of specific gravity ·8. To find the specific gravity for which the hydrometer reads 50.

Let O, Q be the points marked 0, 100; P the point marked 50.

Let V be the volume of water whose weight is equal to that of the hydrometer. Then V is the volume of water displaced when the hydrometer floats in water.

$\therefore$ volume displaced by portion below $O = V$.

When the hydrometer floats in the lighter liquid of specific gravity ·8, it displaces an equal weight, and therefore a greater volume, of liquid.

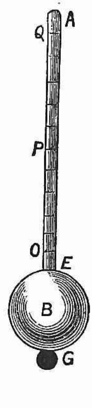

Fig. 12.

$\therefore$ volume displaced by portion below $Q = V \div \cdot 8 = 1 \cdot 25 V$;
volume of stem $OQ = 1 \cdot 25 V - V = \cdot 25 V$;
that is, volume of 100 divisions of stem $= \cdot 25 V$;
$\therefore$ volume of 50 divisions of stem $= \cdot 125 V$;
$\therefore$ volume displaced by portion below $P = V + \cdot 125 V = 1 \cdot 125 V$.

This is the volume displaced by the hydrometer in the given liquid, and its weight is equal to the weight of water displaced.

wt. of vol. $1 \cdot 125 V$ of given liquid = wt. of vol. V of water;
$\therefore$ wt. of vol. 1 of given liquid = wt. of vol. $1 \div 1 \cdot 125$ of water;
$\therefore$ required specific gravity of liquid $= 1 \div 1 \cdot 125 = \frac{8}{9} = \cdot \dot{8}$.

[NOTE.—Although the mark 50 is midway between the marks 0 and 100, the required specific gravity is NOT midway between the corresponding specific gravities, for its value is $\cdot \dot{8}$, and *not* $\cdot 9$ as might on first thoughts be expected.]

(4) With the data of the last example, to find the specific gravity of a liquid whose reading is 28.

We have seen that

volume of 100 divisions of stem $= \cdot 25 V$;
$\therefore$ volume of 28 divisions $= \frac{28}{100} \times \cdot 25 V = \cdot 07 V$;
$\therefore$ volume displaced by hydrometer in given liquid $= 1 \cdot 07 V$;
$\therefore$ weight of volume $1 \cdot 07 V$ of liquid = weight of volume V of water;
specific gravity of liquid $= 1 \div 1 \cdot 07 = \cdot 9346$, nearly.

61. **Beaumé's and Twaddell's Hydrometers.** — The stem of a hydrometer is usually divided into a number of equal parts, very often 100. Thus Beaumé's hydrometer for fluids lighter than water has the stem graduated from 10 up to 70. When plunged into water it reads 10, and the lighter the liquid the higher the reading. Another hydrometer was used by Beaumé for fluids heavier than water.

Twaddell's hydrometers for fluids heavier than water are a set of six. The first is graduated from 0 to 24, and indicates 0 when placed in water. The second sinks to the highest mark on the stem in a liquid in which the first rises to the lowest mark, and is therefore used for rather heavier liquids, and so on.

Tables have been constructed giving the specific gravity corresponding to any reading. In commerce, however, it is very customary to specify the specific gravity of a liquid by its hydrometer reading, thus: "$10\frac{1}{2}$° Twaddell."

62. **The Lactometer** is a common hydrometer adapted for testing whether milk has been adulterated with water. The extremities of the scale are the points to which the hydrometer sinks in pure water and pure milk respectively, and the intermediate divisions indicate the proportions of milk and water occurring in a mixture. Lactometers may now be purchased for a very small sum.

*63. **Sikes' Hydrometer** is similar in construction to the Common Hydrometer, the only essential difference being that the bulb B and the counterpoise G are separated by a thin conical stem, on which may be placed different weights of the form W (Fig. 13). The slot in each weight is just wide enough to go over the thinnest or upper part of the stem C, while the central hole just fits on to the lower part.

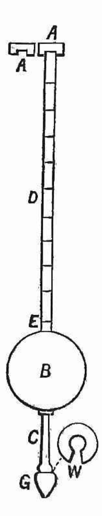

Fig. 13.

The scale of the hydrometer is divided into ten equal parts or degrees, numbered from the top downwards, and each degree is subdivided into fifths. Nine different weights are supplied with the instrument, and these are numbered 10, 20, ... 90, respectively. The smallest weight 10 is such as to sink the hydrometer from the mark 10 to the mark 0 in a liquid of the proper density.

In addition there is another weight A which can be placed on the top of the stem when the hydrometer is employed for liquids heavier than water.

In using Sikes' hydrometer, the number on the weight is added to the reading of the scale. Thus in water (specific gravity 1) the scale reads 10 when the weight 90 is attached, and the hydrometer reading is therefore 100. With the upper weight A attached, the hydrometer reading for water is zero.

The advantage of Sikes' hydrometer is that the moveable weights allow it to be used for a large range of different densities, a result that could not otherwise have been effected *except by making the stem 20 times as long*, which would be very inconvenient, or by making the divisions 20 times as close together, in which case the hydrometer would be much less sensitive, or by having a number of different hydrometers, as in the case of Twaddell's hydrometers.

Summary.

1. *Nicholson's Hydrometer* has scale-pan above and cup below bulb, and is always sunk to a fixed mark on stem.

To find sp. gr. of liquid, it is sunk in liquid by weights in upper pan. These wts. + wt. of hydrom. = wt. of liquid displaced by hydrom. Similarly, wt. of equal vol. of water is found ; hence sp. gr.

To find sp. gr. of solid, it is sunk in water as before. Wt. of solid (W) = wt. subtracted when solid is placed in upper pan. Wt. of water displaced (w) = wt. added when solid is transferred from upper to lower pan ; and sp. gr. of solid $= W \div w$.

2. *The Common Hydrometer* has graduated stem, no scale-pan.

The sp. gr. of a liquid is given by $W = wsV$; $\therefore s = W \div (wV)$, where W = wt. of hydrom., w = sp. wt. of water, V = vol. submerged in liquid.

3. *Sikes' Hydrometer* combines a graduated stem with moveable weights.

EXAMPLES VII.

1. A Nicholson's hydrometer, whose own weight is $4\frac{3}{4}$ oz., requires weights of 2 and $2\frac{3}{8}$ oz., respectively, to sink it to the fixed mark in two different fluids. Compare the specific gravities of the fluids.

2. A Nicholson's hydrometer weighs $3\frac{3}{4}$ oz., and requires a weight of $1\frac{3}{4}$ oz. to sink it to the fixed mark in water. What weight will be required to sink it to the fixed mark in a liquid whose density is 2·5?

3. A Nicholson's hydrometer of weight $4\frac{1}{4}$ ozs. requires a weight of $2\frac{1}{2}$ oz. to sink it to the fixed mark in a fluid whose specific gravity is 1·35. What weight will sink it to the fixed mark in water?

4. A solid is placed in the upper cup of a Nicholson's hydrometer, and it is found that 12 grs. are required to sink the instrument to a certain depth; when the solid is in the lower cup, 16 grs. are required, and, when the solid is removed, 22 grs. are required. What is the specific gravity of the solid?

5. A solid of specific gravity 8 is placed in the upper cup of a Nicholson's hydrometer, and it is found that 12 gm. are required to

sink the instrument to a fixed mark on the stem, and when the solid is removed, it is found that 28 gm. are required. What weight must be placed in the upper cup, when the solid is in the lower cup, in order to sink the hydrometer to the fixed mark?

6. A piece of marble weighing 142 grs. is placed in the upper dish of a Nicholson's hydrometer, and it is found that an additional weight of 40 grs. is required to sink the hydrometer to a fixed mark in its stem. When the marble is placed in the lower dish, it is found that 90 grs. are necessary. What is the specific gravity of the marble?

7. A body weighing 120 gm. is placed in the upper portion of a Nicholson's hydrometer, and it is found that an additional weight of 30 gm. is necessary to sink the hydrometer to the fixed mark on the stem. When the substance is placed in the lower dish, 72 gm. are necessary. What is the specific gravity of the substance?

8. Explain the principle of the common hydrometer, and show that the volume of the part immersed is inversely proportional to the density of the liquid.

9. When the common hydrometer floats in water, $\frac{9}{10}$ of its volume is immersed; and when it floats in milk, $\frac{90}{103}$ of its volume is immersed. Find the specific gravity of milk.

10. The volume of a hydrometer is 10 cub. cm. and its weight $6 \cdot 5$ gm. Find how much of it will be immersed when it is set to float in a liquid of specific gravity $\cdot 88$.

11. The whole volume of a common hydrometer = 6 cub. ins., and its stem, which is square, is $\frac{1}{8}$ in. in breadth; it floats in one liquid with 2 ins. of stem above surface, and in another liquid with 4 ins. of stem above surface. Compare the specific gravities of the two liquids.

12. A common hydrometer floats in water with $\frac{6}{7}$ of its volume immersed. How much of its volume will be immersed when it floats in oil of specific gravity $\cdot 9$?

13. A common hydrometer weighs 2 oz., and is graduated for specific gravities varying from 1 to $1 \cdot 2$. What should be the volume in cubic inches of the portion of the instrument below the graduations 1, $1 \cdot 1$, $1 \cdot 2$, respectively, it being assumed that a cubic foot of water con 1000 oz.?

14. The stem of a common hydrometer is cylindrical, and the highest graduation corresponds to a specific gravity of 1, and the lowest to 1·3. What specific gravity corresponds to a point exactly midway between these divisions?

15. Having given the positions of the marks on a common hydrometer corresponding to the specific gravities 1 and ·8, show how to find the points to which the hydrometer will sink when plunged in liquids of specific gravities ·85 and 1·1, respectively.

16. The stem of a common hydrometer is divided into 100 graduations, beginning from the top; when it is placed in a fluid of specific gravity 1·5, the surface of the fluid is at the graduation 20; when in a fluid of specific gravity 1·6, it is at the graduation 56. What is the specific gravity of a fluid of which the surface is at the graduation 96?

17. What is meant by the "specific gravity" of a substance? A body floats with one-tenth of its volume above the surface of pure water. What fraction of its volume would project above the surface if it were floating in a liquid of specific gravity 1·25?

18. A cube of wood, whose edge is 4 ins. and specific gravity ·72, floats in oil of specific gravity ·9. What weight must be placed on it in order to just totally immerse it?

19. A cylinder, loaded so as to float vertically, and weighing 2 gm. altogether, just sinks overhead in water when $\frac{1}{2}$ gm. extra is put on its top; otherwise it protrudes 7 cm. above the surface. What length will protrude above the surface of a liquid whose density is five-sixths that of water, if the cylinder be set floating in it without the extra load?

20. A solid cylinder of uniform material will float in water with its axis vertical and 2 ft. of its length immersed; or, again, in oil of specific gravity ·8, with 9 ins. more than half of its length immersed. Find its length and specific gravity.

EXAMINATION PAPER III.

1. How is the specific gravity of a body lighter than water found by means of the hydrostatic balance?

2. A piece of iron weighs 32·64 gm. in air, and 28·288 gm. in water; find its specific gravity.

3. A body weighs 60 gm. in air, and to sink it a piece of iron (specific gravity = 7·5) weighing 300 gm. in air is attached to it. The two together weigh 220 gm. in water. Find the specific gravity of the body.

4. A body weighing 30 oz. in air weighs 22·8 oz. in a liquid of unknown specific gravity, and 22·5 oz. in water. Find the specific gravity of the liquid.

5. How would you determine the specific gravity of a body which is soluble in water?

6. A body which is soluble in water weighs 27 gm.; and when weighed in oil of specific gravity ·9, its weight is $20\frac{1}{4}$ gm. Find its specific gravity.

7. Describe the common hydrometer.

8. Give an account of Nicholson's Hydrometer. How is it used for finding the specific gravities of solids and liquids?

9. A piece of crystal weighing 28 grs. is placed in the upper cup of a Nicholson's hydrometer, and 205 grs. are required to sink it to the fixed mark. When it is placed in the lower cup, 213 grs. are needed. Determine the specific gravity of the crystal.

10. A Nicholson's hydrometer weighing 50 gm. requires 270 gm. to sink it to the given level in water, and 238 gm. when immersed in a given liquid. Find the specific gravity of the liquid.

PART II.

FLUID PRESSURE.

CHAPTER VIII.

DEFINITIONS AND PROPERTIES OF PRESSURE.

64. **Thrust.**—DEF.—When two bodies in contact or two parts of a body press against each other, the forces which act between them are said to constitute a **thrust.**

We will now examine how the thrust of a fluid on any body is distributed over different portions of the body's surface.

If one or more holes be made anywhere in the side or bottom of a vessel full of water, the water will run out through them, provided they are below its surface. If the holes are stopped up with plugs, a certain force will have to be applied to *each plug* in order to prevent their being pushed out. Hence the water exerts a thrust on every portion of the surface of the vessel with which it is in contact, instead of its action being applied at one or more separate points. Such a distribution of thrust over a surface is called a **pressure.**

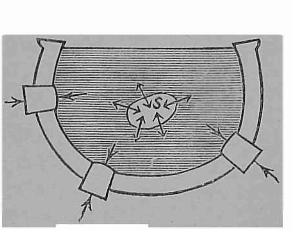

Fig. 14.

Pressure is not confined to the boundaries of a fluid; every portion exerts a pressure on the adjacent portions. For, if the portion S were removed from the interior, the surrounding fluid would rush in on all sides and fill the cavity thus formed. Hence the fluid inside S must exert pressure on that outside. And, since action and reaction are equal and opposite, the fluid outside S must exert pressure on that inside.

65. Fundamental Property of a Fluid.—We have defined a fluid as a substance which yields continually to any force, however small, tending to produce motion of its parts amongst themselves.

From this definition may be deduced the following

FUNDAMENTAL PROPERTY OF A FLUID,

The pressure of a fluid at rest on any surface is everywhere perpendicular to that surface.

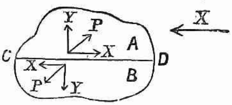

Fig. 15.

[For take any mass of fluid, and suppose it cut into two parts A, B by the plane CD. If the force which B exerts on A is not perpendicular to CD, let it be a force P in another direction. Then P can be resolved into components—one (X) along CD, and the other (Y) perpendicular to CD. And if we were to try to make the part A slide along the part B in the direction DC, we should have to exert a force equal to the resistance X before it would move at all, which would be contrary to the supposition that the fluid yields to any force, however small, tending to move the two portions separately. Hence the actions between the portions A, B must be perpendicular to the surface of separation CD.]

Example.—In raising a vertical sluice-gate, the force that must be used to lift it does not depend on the pressure of the water against the gate. For the action of the water is perpendicular to the gate, and is therefore horizontal. Hence it cannot affect the vertical lifting force applied to the gate.

*66. **Distinction between perfect and viscous fluids.**—The above proof may be employed to show that the action exerted by a *perfect* fluid on any surface is always perpendicular to the surface *whether the fluid is at rest or in motion.* But a *viscous* fluid tends to *retard* motion of its parts. Hence the perpendicularity of pressure to the surface does not necessarily hold in the case of viscous fluids, except when they are at rest.

We will now define "pressure," and state how it is measured.

67. DEF.—**A Pressure** is a distribution of thrust over a surface.

Pressure is measured by the amount of the thrust *per unit area* of a plane surface exposed to it.

68. Uniform Pressure.—DEF.—A pressure is said to be **uniform** when the thrusts exerted on two equal plane areas, however small, are equal, no matter where these areas be situated.

Uniform pressure is measured by the thrust exerted on every unit of area of any plane to which it is applied.

Thus, if a fluid exerts a thrust of 15 lbs. weight on every square inch of its surface, the pressure is said to be a uniform pressure of 15 lbs. per square inch.

It is clear that the whole force exerted on 2 sq. ins. of surface is twice as great as on 1 sq. in., and is, therefore, 30 lbs. weight; on 3 sq. ins., it is three times as great, or 45 lbs. weight; and so on. But the *pressure* is the same in each case, for pressure is measured, not by the force on the whole surface, but by that on a unit area of the surface. Thus, pressure is a different kind of quantity from force.*

69. Unit of Pressure.—The **unit of pressure** is that pressure which exerts a unit of force on every unit of area. Thus, if forces are measured in pounds weight and lengths in feet, the unit of pressure is a pressure of **1 lb. per square foot** (now sometimes written **1 lb./ft^2**).

A pressure of 15 lbs. per square inch is called an *atmosphere*, being the average pressure of atmospheric air, and for certain purposes this is adopted as the unit of pressure.

It is often convenient to measure pressures in pounds per square inch or ounces per square foot.

In the C.G.S. dynamical system, where a centimetre and a dyne are the units of length and force, the unit of pressure is a pressure of one dyne per square centimetre.

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* The word pressure is still sometimes used in Mechanics to denote a *force*; for instance, the "pressure of a body on an inclined plane" or the "pressure of a chair on the floor." But it is incorrect to call a force a pressure under any circumstances. It is far better to speak of such a force as a thrust, though it may, if preferred, be called a *force of pressure*.

A "*C.G.S. atmosphere*" is the name given to a pressure of on million dynes per square centimetre, and is nearly, but not quite equal to the ordinary atmospheric pressure.

70. The resultant thrust on any plane area exposed to uniform fluid pressure is equal to the product of the pressure into the area.

Let p be the pressure, A the area of the surface.

Let the area be divided into a number of portions each of unit area.

Then since the pressure is p,

the thrust on each unit of area is p.

Now the number of such units of area is A, and, since the whole area is plane, the forces on them are parallel, hence their resultant is equal to their sum. Hence, if P denote the resultant thrust,

$$P = pA;$$

that is, **resultant thrust = (pressure) × (area).**

Hence, also, $$p = \frac{P}{A};$$

so that the pressure is measured by the resultant thrust divided by the area.

Examples.—(1) If a ton of water is contained in a rectangular tank whose base is 4 ft. by 2 ft., the whole thrust on the base

$$= 1 \text{ ton} = 2240 \text{ lbs.},$$

and the area over which it is distributed = 8 sq. ft.;

$\therefore$ pressure on base of tank $= \frac{2240}{8} = 280$ lbs. per square foot

$= \frac{280}{144} = 1{\cdot}94$ lbs. per square inch.

(2) If the pressure of the steam inside a boiler is 140 lbs. to the square inch, to find the thrust supported by the ends of the boiler, given that they are circular and 6 ft. in diameter.

Here the area of either end $= \frac{22}{7} \times (\frac{6}{2})^2$ sq. ft. $= \frac{198}{7}$ sq. ft.

$= \frac{28512}{7}$ sq. ins.,

and the pressure = 140 lbs. per sq. in.;

$\therefore$ thrust on either end $\frac{28512}{7} \times 140 = 28512 \times 20$

$= 570240$ lbs. weight $= 254\frac{4}{7}$ tons.

71. Change of Units.—When a given pressure is expressed in terms of one system of units its measure in terms of any other unit may be found in the manner illustrated in the following examples :—

Examples.—(1) To express a pressure of 15 lbs. per square inch in (i.) tons per square foot, (ii.) poundals per square foot.

On 1 sq. in. the pressure produces a thrust of 15 lbs. weight.

Therefore, on 1 sq. ft. (= 144 sq. in.) the pressure produces a thrust of 15 × 144 lbs. weight.

$$\therefore \quad \text{pressure} = 15 \times 144 \text{ lbs. per square foot}$$
$$= \frac{15 \times 144}{2240} \text{ tons per square foot}$$
$$= \tfrac{27}{28} \text{ of a ton per square foot.}$$

(ii.) Taking the acceleration of gravity as 32,

$$1 \text{ lb. weight} = 32 \text{ poundals};$$
$$\therefore \quad \text{pressure} = 15 \times 144 \times 32 \text{ poundals per square foot}$$
$$= 69120 \text{ poundals per square foot.}$$

(2) To express a pressure of 1000 oz. per square foot in pounds per square inch.

On 1 sq. ft. (= 144 sq. ins.) the pressure exerts a force = 1000 oz.

Therefore, on 1 sq. in. the pressure exerts a force

$$= \frac{1000}{144} \text{ oz.} = \frac{1000}{144 \times 16} \text{ lbs.}$$

$$\therefore \quad \text{pressure} = \frac{1000}{144 \times 16} = \cdot 434028 \text{ lbs. per square inch.}$$

(3) To express a pressure of 1 kilog. per square metre (i.) in grammes per square centimetre, (ii.) in C.G.S. dynamical units.

On 1 sq. metre (= 100^2 sq. cm.) the pressure exerts a force

$$= 1 \text{ kilog.} = 1000 \text{ gm.}$$

Therefore, on 1 sq. cm. the pressure exerts a force

$$= \frac{1000}{100^2} \text{ gm.} = \cdot 1 \text{ gm.}$$

Hence pressure = ·1 gm. per square centimetre.

The C.G.S. dynamical unit of pressure is a pressure of 1 dyne per square centimetre.

Now the acceleration of gravity = 981 cm. per second per second ;

$$\therefore \quad \text{weight of a gramme} = 981 \text{ dynes};$$
$$\therefore \quad \text{given pressure} = 981 \times \cdot 1 = 98 \cdot 1 \text{ dynes per square centimetre.}$$

(4) *The measure of a pressure in terms of certain units of length and force is p. To find its measure when the unit of length is increased L times, and the unit of force is increased F times.*

The new unit of area = area of square on new unit of length = area of a square whose side contains L old units = L^2 old units of area.

Hence the thrust on the new unit of area

$$= pL^2 \text{ old units of force}$$

$$= \frac{pL^2}{F} \text{ new units of force ;}$$

and, therefore, the measure of the pressure

$$= \frac{pL^2}{F} \text{ new units of pressure.}$$

72. Pascal's Law.—This law, which is also known as the **Principle of Transmission of Fluid Pressure,** may be stated thus :

When any pressure is applied to any part of the surface of a fluid, an equal and uniform pressure is transmitted over the whole fluid.

73. Experimental Verification.—Let a closed vessel of any shape be filled with water, or other fluid (Fig. 16). Let short tubes of equal sectional area (say 1 sq. in.) be attached to openings made in different parts of the walls of the vessels, and let these tubes be closed with tight-fitting plugs or pistons, acted upon by such forces as support the weight of the fluid. If now an *additional* force, say of 1 lb., be applied to any one of the plugs (say A) it will be necessary to apply an additional force of 1 lb. to each of the other plugs B, C, D, to prevent their coming out ; similarly, if the force on A be increased by 2 lbs. or any other amount, the force applied to each of the other plugs will also have to be increased by 2 lbs. Hence a pressure of 1, 2 or more pounds per square inch imparted to the surface of A gives rise to an equal pressure over every other square inch of the surface.

Observation. — This experiment would be very difficult to arrange in practice.

But, without actually performing the experiment, the law may be deduced from the principle of "Conservation of Energy," as follows :—*

* Another proof will be given in §§ 79 80.

***74. Proof of Pascal's Law.**—Let a quantity of fluid be contained in a vessel furnished with projecting tubes of sectional areas A, B, C, D, along which tight-fitting pistons A, B, C, D can slide. Suppose the fluid without weight, so that the only forces acting on it are thrusts P, Q, R, S applied on the pistons.

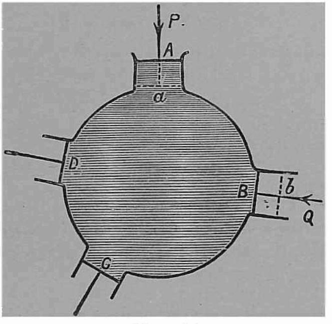

Fig. 16.

Let the piston A be pushed in to a, and let it push the piston B out to b, so that the volume of the fluid is unaltered and the other pistons remain where they were.

Since a fluid offers no resistance to changes of shape which do not alter its volume, therefore no work is done on the fluid itself in moving the pistons.

Therefore the work done by P is equal to the work done against Q. $\therefore \quad P \times Aa = Q \times Bb$ .... (i.).

Again, the volume of fluid forced out of the tube Aa is equal to the volume forced into Bb; that is,

$$Aa \times (\text{area } A) = Bb \times (\text{area } B) \quad \text{........} \quad \text{(ii.)}$$

From (i.) and (ii.) we have

$$\frac{P}{A} = \frac{Q}{B},$$

But $P \div A$ and $Q \div B$ are the pressures on the pistons A, B (§ 70).

Therefore these pressures are equal, and similarly the pressures on the other pistons are also equal.

75. Variable Pressure.—Def.—When the thrusts of a fluid on equal plane areas are not equal, the pressure of the fluid is said to be **variable.**

When a fluid is subjected to forces (such as that due to its weight) which act on its substance and not merely on its bounding surface, the pressure of the fluid is in general variable.

Variable pressure cannot in general be measured by the thrust *actually* exerted on a unit of area, but it may be said to be measured by the thrust **per** unit of area. By "the thrust per unit area" is meant the thrust which would be produced on a unit area by a uniform pressure of the same intensity.

76. **Average Pressure.** — Def. — The **average pressure** of a fluid over any plane area is measured by the resultant thrust of the fluid divided by the area.

Examples.—(1) If a fluid exerts a thrust of 144 lbs. on a square whose area is 9 sq. ins., the average pressure is 144/9 or 16 lbs. per square inch. The same thrust would be exerted on the area by a *uniform* pressure of 16 lbs. per square inch over the area.

(2) If the thrust on an area of $\frac{1}{100}$ sq. in. is $\frac{1}{20}$ lb., the average pressure $= \frac{1}{20} \div \frac{1}{100} = 5$ lbs. *per* square inch.
The same thrust would be produced on the same area by a uniform pressure of 5 lbs. per square inch.

The word "per" thus implies that the area actually exposed to fluid pressure is not necessarily equal to the unit of area.

From § 70, it appears that, when the pressure of a fluid is uniform, the average pressures on different areas are all equal to the pressure of the fluid. When the pressure on any area is variable, the average pressure measures the pressure that, acting uniformly over the area, would produce the same thrust as the given pressure.

77. If we consider the action of fluid pressure over a *sufficiently small* plane area, the pressure will not vary appreciably in the very small distance separating two different parts of this area, and it may therefore be regarded as *practically a uniform pressure.* This pressure is equal to the average pressure over the whole of the little area, and, since it is uniform, it may be said to be the pressure *at any point* of the area.

78. PRESSURE AT A POINT.

DEFINITION.—*The* **pressure at a point** *of a fluid is the average pressure (or average thrust per unit area) taken over any very small plane area enclosing that point.*

The area in question must be so small that the pressure all over it is sensibly uniform (§ 77).

OBSERVATION.—It is advisable to regard "pressure at a point" as an abbreviated expression for "pressure in the immediate neighbourhood of a point." It would, of course, be absurd to imagine that fluid pressure could have any effect on a mere mathematical point, for pressure could produce no thrust if it had nothing to act on.

79. FUNDAMENTAL LAW OF HYDROSTATICS.—The pressure at any point of a fluid at rest is the same in all directions.

***Proof of the Law.** — Let A be any point in a fluid. Let a wedge in the form of a triangular right prism be constructed in the fluid at A (Fig. 17), having its faces in any given directions.

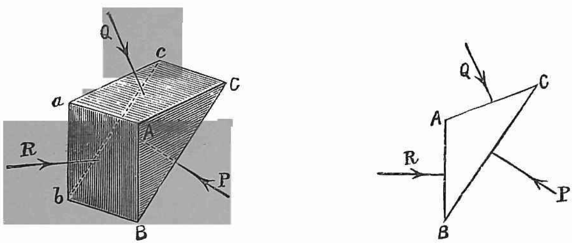

Fig. 17.

Then the fluid inside the wedge is kept in equilibrium by (i.) the thrusts of the fluid on its faces, and (ii.) its own weight. But, if the wedge is very small, its weight may be shown to be very small compared with the thrusts on its faces; hence, by taking the wedge small enough, we may neglect its weight altogether.

Let the forces on the rectangular faces Bc, Ca, Ab be denoted by P, Q, R. These three forces must be in equilibrium among themselves, since the only other forces on the wedge—viz., the thrusts perpendicular to the triangular faces—are perpendicular to P, Q, R.

Now the forces P, Q, R are perpendicular to BC, CA, AB, the sides of the triangle ABC; therefore, by the "Perpendicular Triangle of Forces," these forces are proportional to the sides;

$$\therefore \frac{P}{BC} = \frac{Q}{CA} = \frac{R}{AB};$$

But Aa, Bb, Cc, the heights of the faces, are equal.

$$\therefore \frac{P}{\text{rectangle } Bc} = \frac{Q}{\text{rectangle } Ca} = \frac{R}{\text{rectangle } Ab};$$

average pressure on face Bc = average pressure on Ca
= average pressure on Ab.

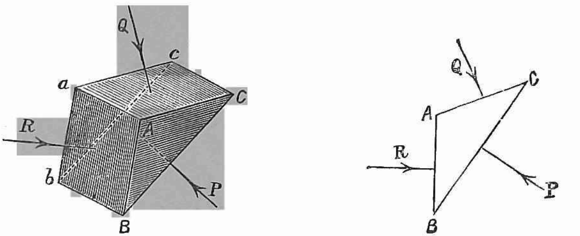

Fig. 17.

And since the areas have been taken very small, these average pressures are the pressures **at** the point A in the directions perpendicular to the planes Bc, Ca, Ab, which are therefore equal.

In the same way, it may be shown that the pressures at A in any other directions are equal.

[That the weight of the fluid may be left out of account in considering the equilibrium of the very small wedge may be shown as follows:—Let the wedge be inverted; then, if the wedge is very small, the forces P, Q, R arising from the fluid pressures on its faces are reversed in direction without being sensibly altered in magnitude. But the direction in which the weight acts is *not* reversed. Hence the weight of the fluid cannot sensibly affect the conditions of equilibrium of the wedge, for, if it did, the wedge would no longer be in equilibrium in its inverted position.]

Observations. — Since the pressure at a point is the same in all directions, we speak of the "pressure at a point" *in a fluid* without specifying its direction.

When, however, the fluid pressure acts *on the surface* of any solid body, its direction is fully specified, being perpendicular or *normal* to the surface.

***80. Deduction of Pascal's law for a weightless fluid.** *When no forces act on the mass of a fluid the pressure is the same in different parts.*

Consider a rectangular column of the fluid taken in any direction, whose ends $ABCD$ and $abcd$ are each, say, 1 in. square. In order that this column may be in equilibrium, the forces on these two ends

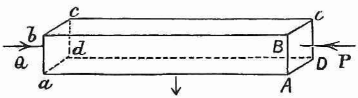

Fig. 18.

must be equal; for the only other forces are due to the pressures on the other faces, and are perpendicular to them. But the areas of the faces $ABCD$ and $abcd$ are equal; therefore the pressures on them are equal.

And, from § 79, we see that the pressures in the neighbourhood of A, a are the same in all directions. Hence the pressure is the same throughout the whole of the fluid.

SUMMARY.

1. *Definitions of Pressure.*—If P is the thrust of a fluid against a plane surface of area A, the fraction $\dfrac{P}{A}$ measures—

   (i.) The *pressure* of the fluid, if this pressure be *uniform*.

   (ii.) The *average pressure* on the area, if the pressure be *variable*.

   (iii.) The *pressure at a point*, if the area A be a *very small* plane area containing that point.

2. *Laws of Fluid Pressure.*—The pressure of a fluid at rest

   (i.) Is perpendicular to the surface on which it acts.

   (ii.) Is the same in all directions at a given point.

3. *Pascal's Law.*—When no forces (such as that due to gravity) act on the fluid particles themselves, the pressure is the same throughout the fluid.

EXAMPLES VIII.

1. How is fluid pressure measured when uniform? Compare the pressures of 15 lbs. on a square inch and of 1000 oz. on a square foot.

2. Compare (*i.e.*, find the ratio of) the following pressures :—

(i.) 14 lbs. per square inch and 8 tons per square yard ;

(ii.) 28 lbs. per square inch and 16·2 tons per square foot ;

(iii.) 28 gm. per square centimetre and 16·1 kilog. per square metre.

3. The pressure of the atmosphere is 15 lbs. per square inch. Express this pressure—

(i.) in ounces per square foot ;

(ii.) in poundals per square foot ;

(iii.) in tons weight per square yard ;

(iv.) in grains per square line (1 lb. = 7000 grs., 1 in. = 12 lines).

4. The pressure of the atmosphere is 103 gm. per square centimetre. Express this (i.) n kilogrammes per square metre, (ii.) in tonnes per square kilometre, (iii.) in milligrammes per square millimetre, (iv.) in dynes per square centimetre.

5. Taking the pressure of the atmosphere as equal to $14\frac{1}{2}$ lbs. per square inch, find its value in dynes per square centimetre, assuming that a gramme is ·0022 lb., and that a metre is 39 ins.

6. A piston 6 sq. ins. in area is inserted into one side of a closed cubical vessel measuring 10 ft. each way, filled with water; the piston is pressed inwards with a force of 12 lbs. Find the increase of thrust produced on the face of the vessel.

7. The neck and bottom of a bottle are $\frac{1}{2}$ in. and 4 ins. in diameter, respectively. If, when the bottle is full of water, the cork is pressed in with a force of 1 lb., what force is exerted upon the bottom of the bottle?

8. Explain what is meant by the *pressure at a point in a fluid.*

A prism, whose height is 10 mm. and whose base is an isosceles triangle with sides 10, 10, 12 mm. and altitude 8 mm., respectively, is placed in a fluid where the average pressure is 100 gm. per square centimetre. Find the thrusts on the respective faces, and the ratios to them of the weight of water required to fill the prism.

9. If all the dimensions of the prism (see the last question) be reduced to one-tenth of the above measurements, show that these ratios will be one-tenth of their previous values. Hence show that, if a prism be taken sufficiently small, the weight of the fluid in it can be neglected in comparison with thrusts of the fluid on its faces.

CHAPTER IX.

APPLICATIONS OF FLUID PRESSURE.

THE BRAMAH PRESS.

81. The Bramah, or Hydrostatic, Press. — The hydraulic press, used for subjecting bales of cotton, sheets of paper for printing, and other goods, to great pressure, affords an excellent illustration of Pascal's Law of transmission of fluid pressure.

It consists, essentially, of a large cylinder A and a small cylinder G, filled with water and connected by a pipe. Both contain pistons or plungers B, K, which can slide up

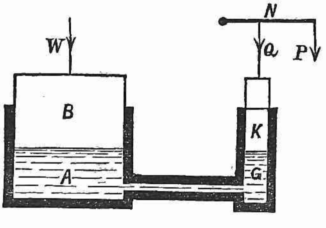

Fig. 19.

and down in them, the larger one B being called the *press-plunger* and the smaller the *pump-plunger*.

The goods to be compressed are placed on a platform attached to the press-plunger B, above which is a fixed framework. To work the machine a force is applied to push down the pump-plunger K. The pressure thus produced raises the press-plunger B and compresses the goods between the platform and the framework.

By making the plunger B very large and the plunger K very small, a small downward force applied to K will produce a very great upward force on the platform. For the pressures of the water over the two plungers are equal. Hence the thrusts on them are proportional to their areas.

Example.—If the area of the pump-plunger is 1 sq. in., and the area of the press-plunger is 100 sq. ins., then a force of 1 lb. on the former will produce a pressure in the fluid of 1 lb. per square inch.

And this pressure, acting over the whole area of the press-plunger (100 sq. ins.), will produce a thrust of 100 lbs. on the platform. Thus, by applying a force of 1 lb., we can lift a weight or overcome a resistance of 100 lbs.

The following additional details are required to complete the actual working machine, represented in Fig. 21 :—

82. **Water-tight Collar.** — To prevent the water from escaping, the space between each plunger and its containing cylinder is closed with a packing or collar of the form shown in section in Fig. 20. A ring of leather is folded over the rim of the cylinder so that its section resembles an inverted **U**, and this leather is forced against the plunger by the pressure of the water underneath. The greater this pressure the more tightly does the leather fit round the plunger.

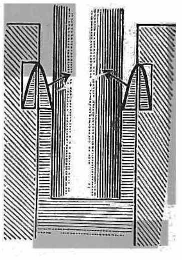

Fig. 20.

83. **Pump Action.**—When the pump-plunger is pushed down through the whole length of the cylinder containing it, the press-plunger only rises through a very small distance. In order to lift the press-plunger through the required height, the pump-plunger is arranged to work up and down as a forcing-pump, as shown in Fig. 21. When the pump-plunger is pushed down, the valve *V* closes and the valve *F* opens, and the water lifts the press-plunger. At the end of the stroke the small piston is again raised ready for a second stroke. The valve *F* is

now closed by the pressure in front, and prevents the escape of the water from the large cylinder, while a fresh supply of water is admitted from a reservoir *I* to the small cylinder by means of the valve *V*. At the next down-stroke this water is forced into the large cylinder. Thus the large piston is raised at each stroke of the pump.

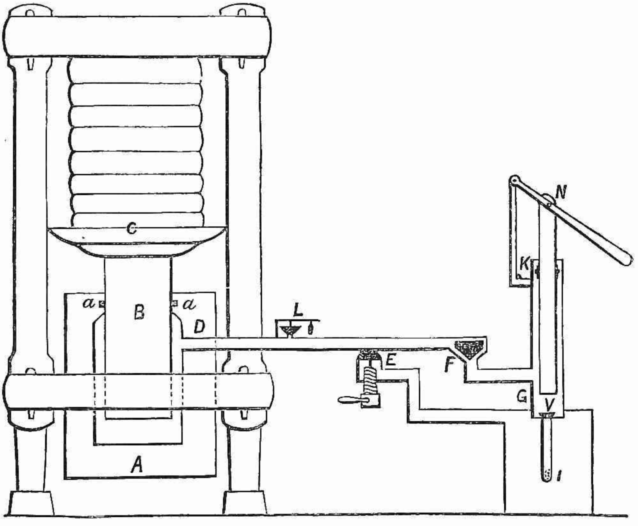

Fig. 21.

When the bales have been compressed, the water is allowed to return from the large cylinder to the reservoir by turning on a tap *E*, and the piston descends by its own weight ready for another load.

84. **Lever.**—Instead of operating directly on the small piston, it is usually raised and lowered by a lever *N*, and this serves to still further increase the mechanica advantage of the apparatus.

Examples.—(1) If the pistons are circular, and of diameters 1 in. and 2 ft., respectively, to find the force required to overcome a resistance of 9 tons.

Here the areas of the circular pistons are, respectively, $\frac{1}{4}\pi \times 1^2$ and $\frac{1}{4}\pi \times 24^2$ sq. ins. Also the thrust of the fluid on the larger piston is required to be 9×2240 lbs.

Hence the pressure of the fluid is

$$\frac{9 \times 2240}{144\pi} \text{ lbs. per square inch,}$$

and, since the area of the smaller piston is $\frac{1}{4}\pi$ sq. in., the force on it must be

$$= \frac{9 \times 2240 \times \frac{1}{4}\pi}{1444\pi} = \frac{9 \times 560}{144} = 35 \text{ lbs.}$$

[N.B.—The numerical value of π should *not* be substituted.]

(2) If the areas of the two plungers are $\frac{1}{4}$ sq. in. and 10 sq. in., and the pump-plunger is worked by a lever whose arms are 2 ins. and 28 ins., to find the resistance that can be overcome by applying a force of 15 lbs. to the end of the longer arm of the lever.

Let Q be the resultant thrust on the plunger. For the equilibrium of the lever, we have, by taking moments about the fulcrum;

$$Q \times 2 = 15 \times 28\,;$$

$$\therefore \quad Q = 15 \times 14 = 210 \text{ lbs.}$$

This force of 210 lbs. is distributed over the area of the small plunger, which is $\frac{1}{4}$ sq. in.

. . pressure produced $= 210 \div \frac{1}{4} = 210 \times 4 = 840$ lbs. per square inch.

This pressure is transmitted to the surface of the large plunger, whose area is 10 sq. ins.;

$$\therefore \quad \text{upward thrust on large plunger} = 840 \times 10 \text{ lbs.}$$

Hence the press can overcome a resistance of 8400 lbs., that is, $3\frac{3}{4}$ tons weight.

(3) If, in the last example, the end of the lever is raised and lowered through 1 ft. at every stroke, to find the number of strokes requisite to raise the press-plunger through 1 in.

Since the arms of the lever are 28 and 2 ins., respectively, therefore, when the end of the longer arm is lowered through 1 ft., the pump-plunger falls through $\frac{1}{14}$ ft., *i.e.*, $\frac{6}{7}$ in.

Hence the volume of water forced out of the pump cylinder

$$\tfrac{1}{4} \times \tfrac{6}{7} \text{ cub. in.} = \tfrac{3}{14} \text{ cub. in.}$$

This volume is forced into the press cylinder; hence the press-plunger rises at each stroke through $\frac{3}{14} \div 10$ ins., *i.e.*, through $\frac{3}{140}$ in.

Hence the number of strokes required to raise it through 1 in. is

$$= \tfrac{140}{3} = 46\tfrac{2}{3}.$$

Hence 46 complete strokes must be made, and the lever must be pressed two-thirds down in the 47th stroke.

85. Mechanical advantage of Bramah's Press without a lever.—Let A, B be the areas of the large and the small plunger, Q the effort or force applied directly to the small plunger, W the resistance to be overcome.

Then, since the pressure due to the force Q distributed over the area B is equal to that due to the force W distributed over the area A (by Pascal's Law),

$$\therefore \quad \frac{Q}{B} = \frac{W}{A};$$

$$\therefore \text{ mechanical advantage } \frac{W}{Q} = \frac{A}{B} = \frac{\text{area of press-plunger}}{\text{area of pump-plunger}}.$$

In practice the plungers are always **circular.**

Let a, b be their diameters; then

$$A = \tfrac{1}{4}\pi a^2, \quad B = \tfrac{1}{4}\pi b^2;$$

$$\therefore \quad \text{mechanical advantage } \frac{W}{Q} = \frac{\frac{1}{4}\pi a^2}{\frac{1}{4}\pi b^2} = \frac{a^2}{b^2}.$$

86. Mechanical advantage taking account of the lever.—Next suppose the pump worked by means of a lever whose arms are x, y, the effort used to work it being a force P applied at the end of the arm x. Then, if Q denote as before the thrust acting on the pump-plunger applied at the end of the arm y, we have, by taking moments,

$$P \times x = Q \times y,$$

or

$$\frac{Q}{P} = \frac{x}{y};$$

whence, by the last article,

$$\therefore \text{ mech. advantage } \frac{W}{P} = \frac{W}{Q} \times \frac{Q}{P} = \frac{A}{B} \times \frac{x}{y} = \frac{a^2}{b^2} \times \frac{x}{y}$$

$$= (\text{mechanical advantage of lever}) \times (\text{that of press}),$$

as it obviously should be.

87. **The so-called "Hydrostatic Paradox"** consists in the fact that a small force may be made to overcome a far greater resistance by means of a hydraulic press. There is really nothing paradoxical in this, for the simple machines or "mechanical powers" described in text-books in Mechanics all have the same property.

The Principle of Conservation of Energy is satisfied in all cases. In the hydraulic press, in which a small force applied to the small plunger overcomes a large resistance applied to the large plunger, the former plunger has to be moved through a considerable distance in order to move the latter through a small distance, and the work done by the effort always equals that done against the resistance.

Examples.—(1) To verify the principle of Conservation of Energy for the Example of § 81 (p. 82).

The area of the large piston is 100 sq. ins.; hence, if we want to raise it through $\frac{1}{100}$ in. we must drive 1 cub. in. of water from the small cylinder into the large one, and to do this the small piston must be pushed down 1 in.

But the work done by 1 lb. in moving through 1 in. is equal to the work done by 100 lbs. in moving through $\frac{1}{100}$ in. Hence the work done by the effort is equal to the work done against the resistance.

(2) In the press of Examples (2), (3), p. 84, the work done by the effort (15 lbs.) in $46\frac{2}{3}$ strokes of the pump

$$= 15 \times 46\tfrac{2}{3} \text{ ft.-lbs.} = 700 \text{ ft.-lbs.}$$

The work done against the resistance of 8400 lbs. in raising the platform through 1 in.

$$= 8400 \times \tfrac{1}{12} \text{ ft.-lbs.} = 700 \text{ ft.-lbs.}$$

These works are equal, thus verifying the principle.

88. **To verify the Principle of Conservation of Energy for the hydraulic press generally,** the process is the converse of § 74, where the principle was used to *prove* Pascal's Law by means of a similar contrivance. The proof is left as an exercise to the student.

89. **The Safety-Valve.**—The boiler of every steam-engine is furnished with at least one **safety-valve** (more often two), which prevent the pressure from becoming sufficient to burst or injure the boiler. A safety-valve is also attached to the hydraulic press at L (Fig. 21) for a similar purpose.

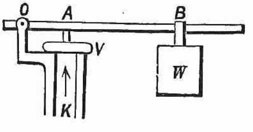

Fig. 22.

The tube K (Fig. 22) is connected with the fluid under pressure, and is closed by the valve V, which is held down by an adjustable weight W. The pressure required to lift the valve must not exceed the greatest pressure of fluid consistent with safety. For less pressures the valve remains closed; for greater pressures the valve rises and fluid escapes, so that no further increase of pressure can take place.

The figure shows the most common form of safety-valve, in which the weight is attached to a lever OB, operating on the valve at A. By varying the weight, or sliding it along the lever, the valve may be made to open at any desired pressure.

Examples.—(1) If the section of the tube of the safety-valve is a square of side $\frac{1}{3}$ in., to find the weight which must be placed on it so that it opens when the pressure exceeds 135 lbs. per square inch.

Here the area of the valve exposed to pressure is $\frac{1}{9}$ sq. in. Hence the upward force on it at the given pressure

$$= 135 \times \tfrac{1}{9} \text{ lbs.} = 15 \text{ lbs.};$$

therefore the valve must be loaded with a weight of 15 lbs.

(2) Suppose the tube is circular and $\frac{1}{2}$ in. in diameter, the maximum pressure 140 lbs. per square inch, and the valve is held down by a lever carrying a moveable weight of $5\frac{1}{2}$ lbs., to find where this weight must be placed.

The thrust required to lift the lever must

$$= 140 \times \tfrac{22}{7} \times (\tfrac{1}{4})^2 \text{ lbs.} = \tfrac{55}{2} \text{ lbs.}$$

Hence, if O is the fulcrum, A the centre of the valve, and B the point where the weight is attached, we have, by taking moments,

$$\tfrac{55}{2} \times OA = \tfrac{11}{2} \times OB,$$

$$\therefore \quad OB = 5 . OA;$$

that is, the distance of the weight from the fulcrum must be five times the distance of the centre of the valve.

(3) The area of a safety-valve exposed to pressure is A, and the valve is held down by a weight W which can slide along a lever, the distance OA of the centre of the valve from the fulcrum being known. To find where the weight must be placed, if the maximum pressure that the boiler will stand is p.

Let B be the required point at which the weight must be hung from the lever.

When the pressure is p, the force acting on the lever at A is $p \times A$. Therefore, by taking moments about O,

$$W \times OB = pA \times OA;$$

whence the required distance $OB = \dfrac{pA}{W} \times OA$.

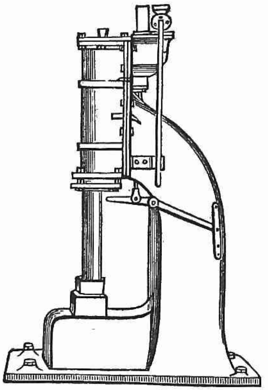

Fig. 23.

90. The Steam Hammer (Fig. 23) consists of a heavy metal hammer attached to a piston, which is forced down by steam pressure applied to its upper surface. On striking the bar of iron or other object to be forged, the energy of its motion is converted into useful work.

A similar machine is used in driving rivets or bolts through metal plating in ship-building, &c.

SUMMARY.

1. The principal parts of *Bramah's Press* are—

Large cylinder containing press-plunger, attached to platform on which goods are compressed against framework;

Small cylinder containing pump-plunger, operated by lever;

Water-tight collars fitted round plungers;

Valves allowing small plunger to be worked as a forcing-pump.

2. By Pascal's Law,

pressure over small plunger = pressure over large one;

hence the thrusts on them are proportional to their areas.

3. The *safety-valve* and *steam hammer* also work by fluid pressure.

EXAMPLES IX.

1. In a hydraulic press the pump-plunger is a cylinder 1 cm. in diameter, and makes a stroke 7 cm. long. The plunger of the press is 20 cm. in diameter. Calculate (*a*) the pressure in the press when a weight of 100 lbs. is applied to the pump-plunger (ignoring friction); (*b*) the force acting on the press-plunger; (*c*) the number of strokes which the pump must make in order to raise the press-plunger 10 cm.

2. In the Bramah press the areas of the two cylinders are $\frac{1}{8}$ sq. in. and 5 sq. ins., and the lengths of the arms of the lever by which it is worked are 36 ins. and $1\frac{1}{8}$ ins. How much thrust is obtained by applying to the end of the longer arm a force of 15 lbs.?

3. In the Bramah press the areas of the two pistons are $\frac{1}{4}$ sq. in. and 16 sq. ins., respectively. If the lengths of the arms of the lever are in the ratio of 20 : 1, what force must be applied at the end of the lever in order to produce a thrust of 16,000 lbs.?

4. If a resistance of 1 ton is overcome by a force of 5 lbs. applied to a Bramah press, and the diameters of the pistons are in the ratio of 8 to 1, find the ratio of the arms of the lever employed to work the piston.

5. If the lengths of the arms of the lever in a Bramah press are 30 ins. and 2 ins., respectively, and area of the smaller piston be $\frac{1}{8}$ sq. in., what must be the area of the larger piston in order that a force of 10 lbs. applied at the end of the lever may produce a thrust of 9000 lbs.?

6. Verify the Principle of Conservation of Energy for Bramah's press.

7. A safety-valve whose area is $1\frac{1}{4}$ ins. is held down by a weight of 28 lbs. attached to the longer arm of a lever whose arms are 2 ins. and 2 ft. What pressure will just lift the valve?

8. Supposing the tube to be circular and $\frac{2}{5}$ in. in diameter, and the maximum pressure to be 350 lbs. per square inch, find the load which must be placed on the valve.

9. The piston of a steam hammer is $\frac{3}{4}$ sq. ft. in area, and it is forced down through 18 ins. by a steam-pressure of 240 lbs. per square inch. How many foot-pounds of work have been done on it?

10. A steam hammer weighing 1 ton, and the diameter of whose piston is 14 ins., is forced down by steam at a pressure of 30 lbs. per square inch, and on striking a piece of iron compresses it by $\frac{1}{2}$ in. If the total distance fallen by the hammer is 2 ft., find the average resistance of the iron.

[Assume that the whole work done on the hammer is expended in compressing the iron.]

EXAMINATION PAPER IV.

1. What is meant by the *pressure of a fluid?* How does it differ from ordinary statical pressure?

2. Estimate a pressure of 15 lbs. weight per square foot in dynes per square centimetre.

3. Show that any pressure applied to the surface of a fluid is transmitted equally in all directions.

4. A thrust of 15 lbs. is applied to a square piston whose edge is 5 ins., fitting into a vessel containing liquid. What pressure per square inch is transmitted to the liquid?

5. Explain the Hydrostatic Paradox.

6. Two communicating cylinders, the diameters of whose bases are 3 ins. and 8 ins., respectively, are fitted with pistons. If a weight of 27 lbs. be placed on the smaller piston, what weight must be placed on the larger to keep it at rest?

7. Describe Bramah's Press. Upon what principle does its action depend?

8. Find the thrust that can be produced in a Bramah's press, the areas of whose pistons are as 100 : 1, by a force of 16 lbs. applied at the end of a lever 28 in. long, and at a distance of 24 ins. from the point of attachment of the piston rod. (In this and the two following examples the lever is of the second class.)

9. If the areas of the pistons in a Bramah's press are as 8 : 1, what force must be applied at the end of a lever 21 ins. long and at a distance of 18 ins. from the piston rod to produce a thrust of $2\frac{1}{2}$ tons?

10. Find the ratio of the areas of the pistons if a force of 12 lbs. produces a thrust of 3 tons, the lever being 28 ins. long and the force applied at a distance of 24 ins. from the fulcrum.

CHAPTER X.

PRESSURE IN A LIQUID ARISING FROM WEIGHT.

Having defined the *pressure at a point* of a fluid in Chap. VIII., § 78, we now proceed to consider the pressures at different points of a liquid due to its weight, and we shall first show that

91. The pressure of a heavy liquid at rest is the same at all points in the same horizontal plane.

Consider the equilibrium of a long thin rectangular portion of liquid, whose faces $ABCD$ and $abcd$ are

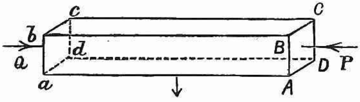

Fig. 24.

vertical, and whose edges Aa, Bb, Cc, Dd are horizontal. The forces acting on this portion are—

(i.) The weight of the liquid acting vertically;

(ii.) The thrusts of the liquid on the six faces acting perpendicular to them.

Now the thrusts on the ends $ABCD$ and $abcd$ are parallel to Aa and perpendicular to all the other forces, none of which can therefore affect their equilibrium.

Hence the thrusts on $ABCD$ and $abcd$ must be equal.

But the areas are equal.

Therefore the average pressures on them are equal.

And, since the areas $ABCD$, $abcd$ may be taken to be very small, it follows that the pressure *at* the point A is equal to the pressure at the point a.

Similarly, the pressures at any other points in the same horizontal plane are equal, as was to be proved.

COR. Hence, if a plane area be placed horizontally in heavy liquid, the pressure over its face is uniform.

92. The pressure of a uniform heavy liquid is proportional to the depth below the surface and to the density of the liquid.

Consider a rectangular column AB whose square base is the unit of area, extending from the surface of the liquid down to any given depth. The liquid inside this column is kept in equilibrium by the following forces:—

(i.) Its weight acting vertically downwards;

(ii.) The upward vertical thrust of the adjacent liquid on its base at B;

(iii.) The horizontal thrusts on the vertical faces.

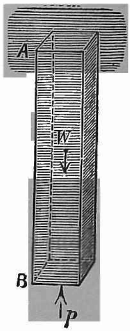

Fig. 25.

The first two must be equal, since the other forces are all perpendicular to them. Hence the thrust on the base is equal to the weight of the liquid column AB.

But, since the base is of unit area and the pressure is uniform over it, this thrust is equal to the pressure at B.

And the weight of the column is proportional to its volume, and therefore to its height. Hence the pressure is proportional to the depth below the surface. Also the weight of the column is proportional to the density of the liquid; therefore the pressure is proportional to the density.

Cor. *Hence we have the following very important result:—*

The pressure of a heavy liquid at a given depth is measured by the weight of a column of liquid whose height is equal to the given depth, and whose base is the unit of area.

This result enables us to deduce, from first principles, the pressure in water or any other liquid at any given depth measured in feet or centimetres.

Example. — To find the pressure in water at a depth of 6 ft.

Let B be a point 6 ft. below the surface.

Construct a rectangular column whose base is 1 ft. square, extending to the surface at A. Since the height is 6 ft., the column can be divided into 6 cubes, each containing a cubic foot;

∴ volume of column = 6 cub. ft.

Now a cubic foot of water weighs 1000 oz.;

∴ weight of column = 6000 oz.

∴ thrust on base at B = 6000 oz. weight;

∴ pressure ,, ,, = 6000 oz. per square foot

= 375 lbs. per square foot.

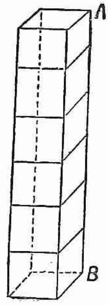

Fig. 26.

93. **To find expressions for the pressure of a given liquid at a depth of h feet.** — Firstly, let the liquid be water. Construct a rectangular column whose base is a foot square, extending from the surface down to the depth h feet; then the weight of this column is supported by the force on its base, and therefore measures the pressure per square foot.

By dividing the height into h portions of 1 ft. high, the column may be divided into h cubes, each measuring 1 cub. ft. Hence the volume of the column is h cub. ft., and the weight of the water contained in it = $1000h$ oz.;

∴ **pressure in water at depth of h ft.**

= $1000h$ oz. weight per square foot.

If the liquid be of specific gravity s, the weight of the column is s times as great as the weight of the corresponding column of water.

∴ **pressure at depth h ft. in liquid of specific gravity s**

= $1000hs$ oz. per square foot.

COR. 1. *The pressure in water increases by* 1000 *oz. per square foot for every foot of increase of depth.*

COR. 2. *If the specific gravity of a liquid be s, the increase of pressure for every foot increase of depth is s times as great as for water.*

94. **To find the pressure of a given liquid at a depth of h centimetres.**—Construct a column whose base is 1 cm. square, and whose height is h cm.; the weight of the liquid in this column measures the pressure at its base per square centimetre.

Now the volume of the column is h cub. cm.

But a cubic centimetre of water weighs a gramme.

Therefore, if the liquid be water, the weight of the column is h gm., and if the liquid be of specific gravity s, its weight is hs gm.;

$\therefore$ **pressure at depth of h cm.**

$= h$ grammes per square centimetre for water

$= hs$ grammes per square centimetre for liquid of specific gravity s.

COR. *The pressure in water increases by* 1 *gm. per square centimetre for every centimetre increase in depth.*

Examples.—(1) A corked-up bottle is lowered to a depth of 28 ft. in water, and the cork is $\frac{1}{16}$ ft. in diameter. What is the force tending to drive the cork in?

The pressure at depth of 28 ft. = 28,000 oz. per square foot.

Also the diameter of the cork $= \frac{1}{16}$ ft.;

$\therefore$ its area $= \frac{22}{7} \times (\frac{1}{32})^2$ square feet,

and the force on the cork

$$= \frac{22}{7} \times \frac{1}{32} \times \frac{1}{32} \times 28{,}000 \text{ oz.} = \frac{1375}{16} \text{ oz.} = 92\tfrac{3}{16} \text{ oz.}$$
$$= 5 \text{ lbs. } 12\tfrac{3}{16} \text{ oz. weight.}$$

(2) A penny sinks to the bottom of a lake 100 metres deep. To find the force which the pressure of the water exerts on either face of the penny.

The pressure at a depth 100 metres, or 10,000 cm.,

= 10,000 gm. per square centimetre

= 10 kilog. per square centimetre.

The diameter of a penny is 3 cm.;

$\therefore$ its area $= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2}$ sq. cm., or $\frac{99}{14}$ sq. cm.;

$\therefore$ the force on either face $= \frac{990}{40}$ kilog. weight $= \frac{495}{7}$ kilog. weight

$= 70\frac{5}{7}$ kilog. weight.

OBSERVATION.—*This force does not represent the resultant force on the penny as a whole*, because both sides of the penny are exposed to pressure, and these pressures act in opposite directions. The only *resultant* force on the penny is equal to the weight of water displaced (Chap. V.).

We may find in like manner the difference between the pressures at two different depths of a liquid.

Example.—To find the difference of pressure at the top and bottom of a vertical tube 760 mm. long filled with mercury.

The specific gravity of mercury is 13·6.
Therefore 1 cub. cm. of mercury weighs 13·6 gm.
Therefore difference of pressure for 1 cm. of height

$= 13 \cdot 6$ gm. per square centimetre.

Therefore difference of pressure for 76 cm. of height

$= 13 \cdot 6 \times 76 = 1033 \cdot 6$ gm. per square centimetre.

95. To find a general expression for the increase of pressure in a heavy liquid corresponding to a given increase of depth.

Let any units of weight and length be chosen. Let A, B be two points in the liquid in the same vertical line. Let p be the pressure at A, P the pressure at B, h the vertical distance AB, w the weight of a unit volume of liquid.

Describe any rectangular or cylindrical column of liquid whose height is AB, and let A denote the area of its base; the volume of the column is therefore Ah.

Then the only vertical forces acting on the column are—

(i.) the pressure pA acting downwards at A;

(ii.) the pressure PA acting upwards at B;

(iii.) the weight of the column wAh acting downwards.

Fig. 27.

Therefore, for the equilibrium of the column,

$$PA = pA + wAh;$$

$$\therefore \quad P = p + wh,$$

or

$$P - p = wh.$$

In other words, the increase of pressure $P-p$

= (increase of depth) × (weight of unit volume of liquid).

96. Experimental Illustrations. — The properties proved above may be verified by means of the apparatus shown in Fig. 28. A cylindrical tube has its lower end closed by a flat plate which can be held up by means of a string. On lowering the cylinder in water it will be found that, after a certain depth has been reached, the string may be let go without the plate sinking. The exact depth at which this happens may be found by again raising the tube slowly until the plate just sinks; the force produced by the pressure on the under side is then just equal to the weight of the plate. Now repeat the experiment with different weights placed on the plate. If the added weight is equal to the weight of the plate (so that the total weight supported is doubled) it will be found that the depth at which the string may be let go is also doubled. That is, if the depth be doubled, the pressure is doubled.

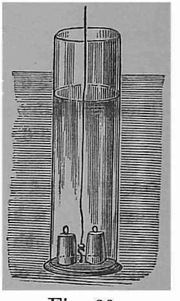
Fig. 28.

Similarly, if the depth of immersion be increased threefold, the total weight when the plate sinks is also increased threefold, and so on. Hence the pressure is proportional to the depth.

Next, let the experiment be repeated with liquids of different densities. It will be found that, if the cylinder be always immersed to the same depth, the total weight required to sink the plate (including, of course, the weight of the plate itself) is proportional to the specific gravity, and therefore to the density, of the liquid.

97. To show that the free surface of a heavy liquid at rest is horizontal.

Take any two points P, Q in the liquid, such that the line PQ is horizontal, and lies entirely in the liquid. Let the verticals through P, Q meet the surface in A, B.

Since PQ is horizontal,

$\therefore$ pressure at $P =$ pressure at Q. (§ 91.)

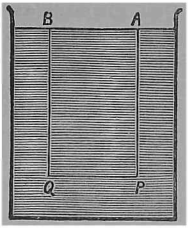

Fig. 29.

But the pressures at P, Q are proportional to AP, BQ, the depths below the surface;

$$\therefore \quad AP = BQ;$$

$\therefore$ AB is parallel to PQ.

But PQ is horizontal;

$\therefore$ AB is horizontal.

And, similarly, any line drawn in the surface is horizontal; therefore the surface is horizontal.

98. The surface of a liquid at rest rises everywhere to the same level.

Experimental Illustration.—This well-known property may be verified experimentally by constructing an apparatus such as that shown in Fig. 30, in which several open vessels of different shapes and sizes D, E, F communicate freely with one another. If water or any other liquid be poured into one of them, it will rise to the same level in them all.

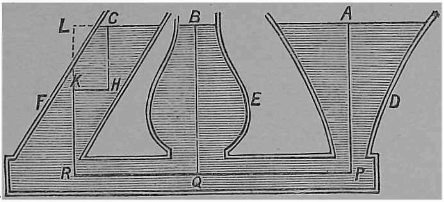

Fig. 30.

The proof of § 97 holds when the liquid is contained in two or more communicating vessels such as D, E. In this case all the free surfaces are at the same level, and form part of one and the same horizontal plane. If the liquid is contained in a vessel such as that shown at F the proof fails, for we cannot construct a vertical column whose base is at a point R of the bottom without passing out of the liquid.

But we can always connect any point R with the surface by means of a zigzag of alternately vertical and horizontal straight lines CH, HK, KR, and can find the difference between the pressures at any two points on this zigzag by § 95.

Thus, since CH is vertical, we have

$$\text{pressure at } H = w \times CH \text{ .... (i.)},$$

where w is the weight of unit volume of liquid.

Since BC is horizontal,

$$\text{pressure at } K - \text{pressure at } H = 0 \text{ ...... (ii.)}.$$

Since KR is vertical,

$$\therefore \text{ pressure at } R - \text{pressure at } K = w \times KR \text{ ... (iii.)}.$$

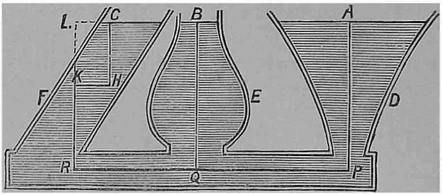

Fig. 30.

Therefore, by adding (i.), (ii.), (iii.),

$$\text{pressure at } R = w \times (CH + KR)$$
$$= w \times \text{depth of } R \text{ below surface.*}$$

We may now show that the liquid in the vessel F reaches the same level as in D. For, if R is on the same level as P, the pressures at R, P are equal, and therefore the depth of R below the surface at C is equal to that of P below the surface at A. Hence AC is horizontal.

* For, produce RK up to meet the horizontal plane through C in L. Since CL, HK are parallel, $\therefore$ $CH = LK$, and $\therefore$ $CH + KR = LR$ = depth of R below level of surface.

99. **The Water Level,** an instrument used in surveying, is based on this principle. It consists of two vessels D, E communicating by means of a tube, and partially filled with water. The water of course rises to the same level in both, and enables us to find the horizontal direction. To facilitate its use, the two vessels contain floats A, B having "sights" X, Y fixed on them, which rise to equal heights above the level of the water. Since the water reaches the same level in the vessels D, E, the sights are also on the same level, and the line joining them is horizontal. If now a distant object be observed to be in a line with the two sights, we know that the object and the sights are on the same level. We are thus able to find any number of different points on the same level, and hence to determine the difference of level of two different places.

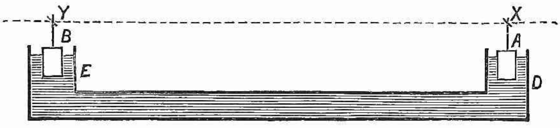

Fig. 31.

100. **The water supply of towns** affords an excellent illustration of these principles. The water is brought from a reservoir above the town by means of a series of mains and pipes, and, whatever be the arrangement of these mains, the water everywhere tends to rise to the level of its surface at the reservoir. When at rest, the pressure of the water at any point is proportional to the vertical depth of that point below the reservoir.

[*Practically*, however, the reservoir must be placed somewhat *above* the highest point to be supplied, in order that the water may *flow through* the pipes sufficiently rapidly to supply the town.]

101. DEF.—The **head** of liquid means the height of the column of liquid to which the pressure at any given point is due.

Thus, at a point 100 ft. below the level of the reservoir there is a *head* of water of 100 ft.

102. **The thrust exerted by a liquid of given depth on the base of its containing vessel is independent of the shape of the remaining portion of the vessel.**

For the pressure at any point of the base depends only on the depth of the liquid and the density, and not on the shape of the other part of the containing vessel; and the thrust upon the base depends only on this pressure and the area of the base.

103. **Pascal's Vases.**—The above properties may be illustrated by taking a number of vessels of conical, cylindrical, and other shapes (Figs. 33, 34, 35), the apertures at the bottom of which are of the same size and can be closed by a circular disc. One of the vessels is fixed upright, and the disc closing it suspended from a hydrostatic balance by a string fastened to a hook on its upper side (Fig. 32).

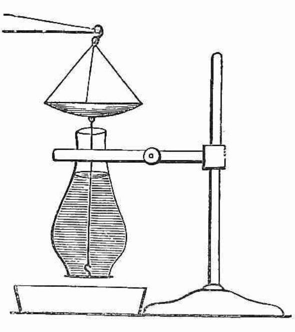

Fig. 32.

In the other scale-pan are placed weights by which the disc is held up against the bottom rim of the vessel.

Now, let water be poured into the vessel. The disc will fall, and the water will escape as soon as the weight of the disc and the thrust of the water on it together exceed the weights in the opposite scale.

Let the experiment be repeated, using the same weights and one of the other vessels. When the water has been poured in to the same height as before, it will again escape. Hence the thrust on the disc is the same in each case, provided that the depth of water and the area of the base are the same.

104. To explain why the thrusts on the bases of Pascal's Vases are not always equal to the weight of the contained liquid.

CASE I. — *In a cylindrical vessel* $ABCD$ (Fig. 33) the reactions of the sides AD, BC are horizontal, and therefore the thrust on the base *equals* the weight of the liquid.

CASE II. — *If the sides slant upwards from the base* (Fig. 34), the thrust on the base (being independent of the shape of the vessel) is the same as in a cylindrical vessel $ABEF$ with the same base and altitude, and is therefore *less* than the total weight of liquid.

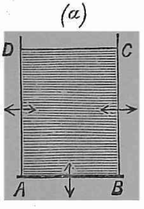

Fig. 33.

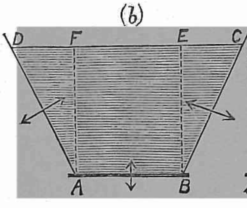

Fig. 34.

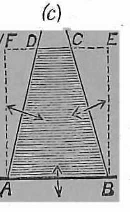

Fig. 35.

Here, however, the pressures of the liquid acting perpendicularly to the sides of the vessel have a vertical component which produces a downward thrust on these sides.

Considering separately the equilibrium of the liquid outside and inside the cylinder $ABEF$, and observing that the pressures of these portions on each other have no vertical component, we have

vertical thrust on *sides* AD, BC = weight of liquid *outside* $ABEF$;

,, ,, *base* AB = ,, ,, *inside* $ABEF$.

Adding these together, we find, as we might expect, that

vertical thrust on *whole vessel* $DABC$ = weight of *whole* of liquid.

Case III.—*If the sides slant inwards from the base* (Fig. 35), the vessel contains less liquid than a cylinder $ABEF$ on the same base; hence the thrust on its base is *greater* than the weight of the contained liquid.

(c)

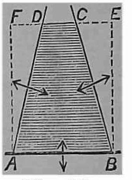

Fig. 35.

Here, however, the pressures on the sides AD, BC have an upward component, and therefore the liquid tends to lift the sides.

Suppose liquid poured into the cylinder $ABEF$ to the same height outside $ABCD$ as inside. The pressures on the inside and outside of the faces AD, BC will now balance each other. Hence

upward thrust on *sides* AD, BC = weight of liquid *outside* $ABCD$.

But

downward ,, ,, *base* AB = ,, ,, *in cylinder* $ABEF$.

Subtracting these, we find, as we might expect, that

resultant thrust on *whole vessel* $DABC$ = weight of *contained* liquid.

105. **The Hydraulic Lift.**—In this, as in the Hydrostatic Press, very large weights are raised by the pressure of water on the under surface of a large piston; but this pressure is produced by the weight of a column of water instead of by a force applied to a small plunger.

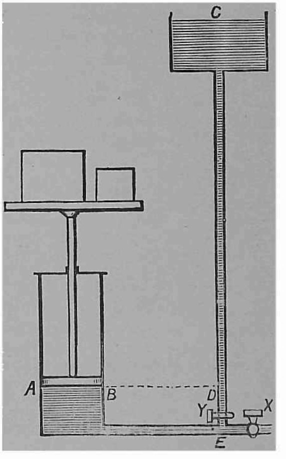

Fig. 36.

Thus, if CE is a vertical column of water connected with a cylinder which contains the piston AB, and if the surface AB produced meets the column CE in D, the pressure per square inch at the level ABD is equal to the weight of a column of height CD and sectional area 1 sq. in. Hence the *thrust* on AB is equal to the weight of a cylinder of water of height CD, whose base is the area AB.

And, by making the piston AB very large, this upward force can be made as large as we like without altering the weight of fluid in CD which produces it.

Thus any quantity of fluid, however small, may be made to lift any weight, however large.

When the lift is raised, water flows into the cylinder AB, and the tube is kept filled from a high reservoir. To lower the lift, the cylinder is disconnected from the tube CE, and the water allowed to escape by the tap X, the piston descending by its own weight.

*106. **Effect of variations in the direction of gravity.**—The proof that the surface of a liquid is a horizontal plane is only true when the body of fluid considered is so small that the "verticals" or directions of gravity are parallel at all points of the fluid. But in a large body of water, such as a lake or ocean, the verticals at different points cannot be regarded as parallel, since they meet in the centre of the Earth, and the proofs of §§ 97, 98 no longer hold. Here the surface of the water is not plane, but convex.

The surface of water in the neighbourhood of any point is still horizontal, if by "horizontal" we mean everywhere perpendicular to the vertical or direction of gravity. Combining this with the fact that the verticals at different places meet approximately at the centre of the Earth, it is possible to show that the surface of the ocean is approximately spherical, its centre being at the centre of the Earth; as is easily verified by observation.

*107. **The intensity of gravity** g is known to vary slightly in different latitudes, and this produces slight local variations in the pressure due to the weight of a column of liquid of given depth and density. Employing the formula $p = wh$ for the increase of pressure in depth h, we notice that w is proportional to g; hence the pressure p is proportional to g. If w and p be measured in dynamical units of force, and if d be the density, then, since the weight of a unit volume in dynamical units is g times its mass

$$\therefore \quad w = dg;$$

$$\therefore \quad p = dgh \text{ dynamical units of pressure.}$$

If the experiments of §§ 96, 103 were repeated in different latitudes—say at the equator and near the pole—no difference would be observed, because in these experiments the pressure is made to balance the weights of masses (*i.e.*, the set of weights employed), and these also undergo the same proportional variations according to the value of "g."

Thus we could prove by experiment that the pressure due to a given depth of given liquid is proportional to g.

Experiment also shows that the intensity of gravity decreases if we go from the surface towards the centre of the Earth. Hence in the case of a very deep ocean, the pressure is no longer strictly proportional to the depth.

*108. **Effect of compressibility of liquid.**—Another reason why the pressure at great depths is not strictly proportional to the depths is that all liquids are slightly compressible. The lower portions are squeezed down by the pressure due to the weight of the liquid above, and occupy slightly less bulk than they would do if the pressure were removed. Hence, in a column of liquid, the density increases slightly with the depth. For this reason, also, the weight of the column, and therefore the pressure at great depths increases a little more rapidly than it would if the liquid were absolutely incompressible.

*109. **Effect of variations of temperature.**—If a liquid be heated, it expands and occupies a greater volume than before. Hence its density, and therefore the weight of a unit volume, decreases. Therefore the pressure at a given depth also decreases with a rise of temperature. But, if the liquid is contained in a *cylindrical* vessel and is heated, it will rise to a greater height in the vessel, and this will make up for the diminution of density, so that the force on the base will still be equal to the weight of the liquid.

SUMMARY.

1. *The pressure in a liquid arising from its weight*

   (i.) Is the same at all points on the same level;

   (ii.) Is proportional to the depth and the density of the liquid;

   (iii.) Is measured by the weight of a column of unit sectional area extending to the surface.

2. *The pressure at depth* $h = wh$

   $= 1000\,hs$ oz. per square foot if h is measured in *feet*

   $= hs$ gm. per square centimetre if h is measured in *centimetres;*

where $w =$ specific weight of liquid,

$s =$ specific gravity referred to water.

3. *If the surface is at pressure* p, the above pressures must all be increased by p.

EXAMPLES X.

1. Find in pounds per square inch the pressure in water at a depth of 32 ft.

2. Show that the pressure is the same at equal depths in a body of liquid, and find the increase of pressure in pounds per square inch for every foot-depth of water.

3. Find the difference between the pressures at the top and bottom of (i.) a column of water 30 ft. high;
(ii.) a column of air a mile high (specific gravity = ·001);
(iii.) a column of sea-water a kilometre deep;
(iv.) a column of mercury 760 mm. high.

4. How would you show experimentally that the difference in pressure at two points in a heavy liquid is proportional to the difference in depth of the points? Does the pressure at any point depend on anything besides the depth of the point?

5. A vessel, whose shape is that of a pyramid 4 ft. high, has a base 5 sq. ft. in area. Find the pressure and the force on the base when the vessel is filled with water.

6. Three rectangular cisterns are filled with water. One of them is in the form of a cube whose edge is 7 ft.; another is 4 ft. high, 4 ft. wide, and 13 ft. long; and the third is 3 ft. high, 3 ft. wide, and 15 ft. long. Show that the thrust on the base of the second is 5 tons 1800 lbs., the weight of a cubic foot of water being 1000 oz., and that this thrust is equal to the difference between the thrusts on the bases of the other two, the atmospheric pressure not being taken into consideration.

7. A spherical boiler 4 ft. in height is half full of water and half full of steam. What is the difference between the pressure at the top and bottom of the boiler?

8. Find the height of a column standing in water 30 ft. deep, when the pressure at the bottom is to the pressure at the top as 4 to 3.

9. A long glass tube of 1 in. diameter has a disc weighing 2 oz. placed at one end. How far under water must the end of the tube, with the disc below it, be immersed, in order that the disc may not all off.

10. Determine the greatest depth in fathoms at which a submarine diver can work in sea-water, supposing he can bear a pressure of 5 atmospheres, taking an atmosphere to be a pressure of 15 lbs. per square inch.

11. A hole 6 ins. square is made in a ship's bottom 20 ft. below the water-line. What force must be exerted in order to keep the water out, by holding a piece of wood against the hole, if a cubic foot of water weighs 64 lbs.?

12. *A* and *B* are vessels full of water with circular and horizontal bases 12 ins. and 8 ins. in diameter, respectively. *A* is 8 ins., and *B* is 9 ins., high. Compare the pressure on the bases.

13. Two rectangular cisterns standing on a horizontal plane are joined at their bases by a leathern pipe resting on the plane. If one of them be 5 ft. long and 3 ft. broad and the other 4 ft. 6 ins. long and 3 ft. 4 in. broad, and water be poured into either, then, when water is at rest, the thrusts on the bases will be equal.

14. Prove that the surface of a heavy fluid at rest under the action of gravity is a horizontal plane. Why is this not true of very large surfaces of water?

15. Find the pressure at a given depth (z) in a liquid whose specific gravity is s, and whose surface is subject to a given pressure P.

16. Taking 1000 oz. as the weight of a cubical foot of water, and 15 lbs. weight on a square inch as the atmospheric pressure, find, in hundredweights, the thrust on a horizontal area of 7 sq. ft. in water at the depth of 32 ft.

17. Find the pressure at a depth of 96 ft. below the surface of the sea, the pressure of the atmosphere at the surface being 14 lbs. per square inch, the weight of a cubic foot of ordinary water 1000 oz., and the specific gravity of sea-water 1·025.

18. If a cubic foot of sea-water weighs 1025 oz., what will be the pressure on the square inch at the depth of $\frac{1}{2}$ mile? (The pressure of the atmosphere at the surface is to be taken into account.)

19. Show that the effect of an external pressure of $13\frac{8}{9}$ lbs. per square inch may be allowed for when the liquid is water, by supposing a layer of water, 32 ft. thick, to be superposed on the original liquid.

20. The pressure at the bottom of a well is four times that at a depth of 2 ft.; what is the depth of the well if the pressure of the atmosphere is equivalent to 30 ft. of water?

21. If the pressure at a point 5 ft. below the surface of a lake be one-half of the pressure 44 ft. below the surface, account being taken of the atmospheric pressure, find the atmospheric pressure in pounds on the square inch, assuming a cubic foot of water to weigh 1000 oz.

22. The pressure at a point 3 ft. below the surface of a heavy fluid is 30 lbs. per square inch, and at a depth of 7 ft. it is 50 lbs. What is the pressure at the surface?

CHAPTER XI.

PRESSURE DUE TO THE WEIGHT OF SEVERAL DIFFERENT LIQUIDS.

110. We shall now consider the pressures arising from the weight of several liquids of different densities which do not mix, but rest on one another.

The proofs of §§ 91, 98 show that in any *continuous* portion of the same liquid the pressure is always the same at all points in the same horizontal plane, whether this pressure is due to the weight of the liquid itself or to the weight of superincumbent liquids.

[NOTE.—By a *continuous* portion, we imply that any two points can be connected by a zigzag of alternately horizontal and vertical lines, as in § 98, without passing out of the liquid.]

Example.—A vessel 10 cm. deep contains mercury to the depth of 1 cm., and is filled up with water. To find the pressure at the bottom of the vessel, the atmospheric pressure being 1033 gm. per square centimetre.

Construct a rectangular column whose base is 1 cm. square, extending from the bottom to the top of the liquid, and consider the equilibrium of the liquid in this column.

The column may be divided into ten cubes, each 1 cub. cm.: nine filled with water, and one with mercury.

But

weight of 9 cub. cm. of water $= 9$ gm.,

weight of 1 cub. cm. of mercury $= 13{\cdot}6$ gm.,

and thrust on upper end $= 1033$ gm.;

$\therefore$ thrust on base $= 1055{\cdot}6$ gm.,

and pressure at bottom of liquid

$= 1055{\cdot}6$ gm. per square centimetre.

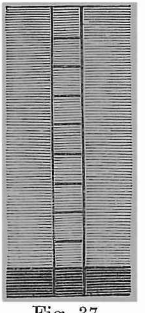

Fig. 37.

111. To find the pressure at any point due to the weight of several liquids which rest one on another without mixing.

Let s_1, s_2, s_3 be the specific gravities of any different liquids that do not mix, w being the weight of unit volume of water.

Let P be the atmospheric pressure at the surface Aa, p the required pressure at any given point R.

Draw $RQPO$ vertical, and construct a rectangular column on a unit of area as base, extending from R to the surface Aa. From the equilibrium of this column, we find—

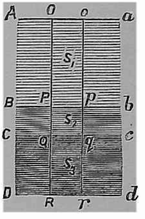

Fig. 38.

Pressure at R = pressure at O + sum of weights of vertical columns of the several liquids having the unit of area for their base, and extending from R to O.

Now the weights of the columns are $ws_1 \,.\, OP$, $ws_2 \,.\, PQ$, $ws_3 \,.\, QR$, respectively.

$$\therefore \quad p = P + w\,(s_1 \,.\, OP + s_2 \,.\, PQ + s_3 \,.\, QR).$$

112. To prove that, when several liquids of different densities do not mix, the common surface of any two of the liquids is horizontal.

Consider two liquids of specific gravities s_1, s_2.

Let M, Q be any two points in their common surface.

Draw the verticals LMN, PQR through M, Q, and on them take points L, P in the upper and N, R in the lower liquid, such that the lines LP and NR are horizontal. Then the pressures at L, P are equal, and the pressures at N, R are equal; therefore pressure due to columns LM, MN = pressure due to columns PQ, QR.

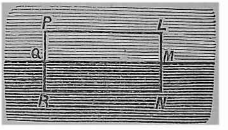

Fig. 39.

Therefore, as in the last paragraph,

$$w\,(s_1 PQ + s_2 QR) = w\,(s_1 LM + s_2 MN)\,;$$

that is,

$$\therefore\quad s_1(PR - QR) + s_2 QR = s_1(LN - MN) + s_2 MN.$$

Also $$s_1 PR = s_1 LN.$$

Hence, by subtraction,

$$s_2 QR - s_1 QR = s_2 MN - s_1 MN\,;$$

i.e., $$(s_2 - s_1)\,QR = (s_2 - s_1)\,MN.$$

But $s_2 - s_1$ is not zero;

$$\therefore\quad QR = MN.$$

Hence QM is parallel to RN, and therefore horizontal.

OBSERVATIONS.—When the liquids are contained in two or more intercommunicating vessels, such as the **U**-tube about to be described, we shall see that the common surfaces of *separate* portions (cf. § 110, note) are *not* necessarily all at the same level.

When several liquids are poured into a vessel they will always arrange themselves in order of their densities, *the heaviest liquid being the lowest*. If the density of any liquid were greater than that of the one next below, the two might for an instant remain in equilibrium with their common surface horizontal, but the least disturbance would turn them topsy-turvy. The equilibrium would, in fact, be "*unstable.*"

113. **The U-tube** is, as its name implies, simply a glass tube bent into the shape of an elongated **U**, one of its uses being to compare the specific gravities of liquids which do not mix. For this purpose, the heavier of two given liquids should first be poured into the bend of the tube, and the lighter then poured down one of the branches.

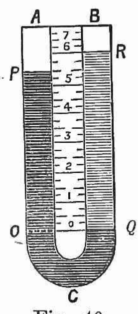

Fig. 40.

The heights of the free surfaces P, R above the surface of separation Q can be measured by a scale of inches or millimetres placed behind the two tubes, and by comparing these heights the densities or specific gravities of the liquids may be compared.

Example.—A U-tube was partly filled with water, and oil was poured into one of the branches to a depth of 6 ins. The surface of the water in the other branch stood 5·49 ins. above the common surface of the oil and water. To find the specific gravity of the oil.

Let PCQ be the water, QR the column of oil (Fig. 40), and let O be the point on the branch PC at the same level as Q.

Since the portion QCO is all filled with the same liquid (water),

$\therefore$ pressure at Q = pressure at O ...... (i.).

Let W be the weight of a cubic inch of water, w that of a cubic inch of oil. Then, since

$$PO = 5 \cdot 49 \text{ ins. and } RQ = 6 \text{ ins.},$$

pressure at O (per sq. in.) $= W \times PO = W \times 5 \cdot 49$,

pressure at Q $= w \times RQ = w \times 6$;

therefore, by (i.), $W \times 5 \cdot 49 = w \times 6$,

and specific gravity of oil $= \dfrac{w}{W} = \dfrac{5 \cdot 49}{6} = \cdot 915$.

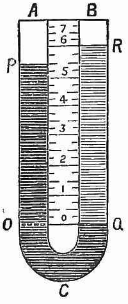

Fig. 40.

114. **When the two branches of a U-tube contain two different liquids which do not mix, their specific gravities are inversely as the heights of their free surfaces above their surface of separation.**

For, let s_1, s_2 be the specific gravities of the liquids in the portions PCQ, QR (Fig. 40), w the weight of a unit volume of water.

Let the horizontal through Q meet the branch AC in O. Then we have for the equilibrium of the liquid QCO

$$\text{pressure at } O = \text{pressure at } Q.$$

$$\therefore \quad ws_1 \times PO = ws_2 \times RQ,$$

whence
$$\frac{s_1}{s_2} = \frac{RQ}{PO};$$

or
$$\frac{\text{specific gravity of liquid in } PQ}{\text{specific gravity of liquid in } RQ} = \frac{\text{height of } R \text{ above } Q}{\text{height of } P \text{ above } Q},$$

as was to be proved.

Example.—If the **U**-tube contains mercury of specific gravity 13·5 to within 5 ins. of the top, to find the height of the column of water which must be poured in to fill one of the branches.

Let p, q be the original surfaces of the mercury. As soon as water is poured into the branch BC, the surface of the mercury will sink in that branch from q (say) to Q, and will rise in the other from branch p to P. We suppose the branches of the tube to be of equal diameter; we shall then have
$$pP = qQ.$$
Let
$$qQ = x.$$
Since $Bq = 5$ ins., we have

height of column of water $BQ = 5 + x$,

height of mercury at P above common surface at Q
$$= Pp + qQ = 2x.$$

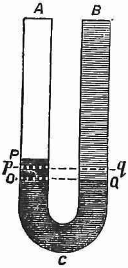

Fig. 41.

Hence, since the pressures at the level of Q are equal in the two branches,
$$\therefore \quad 1 \times (5 + x) = 13{\cdot}5 \times 2x = 27x;$$
$$\therefore \quad 5 = 26x; \quad \therefore \quad x = \tfrac{5}{26};$$
and height of water column $BQ = 5 + x = 5\tfrac{5}{26}$ ins.

SUMMARY.

1. The common surface of two continuous portions of different liquids is horizontal.

2. The pressure at a given depth in one of the lower liquids = sum of pressures due to separate liquids.

3. If the **U**-tube contains two liquids, heights above common surface are inversely proportional to densities.

EXAMPLES XI.

1. Prove that the common surface of two liquids of different density which do not mix is a horizontal plane. Does the argument apply to the parts of the common surface in the two branches of a **U**-tube containing water, when different quantities of oil are poured down the two arms?

2. A vessel whose bottom is horizontal contains mercury whose depth is 20 ins., and water floats on the mercury to the depth of 16 ins. Find the pressure at a point on the bottom of the vessel in lbs. per square inch, specific gravity of mercury being 13·6.

3. A vessel whose base is a square, the side of which measures

6 ins., contains mercury to the depth of 1 in., and water is poured upon the mercury to the depth of $10\frac{1}{2}$ ins. If the specific gravity of the mercury be 13·5 times that of water, find the pressure on the base of the vessel.

4. Find the whole thrust on a square, the length of a side of which is 4 ins., immersed horizontally in oil at a depth of 5 ins., the specific gravity of oil being ·87.

5. A circular cylinder, whose radius is 14 cm. and height 40 cm., is filled half with water and half with oil of specific gravity ·9. Find the pressure anywhere on the curved surface of the cylinder, and also the thrust on the base. (Take $\pi = \frac{22}{7}$.)

6. The two branches of a uniform bent tube are straight and vertical, and the portion of the tube which unites them is horizontal. Water is poured in sufficient to fill 6 ins. of the tube, and then oil, sufficient to occupy 5 ins., is poured in at one end, the specific gravity of the oil being four-fifths that of water. Find the position of the fluids when they are in equilibrium, the horizontal part of the tube being 2 ins. long.

7. Two liquids which do not mix are contained in a **U**-tube. Obtain a relation between their densities and their heights above the common surface.

8. Water is poured into a **U**-tube, the legs of which are 8 ins. long, until they are half full. As much oil as possible is then poured into one of the legs. What length of the tube does it occupy, the weight of the oil being two-thirds that of water?

9. The lower ends of two vertical tubes, whose cross sections are 1 and ·1 sq. ins. respectively, are connected by a tube. The tubes contain mercury. How much water mnst be poured in to raise the level of the mercury in the smaller tube 1 in.?

10. A bent tube containing equal quantities of two liquids which do not mix consists of two branches inclined at angle 60°. When one of the branches is held vertically, the different fluids meet at the angle of the tube. Show that when the tube is held with the two branches equally inclined to the vertical, one-fourth of the liquid contained in the branch which was previously inclined to the vertical flows into the one which was vertical.

CHAPTER XII.

RESULTANT THRUSTS OF HEAVY LIQUIDS ON PLANE AND OTHER SURFACES.

115. Def.—When *one* side of any surface is exposed to pressure, the *force* which that surface experiences owing to the pressure is called the **resultant thrust** or **pressure-resultant** on the surface.*

Its vertical and horizontal components are the **vertical and horizontal thrusts** on the area respectively.

[Note that a **resultant thrust** is a particular force, while a *pressure* is a force per unit area.]

In the present chapter, we shall show how to find—

(i.) The resultant thrust of a heavy liquid on a horizontal plane area.

(ii.) The vertical thrust on any surface.

(iii.) The resultant thrust on a plane area inclined to the horizon or vertical.

116. **To find the resultant thrust on a horizontal plane area.**—When a *horizontal* surface is exposed to the pressure of a heavy liquid, this pressure is *uniform* over the whole surface, and its amount may be found as in Chap. X. Multiplying this pressure by the area of the surface, we have the required resultant thrust on the area.

* When this book was first in manuscript, it was our intention to use the term "pressure-resultant"; but quite recently the term "thrust" has come into very general use in Hydrostatics, and accordingly we have gladly adopted it. The student should, however, take care to be able to identify any of the other terms used in this sense, such as *resultant pressure*, which is still commonly found in books and examination papers, although it would be more correct to speak of *resultant force of pressure.*

117. To find the vertical thrust on any surface.—When any surface S is exposed to the pressure of a heavy liquid, the vertical thrust may be found by drawing verticals, such as AB, CD, from the boundary of S to the surface of the liquid. These verticals, together with the surface itself, will enclose a column of liquid $ABDC$, whose weight is equal to the required vertical thrust on the surface.

Case I.—Suppose that the liquid presses on the upper side of the surface S (Fig. 42).

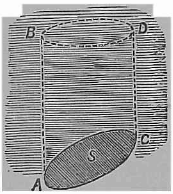

Fig. 42.

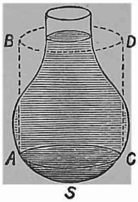

Fig. 43.

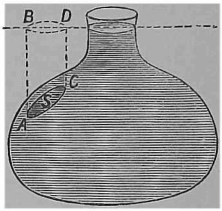

Fig. 44.

Consider the equilibrium of the liquid in this column. The pressures acting perpendicularly on its vertical sides are horizontal. Hence they have no vertical component. If there is no pressure at the upper surface BD, we therefore have

vertical thrust on S = weight of liquid in column $ABDC$.

Case II.—If the sides of the vessel should anywhere fall within the cylinder $ABDC$ (as in Fig. 43), it is only necessary to suppose the vessel replaced by a larger one, whose sides do not fall within the cylinder, the liquid rising to the same level as before. The pressure at every point of S will be unaltered, and therefore the vertical thrust on S will still be equal to the weight of liquid required to fill $ABDC$.

Case III.—Let the liquid press on the lower side of the surface S (Fig. 44). Let the vertical column $ABDC$ be constructed as before, extending from the surface S to the plane of the free surface. If this column be supposed filled with liquid, the pressures on the upper and under sides of S will now be everywhere equal, and therefore the thrusts on the two sides of S will be also equal and opposite. Hence the liquid exerts an *upward* vertical thrust on the under side of the surface S equal to the weight of the liquid required to fill the column $ABDC$.

These results are expressed by the statement that *the vertical thrust on a surface is always equal to the weight of the superincumbent column of liquid.*

Examples.—(1) To find the resultant thrust on the concave surface of a hemispherical bowl 1 ft. in diameter, immersed in water, with its base horizontal, at a depth of 2 ft. below the surface.

Construct a vertical cylinder having the rim of the bowl for its base. The weight of superincumbent water contained between the cylinder and bowl is equal to the resultant thrust on the surface of the bowl.

Now, the radius of the cylinder $= \frac{1}{2}$ ft., and height $= 2$ ft.;

$\therefore$ area of its base $= \frac{22}{7} \times (\frac{1}{2})^2 = \frac{11}{14}$ sq. ft.;

$\therefore$ volume of cylinder $= \frac{11}{14} \times 2$ cub. ft. $= \frac{11}{7}$ cub. ft.;

and $\therefore$ weight of water in cylinder $= \frac{11}{7} \times 1000$ oz.

Again, volume of hemisphere $= \frac{2}{3}\pi r^3 = \frac{2}{3} \cdot \frac{22}{7} \times (\frac{1}{2})^3$ cub. ft.

$\therefore$ weight of water in hemisphere $= \frac{11}{42} \times 1000$ oz.

If the hemisphere is turned *base upwards*, the resultant thrust

$= $ whole weight of water supported $= \frac{11000}{7} + \frac{11000}{42}$ oz.

$= \frac{77000}{42}$ oz. $= 1833\frac{1}{3}$ oz.

If the hemisphere is *base downwards*, the resultant thrust

$$= \frac{11000}{7} - \frac{11000}{42} \text{ oz.}$$

$$= \frac{55000}{42} \text{ oz.} = 1308\tfrac{8}{21} \text{ oz.}$$

(2) A cone, whose height is 3 cm. and the area of whose base is 10 sq. cm., is filled with water and placed vertex upwards on a table. To find the thrust on its base and sides.

Pressure at depth 3 cm $= 3$ gm. per sq. cm.;

$\therefore$ thrust on base $= 30$ gm.

Also vol. of cone $= \frac{1}{3}$ base $\times$ height $= \frac{1}{3} \cdot 10 \cdot 3 = 10$ cub. cm.;

$\therefore$ weight of water in cone $= 10$ gm.

From the equilibrium of the liquid in the cone, we therefore have

vertical thrust on sides of cone $= 30$ gm. $- 10$ gm. $= 20$ gm.,

and this thrust acts upwards, tending to lift the sides.

118. **To find the resultant thrust on any plane area,** it is sufficient to find the *average pressure* on the area. Since the area is plane, the product of this average pressure into the area is equal to the required resultant thrust on the area.

The following example will be an instructive introduction to the method to be followed in the next article.

Example.—To show that in the case of a rectangle $ABCD$ in a vertical plane immersed to any depth *the average pressure is equal to the pressure at the centre of gravity of the area.*

Divide the rectangle into a large number of very thin strips of equal breadth, by ruling a number of equidistant horizontal lines across its face. Let EF be the vertical line bisecting the rectangle, and therefore passing through its centre of gravity G. Consider any pair of strips PQ, RS of equal area a, whose centres H, K are at equal distances above and below G. If O be in the surface of the liquid, we have

pressure at $H = w \times OH$,
pressure at $G = w \times OG$,
pressure at $K = w \times OK$;

$\therefore$ thrust on strip PQ

$= w \times OH \times$ area a,

thrust on strip RS

$= w \times OK \times$ area a.

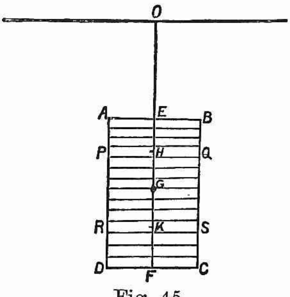

Fig. 45.

Now $HG = GK$;

$$\therefore \quad OH + OK = (OG - HG) + (OG + GK) = 2OG;$$

hence, sum of thrusts on the two strips

$$= w \times (OH + OK) \times a = w \times 2OG \times a$$
$$= w \times OG \times \text{sum of areas of strips};$$

and this is the same as if the pressure on either strip were equal to $w \times OG$, that is, equal to the pressure at G.

In the same way, the sum of the thrusts on every other pair of strips is the same as if the pressures on them were equal to that at G.

Therefore the thrust on the whole rectangle is the same as if the pressure were everywhere equal to that at G.

Hence, the average pressure on the rectangle is equal to the pressure at its centre of gravity G.

The following generalization of this result is most important:—

119. To prove that the average pressure on any plane area immersed in a heavy liquid is equal to the pressure at its centre of gravity.

Divide the area into a very large number n of strips by ruling horizontal lines across it at small distances apart.

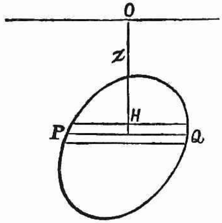

Fig. 46.

Let PQ be any such strip, H its middle point, and let the vertical through H meet the surface in O. Let the area of the strip $= a$, and let $OH = z$.

The pressure at $H = w \,.\, OH$. And since (by construction) the difference of level of the top and bottom edge of the strip is very small, the pressure is sensibly the same all over the strip.

Hence, thrust on strip $= w \,.\, OH \times$ area $PQ = wza$.

Thus, if $z_1, z_2, \ldots z_n$ are the depths and $a_1, a_2, \ldots a_n$ the areas of the strips, the thrust on the n^{th} strip is $w \times z_n a_n$.

Now the thrusts on the different strips are parallel.

$\therefore$ resultant thrust on whole area $=$ sum of thrusts on strips $= w \times (z_1 a_1 + z_2 a_2 + \ldots + z_n a_n)$.

Also whole area $= a_1 + a_2 + \ldots + a_n$;

$$\therefore \text{ average pressure} = w \times \frac{z_1 a_1 + z_2 a_2 + \ldots + z_n a_n}{a_1 + a_2 + \ldots + a_n}.$$

Now it is proved in Statics that if $\bar{z}$ be the depth of the centre of gravity of the areas $a_1, a_2, \ldots a_n$,

$$\bar{z} = \frac{z_1 a_1 + z_2 a_2 + \ldots + z_n a_n}{a_1 + a_2 + \ldots + a_n}.$$

Hence **average pressure on area**

$= w \times \bar{z} =$ pressure at depth $\bar{z}$

$=$ **pressure at C.G. of area.**

COR. 1.—The resultant thrust on any plane area is equal to the product of the area into the pressure at its C.G.

COR. 2.—The resultant thrust on a plane area depends only on the area and the depth of its C.G., and is unaltered by turning the area about its C.G., provided that the whole of the area is kept below the surface.

OBSERVATION.—In the above proof, *the area need not be in a vertical plane*, for the formula giving the depth of its C.G. in terms of the depths of the separate strips holds good whatever be the position or form of the area.

Examples.—(1) To find the resultant thrust on a vertical dock-gate 14 ft. wide if one side is exposed to the pressure of sea-water 10 ft. deep.

The area exposed to pressure $= 14 \times 10 = 140$ sq. ft. ;

the depth of its C.G. $= 56$ ft.

Now a cubic foot of sea-water weighs 64 lbs.,

$\therefore$ pressure at C.G. of area $= 5 \times 64 = 320$ lbs. per sq. ft. ;

$\therefore$ thrust on dock-gate $= 320 \times 140$ lbs. $= 44{,}800$ lbs. $= 20$ tons.

(2) To find the resultant thrust of water on the slanting face of an embankment 100 metres long and 30 metres broad, which shelves down to a depth of 12 metres below the surface at the lowest part.

The area exposed to pressure $= 100 \times 30 = 3000$ sq. metres.

The depth of its C.G. $= \frac{1}{2}$ depth of lowest portion

$= 6$ metres $= 600$ cm. ;

$\therefore$ average pressure $= 600$ gm. per sq. cm.

$= 600 \times 100 \times 100$ gm. per sq. metre

$= 6000$ kilog. per sq. metre. ;

$\therefore$ resultant thrust $= 6000 \times 3000$ kilog. $= 18{,}000{,}000$ kilog.

(3) A hemispherical bowl holding 4 lbs. of a liquid is held with its rim resting against a vertical wall. To find the resultant thrusts of the water (i.) on the wall, (ii.) on the bowl.

Let r be the radius of the bowl, w the specific weight of the liquid. Then weight of liquid in bowl $= \frac{2}{3}\pi r^3 w = 4$ lbs.

Since the area of the base is πr^2 and the depth of its centre of gravity is r, and the base is plane, the thrust on the wall is equal to

$$\pi r^2 w \times r = \pi r^3 w = \tfrac{3}{2} \text{ wt. of liquid} = 6 \text{ lbs.}$$

The three forces acting on the liquid contained in the hemisphere are

(i.) The reaction of the bowl, equal and opposite to the required resultant thrust.

(ii.) The weight of the liquid, which is equal to 4 lbs. and acts vertically downwards.

(iii.) The thrust of the base of the hemisphere, acting horizontally.

Hence the conditions of equilibrium require that the horizontal and vertical components of the resultant thrust on the bowl shall be 6 lbs. and 4 lbs., respectively. These components are at right angles; hence, if R denote the required resultant thrust, we have

$$R^2 = 6^2 + 4^2 = 52\,;$$

$$\therefore \quad \text{resultant thrust } R = \tfrac{1}{3}\sqrt{13}\,.\,\pi r^3 w$$
$$= \sqrt{52} \text{ lbs.} = 7{\cdot}2 \text{ lbs., nearly.}$$

*120. **Centre of Pressure.**—DEF.—The **centre of pressure** of a plane area immersed in fluid is the point in which the line of action of the resultant thrust of the fluid meets the area.

It does not coincide with the centre of gravity unless the pressure be uniform. Thus the centre of pressure of a rectangle with one side in the surface is at a depth equal to two-thirds the depth of the lowest side.

*121. **Whole pressure on a curved surface.**—When an area is not plane—as, for example, any part of the surface of a sphere or cylinder, the product of the area into the pressure at its centre of gravity is no longer equal to the resultant thrust, but is equal to a quantity called the "whole pressure."

When a surface consists of a number of plane areas (such as the six faces of a cube), the *whole pressure* is defined as the sum of the thrusts acting on the several faces. When the surface is curved, it must be divided into a large number of small portions, each portion being so small as to be approximately plane. The sum of the forces acting on all the small areas is defined as the whole pressure on the surface.

From these definitions it may be deduced that the whole pressure on any surface is equal to the product of its area into the pressure at ts centre of gravity.

The "whole pressure" on a *plane* area is the same as the resultant thrust. In all other cases, calculations of the whole pressure on surfaces are devoid of interest, and are not of the slightest practical use. They are, however, sometimes set in examinations.

The quantity found by dividing the whole pressure by the area of the surface is called the **average pressure** on the surface. Hence the average pressure on *any* surface is equal to the pressure at the centre of gravity of that surface.

The average pressure is, therefore, not equal to the *resultant thrust* divided by the area, except when the surface is plane.

Example.—(1) A hemispherical bowl whose diameter is 12 cm. is full of oil whose specific gravity is ·92. Find the whole pressure on the bowl, and the pressure-resultant.

$$\text{Area of surface of bowl} = \tfrac{1}{2} \times 4\pi \times 6^2 \text{ sq. cm.} = 72\pi \text{ sq. cm.}$$

$$\text{Depth of centre of gravity of bowl} = \text{half the radius} = 3 \text{ cm.}$$

$$\begin{aligned} \therefore\quad \text{whole pressure} &= \text{weight of } 3 \times 72\pi \text{ cm. of oil} \\ &= 216\pi \times {\cdot}92 \text{ gm.} = 624{\cdot}3 \text{ gm., nearly.} \\ \text{Pressure-resultant} &= \text{weight of fluid contained} \\ &= \text{weight of } \tfrac{1}{2} \times \tfrac{4}{3}\pi \times 6^3 \text{ cm. of fluid} \\ &= \tfrac{2}{3}\pi \times 216 \times {\cdot}92 \text{ gm.} = 416{\cdot}2 \text{ gm., nearly.} \end{aligned}$$

SUMMARY.

1. *The vertical thrust of a liquid* on any area is equal to the weight of the superincumbent column of liquid (§ 117).

2. *The average pressure of a heavy liquid on any area is equal to the pressure at the* C.G. *of the area* (§ 119).

3. *Hence resultant thrust on any* ***plane*** *area* $A = wzA$, where
   w = specific weight of liquid,
   z = depth of C.G. of area A below surface of liquid.

4. This thrust acts *perpendicular* to plane of area at a point called its *centre of pressure*, which is usually *lower* than its C.G.

5. If the area is not plane, wzA represents the "whole pressure," and not the resultant thrust.

EXAMPLES XII.

(Except where otherwise stated, a cubic foot of water weighs 1000 oz.)

1. A cubical box whose edge measures 1 ft. has a pipe communicating with it which rises to a vertical height of 20 ft. above the lid. It is filled with water to the top of the pipe. Find the upward force on the lid and the downward force on the base, and show that their difference is equal to the weight of water in the box. How do you account for the pressure on the base being greater than the weight of water it has to support?

2. A tapering glass tube, 10 ins. long, 1 in. in diameter at one end and $\frac{1}{2}$ in. at the other, is held vertically and filled with water, a thin plate (the weight of which is negligible) being pressed against the lower end to prevent the water from escaping. Compare the forces with which the plate must be held in its place, according as the larger end of the tube is at the top or bottom.

3. A hollow cone stands with its base on a horizontal table. The area of the base is 100 sq. ins., and the height 8·64 ins.; its weight is equal to the weight of water it will contain. When filled with water, what is the ratio of the pressure of the water on the base to that of the base on the table (supposed uniform)? How do you account for the result? (The volume of a cone is one-third of that of a cylinder with the same base and altitude.)

4. A right circular cone is open at the base and has a small hole at the vertex; it rests on a horizontal plane, the diameter of the base being 1 decim. and the height of the cone 2 decim. Find the weight of the cone that it may be just possible to fill it with water without causing it to lift from the plane.

5. Prove that, in a liquid subject to gravity, the average intensity of the pressure over any plane area is equal to the intensity at the centre of gravity of the area.

Does the line of action of the resultant thrust pass through the centre of gravity?

6. Determine the thrust in pounds on every foot-breadth of a vertical wall of a rectangular reservoir of water 150 ft. deep.

7. A lock-gate, 10 ft. wide and 10 ft. deep, has water on one side 8 ft. deep and on the other 5 ft. deep, in each case measured from the lower edge of the gate. Determine the resultant thrust.

8. Determine the total thrust on one side of a rectangular vertical dock-gate 50 ft. wide immersed in salt water to a depth of 25 ft., having given that the specific gravity of sea water is 1·026.

9. A cube whose edge is 1 ft. is suspended in water with its upper face horizontal, at a depth of $2\frac{1}{2}$ ft. below the surface. Find the thrust on each face of the cube.

10. An artificial lake, $\frac{1}{4}$ mile long and 100 yds. broad, with a gradually shelving bottom varying from nothing at one end to 88 ft. at the other, is dammed at the deep end by a masonry wall across its entire breadth. Find the total thrust on the embankment when the lake is full of water weighing $\frac{3}{4}$ ton to the cubic yard. Find also the total weight of water in the lake.

11. Find the thrusts on the faces of a cube, whose edge is 6 ins. long, immersed in water with its upper face horizontal at a depth of 5 ins.

12. A rectangular box is 18 ins. long, 8 ins. wide, and 12 ins. deep. One of its sides is removed, and a board is nailed on, joining that edge of the bottom from which the side has been removed to the top of the opposite side, and fitting against the ends of the box so as to be water-tight. The box is placed with its base horizontal, and the space between the bottom and the board is filled with water through a small hole made at the top of the board. Find the vertical thrust on the board.

13. A rectangular board is immersed in water with one of its longer edges parallel to the surface and at a given depth. Compare the whole thrust on the board when it is (i.) horizontal, (ii.) vertical and upwards, (3) vertical and downwards.

14. A bowl in the shape of a hemisphere is filled with water. Find the vertical thrust and the horizontal thrust on either of the portions into which it is divided by a vertical plane through its centre.

15. A hollow cone, whose height is 4 ins. and the radius of whose base is 3 ins., is fixed with its base horizontal and its vertex down-

wards. The cone is filled with water; find the resultant thrust on the curved surface, taking $\pi = \frac{22}{7}$.

16. If the cone in the last question be inverted so as to stand on its base, find the increase of the resultant thrust on the curved surface.

17. The surface of a vessel containing liquid consists of a number of plane faces of areas a_1, a_2, a_3, ..., &c., and the centres of gravity of these areas are at depths z_1, z_2, z_3, ..., &c., below the surface of the liquid. Write down the sum of the resultant thrusts on the several faces, the weight of a unit volume of liquid being w, and show that this sum is equal to the product of the whole superficial area of the vessel into the pressure at the centre of gravity of this area.

18. Explain in what respect the quantity called "whole pressure" on a curved surface differs from (i.) a pressure, (ii.) a force. What is the whole pressure on a *plane* surface?

19. A smooth vertical cylinder, 1 ft. in height and 1 ft. in diameter is filled with water and closed by a heavy piston weighing 5 bs. Find the "whole pressure" on its curved surface.

20. A cubic block, each of whose edges is 6 ins., is sunk in water to a depth of 2240 ft. Find the "whole pressure" upon its surface in tons, neglecting the dimensions of the cube in comparison with the depth of its immersion, and supposing a cubic foot of water to weigh 62·5 lbs.

21. A sphere, the radius of which is 4 ins., is totally immersed in water, with its centre at a depth of 6 ins. Find the "whole pressure" and the resultant thrust.

22. A hollow cone, whose axis is vertical and vertex downwards, is filled with water. At what depth is a horizontal plane situated when the "whole pressures" on the portions of the curved surface above and below it are equal?

EXAMINATION PAPER V.

1. Define *pressure at a point* in a fluid. Show that the pressure in a fluid is the same at all points in a horizontal plane.

2. Show that the pressure at any point in a fluid is proportional to the depth of the point below the surface.

3. Find the pressure at a depth of 120 ft. below the surface of a lake (i.) in pounds weight per square foot, (ii.) in dynes per square centimetre; the atmospheric pressure being neglected.

4. Show that the surface of a heavy liquid at rest in a vessel is horizontal.

5. Find the whole thrust on a plane surface immersed in liquid.

6. A cylindrical vessel, the radius of whose base is 4 ins., is placed with its base horizontal, and is filled with water to a height of 14 ins. Find the whole pressures on the base and the curved surface. (Take $\pi = \frac{22}{7}$.)

7. The face of an embankment is a rectangle $\frac{1}{4}$ mile long and 50 ft. wide, and is inclined so that it is completely immersed in water whose depth is 30 ft. Find the total thrust on the embankment.

8. A cube whose edge is 6 ins. is completely immersed in mercury (specific gravity = 13·6), so that its upper face is horizontal and at a depth of 9 ins. below the surface. Find the thrusts on the faces.

9. Find the whole resultant thrust on the surface of a circle whose radius is 4 ins. immersed in water with its centre at a depth of 14 ins.

10. How would you show experimentally that the pressure on the base of a vessel is independent of the shape of the vessel?

CHAPTER XIII.

RESULTANT THRUSTS ON IMMERSED AND FLOATING SOLIDS.

In the present chapter we shall consider more fully the principle of Archimedes which asserts that a body floating or immersed in liquid experiences an upward force equal to the weight of the liquid it displaces.

This force, since it is due to the pressure of the *liquid*, is the *resultant thrust* on the body, and we shall now find its line of action as well as its magnitude.

122. To find the magnitude and line of action of the resultant thrust of a liquid on any floating or immersed body.

Let the submerged portion of the body occupy the space bounded by the surface S (Fig. 5, p. 40).

The pressure at any point of S depends only on the density and depth of the liquid at that point; hence the thrust of the outside liquid on S does not depend on the nature of the substance filling S.

Let the body be removed and let S be filled with liquid similar to that surrounding S. This liquid is called the *liquid displaced* by the body. The liquid inside and outside S is now in equilibrium, since it may be regarded as forming part of the same continuous mass of liquid.

Now the forces acting on the liquid inside S are—

(i.) Its weight, acting vertically downwards through its centre of gravity;

(ii.) The resultant thrust of the surrounding liquid.

Hence the conditions of equilibrium show that—

The resultant thrust of a heavy liquid on any body

(1) **Is equal to the weight of liquid displaced;**

(2) **Acts vertically upwards through the centre of gravity of this displaced liquid.**

Def.—The centre of gravity of the liquid displaced by any body is called the **centre of buoyancy.**

123. Conditions of equilibrium of a floating body. —When a body (*e.g.*, a boat) floats in equilibrium at the surface of a liquid, there are two forces on it, namely, its weight acting at its centre of gravity, and the resultant thrust of the surrounding liquid acting through the centre of buoyancy. If these keep the body in equilibrium, they must be equal and opposite and in the same straight line.

Hence the conditions of equilibrium are—

(1) *The weight of the liquid displaced must be equal to the weight of the solid;*

(2) *The centres of gravity of the solid and of the liquid displaced must lie in the same vertical line.*

Observation.—If the first condition is satisfied but not the second, the two forces acting on the body will constitute a *couple*, which will cause the body to roll over until it comes into a position of equilibrium.

124. **Equilibrium of a Submerged Body.** — If W denote the weight of a submerged body, w the weight of liquid it displaces, it readily follows from § 122 that

Case I.—If $W > w$, the body will *sink* unless it is held *up* by a string whose tension $= W - w$ (Fig 47, A).

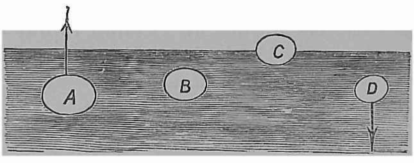

Fig. 47.

Case II. — If $W = w$, it will *rest in any position* (Fig. 47, B).

Case III.—If $W < w$, it will *rise till it floats*, and will then displace less liquid than before (Fig. 47, C), unless it is held *down* by a string whose tension $= w - W$ (Fig. 47, D).

Examples.—(1) A cube of gold (specific gravity 19·35), whose edges are 5 cm. in length, is suspended in mercury with 4 cm. of each of its sides submerged. To find the tension in the supporting string.

Volume of cube $= 5 \times 5 \times 5$ c.c. $= 125$ c.c.,

$\therefore$ weight of cube $= 125 \times 19{\cdot}35$ gm. $= 2418{\cdot}75$ gm.;

volume of mercury displaced $= 5 \times 5 \times 4$ c.c. $= 100$ c.c.;

weight of mercury displaced $= 100 \times 13{\cdot}6 = 1360$ gm.;

$\therefore$ required tension of string $= 2418{\cdot}75 - 1360$ gm. $= 1058\frac{3}{4}$ gm.

(2) If the tension be reduced to 1 kilog., to find how much the cube will sink.

Here the cube sinks until the weight of the *additional* liquid displaced equals the decrease of tension, or $58\frac{3}{4}$ gm.;

$\therefore$ the additional volume displaced $= 58{\cdot}75 \div 13{\cdot}6$ c.c. $= 4{\cdot}32$ c.c.

But the area of the base of the cube $= 25$ sq. cm.,

$\therefore$ increase in depth of immersion $= 4{\cdot}32 \div 25$ cm. $= {\cdot}1728$ cm.
$= 1{\cdot}728$ mm.

(3) To find the weight of a cylindrical cork (specific gravity ·24) which requires a weight of 13 gm. to sink half the length of its axis in water.

Let the volume of the cylinder $= 2v$ cub. cm.

Then the volume of the water displaced $= v$ cub. cm.;

$\therefore$ weight of water displaced $= v$ gm.,

and weight of cylinder $= 2v \times {\cdot}24$ gm. $= {\cdot}48v$ gm.;

therefore, from the equilibrium of the cylinder,

$${\cdot}48v + 13 = v\,;$$

$$\therefore\ {\cdot}52v = 13 \quad\text{or}\quad v = 25 \text{ cc.};$$

$$\therefore\ \text{weight of cork} = {\cdot}48v = 12 \text{ gm.}$$

*125. **The conditions of equilibrium of a solid in liquid suspended by a string** may be *completely* found as follows :—

Let W be the weight of the solid, G its centre of gravity, w the weight of the liquid displaced, H its centre of buoyancy, T the tension of the string, P its point of attachment.

Then the only three forces acting on the body are—

(i.) a downward force W at G,

(ii.) an upward force w at H, and

(iii.) an upward force T at P.

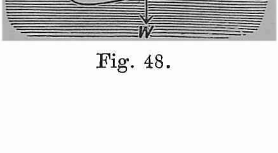

Fig. 48.

Hence (1) The three forces lie in one plane, or the points P, G, H and the string must lie in one vertical plane.

(2) If H, G meet the direction of the string in F, taking moments about F,
$$W \times GF = w \times HF.$$

(3) Lastly, resolving vertically, $T = W - w$, which determines the tension in the string.

126. **To find the conditions of equilibrium of a solid body floating in a series of liquids of different densities that do not mix.**—Let S be any solid floating partly immersed in several different liquids 1, 2, 3, ... bounded by the horizontal planes Aa, Bb, Cc. Then it is clear, as in the foregoing investigations, that the equi-

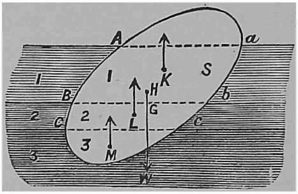

Fig. 49.

librium will be unaffected by removing the solid S and supposing the space $ABba$ filled with the liquid 1, the space $BCcb$ filled with the liquid 2, the space CDc filled with the liquid 3, and so on. The liquids that would fill these spaces are the *liquids displaced* by the solid, and the resultant upward thrust of the liquid on S is the resultant of the weights of the liquids displaced acting vertically through their respective centres of gravity. Hence, for equilibrium, *the weight of the solid must be equal to the sum of the weights of the different liquids displaced.*

The *centre of buoyancy* in this case is the centre of gravity of the whole series of liquids displaced. This point and the centre of gravity of the solid must be in the same vertical line.

The most interesting cases of equilibrium, however, are those in which the solid is symmetrical about a vertical axis—such as a prism, cylinder, or right cone having its axis vertical, a cube with one edge or one diagonal vertical, a sphere, &c. In such cases, the centres of gravity of the different liquids displaced and that of the solid all lie in the same vertical line, namely, the axis of symmetry, and the second condition of equilibrium is necessarily satisfied.

Examples.—(1) Water being poured on the top of mercury (specific gravity 13·6), to find the specific gravity of a body which floats with one-third of its volume above water, one-third immersed in the water, and the remaining third immersed in the mercury.

Here the volumes of the water and mercury displaced are equal, and their densities are as 1 : 13.6.

Let weight of water displaced $= w$.

Then weight of mercury displaced $= 13{\cdot}6w$.

But the solid is floating in equilibrium;

$\therefore$ the weight of the solid

$$= \text{weight of mercury displaced} + \text{weight of water displaced}$$
$$= 13{\cdot}6w + w = 14{\cdot}6w.$$

Again, one-third of the volume of the solid is immersed in water;

$\therefore$ volume of solid $= 3$ volumes of water displaced;

weight of an equal volume of water $= 3w$;

$\therefore$ required specific gravity of solid

$$= \frac{\text{weight of solid}}{\text{weight of equal volume of water}} = \frac{14{\cdot}6w}{3w} = \frac{14{\cdot}6}{3} = 4{\cdot}8\dot{6}.$$

(2) A cone whose specific gravity is 2·575 rests partly immersed in water and partly in mercury. To find what fraction (i.) of its volume, (ii.) of its axis, is immersed in mercury, taking the axis vertical and vertex downwards.

(i.) Let V be the volume of the cone, x that of the portion submerged in mercury.

Then the weights of the cone, the mercury displaced, and the water displaced, are proportional to $2{\cdot}575V$, $13{\cdot}6x$, $V - x$.

For equilibrium the former equals the sum of the two latter weights; $\therefore$ $2{\cdot}575V = 13{\cdot}6x + V - x$;

$\therefore$ $12{\cdot}6x = 1{\cdot}575V$ or $x = {\cdot}125V = \frac{1}{8}V$.

Therefore $\frac{1}{8}$ of the volume is immersed in mercury.

(ii.) This portion is a cone with the same vertical angle as the original cone. Now it is known that *the volumes of two such cones are proportional to the cubes of their heights.*

Therefore $\sqrt[3]{\frac{1}{8}}$ or $\frac{1}{2}$ of the axis is immersed in mercury.

127. **Effect of Immersed Solids on Pressure.**—If solids be lowered into a vessel containing liquid, the level of the liquid will rise owing to the displacement produced by the solids, and therefore there will be an increase of pressure all over the surface of the vessel.

Since the pressure at any point of a heavy liquid depends only on the depth and density, it follows that the pressure on the sides and bottom of the vessel is the same as if the solids were replaced by liquid equal in amount to that which they displace.

Illustrative Examples.—(1) Consider a bucket containing water and suspended by a rope. Now, let any body —say a brickbat—be lowered into the bucket by means of a second rope. The water will rise in the bucket; there will, therefore, be an increase in the pressure all over the bucket, and the tension in the first rope will be greater than before, since it has to support a greater resultant thrust.

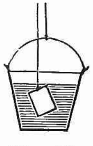

Fig. 50.

In this case, the tension in the rope supporting the bucket

= weight of bucket + weight of water actually contained in it
+ weight of water displaced by brickbat.

Also, we know that

tension in rope supporting brickbat

= weight of brickbat − weight of water displaced by brickbat;

$\therefore$ sum of tensions in the two ropes

= weight of bucket + actual weight of water + weight of brickbat;

as evidently should be the case, for the two ropes together have to support the bucket, the water, and the brickbat.

(2) If, instead, we place in the bucket a body lighter than water—say a block of wood—and allow it to float, it will displace a quantity of water of weight equal to its own weight. As before, we have

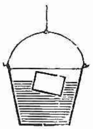

Fig. 51.

tension in supporting rope

= weight of bucket + weight of water actually contained in it
+ weight of water displaced by wood
= weight of bucket + weight of contained water
+ weight of wood;

as evidently should be the case, since the rope has to support the bucket, the water, and the wood.

*(3) Next consider a barge filled with coal, moving slowly along a canal crossing a bridge. The pressure of the water, and therefore the weight supported by the bridge, will be unaltered by its presence. Suppose now that some of the coal is thrown suddenly from the barge on to the towing-path; the barge, being lightened, will rise up out of the water; and, the displacement being reduced, the water will

fall in its immediate neighbourhood. The resultant thrust produced by the pressures on the sides of the canal will, therefore, be instantaneously reduced by an amount equal to the weight of the coal, so that the whole weight supported by the bridge is the same as before. But the lowering of the water level soon causes more water to flow into the canal from the ends of the bridge, and this goes on until the water has reached the same level as before. The bridge, then, has to support the same thrusts due to the water pressure as at the beginning, and has to support the weight of the coal in addition.

Thus the whole weight supported by the bridge increases *slowly*.

Summary.

1. *The resultant thrust of a liquid on any body*

(1) Is equal to the weight of liquid displaced;

(2) Acts vertically upwards through the C.G. of this liquid.

2. *The conditions of equilibrium of a floating body* are

(1) Weight of liquid displaced = weight of body;

(2) C.G. of body and C.G. of liquid in same vertical line.

3. A body tends to *sink*, *float*, or *rise* according as weight of body (W) $>$, $=$, or $<$ weight of liquid displaced (w).

4. *If the body is attached to a string*, the tension = $W - w$ *upwards* in first case and $w - W$ *downwards* in third.

5. *If a body floats partly immersed in each of several liquids*,

weight of body = sum of weights of liquids displaced.

(The other condition is rarely used.)

6. *The increase of pressure on the containing vessel* due to an immersed solid is the same as if the solid were replaced by the liquid it displaces.

EXAMPLES XIII.

1. State the conditions necessary for the equilibrium of a floating body, and discuss the effect of moving a heavy weight across the deck of a ship.

2. A body whose specific gravity is less than that of water is fastened to a string and drawn completely below the surface of the water in a vessel, the string being fastened to the bottom of the vessel. Find the tension of the string. Is there any alteration in the fluid pressure upon the base of the vessel?

3. A body whose specific gravity is greater than that of water is fastened to a string and lowered into a vessel containing water. If it be completely immersed, find the tension of the string. What effect has this on the pressure of the water upon the base of the vessel?

4. A piece of lead and a piece of sulphur are suspended by fine strings from the extremities of a balance beam, and just balance each other in water. Compare their volumes, their densities being respectively 11·4 and 2 gm. per cubic centimetre. Which of them will appear to be the lighter in air, and what weight must be added to it to restore equilibrium?

5. A piece of lead weighing 17 gms. and a piece of sulphur have equal apparent weights when suspended from the pans of a balance and immersed in water. When the water is replaced by alcohol of density 0·9, 1·4 gms. must be added to the pan from which the lead is suspended to restore equilibrium. Determine the weight of the sulphur, the density of lead being 11·333.

6. A block of wood (specific gravity ·75), whose volume is 250 cc., is totally immersed in a liquid of specific gravity 1·25 by means of a string attached to the bottom of the vessel containing the liquid. Find the tension of the string.

7. A block of wood, whose weight is 63 lbs. and whose specific gravity is ·6, is in a pond. If a ball of lead, whose specific gravity is 11·5, be attached to the block by a string, find the least weight which the ball can have so as to keep the block quite under water.

8. Two solids, whose weights are 4 and $6\frac{1}{2}$ lbs., the volume of the former being double that of the latter, are connected by a weightless string passing over a smooth pulley, and rest in equilibrium totally immersed in fluids of specific gravity 1·3 and 3·24 respectively. Find the volumes of the solids.

9. Two pieces of iron (specific gravity 7·7), suspended from the two scale-pans of a balance, the one in water and the other in alcohol of specific gravity 0·85, are found to weigh exactly alike. Find the proportion between their true weights.

10. The edge of a hollow cube of lead (specific gravity = 11·35) is 7 cm.; the thickness of the metal forming the cube is 1 cm. Find the apparent weight of the cube in water.

11. A cube of wood floating in water supports a weight of 480 oz. On the weight being removed, it rises 1 in. Find the size of the cube.

12. A cubical block of wood, specific gravity ·6, whose edge is 1 ft., floats, with two faces horizontal, down a fresh-water river and out to sea, where a fall of snow takes place, causing the block to sink to the same depth as in the river. If the specific gravity of sea-water be 1·025, show that the weight of snow on the block is 15 oz.

13. A square piece of wood of uniform thickness floats in water with two of its sides vertical and with seven-eighths of its surface area immersed. Find how deep it would sink when floating in a similar position in a fluid whose specific gravity is 1·25. Show that, if it float with one of its diagonals vertical in a mixture composed of equal volumes of the fluid and water, then one-third of that diagonal will be above the surface.

14. A wooden cone (specific gravity ·84), whose volume is 36 cub. ins., floats vertex downwards in a liquid, with its base horizontal and two-thirds of its axis immersed. What weight must be placed on the base in order that three-fourths of the axis may be immersed?

15. A right cone, whose weight is W, floats in a liquid, vertex downwards, with one-third of its axis immersed. What additional weight must be placed on the base of the cone so as just to sink it entirely in the liquid?

16. Show that, if the apparent weight of a body, suspended in a mixture by volume of two fluids which mix without contracting, be equal to the arithmetic mean between its apparent weights when suspended in the two fluids separately, the mixture contains equal volumes of the two fluids.

17. A piece of wood floats partly immersed in water, and oil is poured on the water until the wood is completely covered. Explain clearly whether this will make any change (if so, whether there will be an increase or decrease) in the portion of the wood below the surface of the water.

18. A mass composed partly of solid copper, specific gravity 8·8, and partly of solid lead, specific gravity 11·4, floats with two-thirds of its bulk immersed in mercury, specific gravity 13·6, and the

remaining one-third in water. Compare the volumes and weights of the copper and lead in the mass.

19. A lump of metal of specific gravity 4·05 floats partly in oil of specific gravity ·9 and partly in mercury of specific gravity 13·5. What portion of its volume is in each?

20. A block of oak, whose specific gravity is 1·2 and weight 6 lbs., is supported by a string, which cannot bear a strain of more than 1·5 lbs., in a large barrel partly filled with water, in which the block is wholly immersed. Fluid whose specific gravity is ·7 is now poured into the barrel so as to mix with the water, until it is filled. Show that the string will break if the barrel was originally less than two-thirds filled up with water.

21. A uniform rod 10 ins. long floats vertically with ·9 of its length immersed in a cylindrical vessel containing water. If alcohol, specific gravity ·8, be now poured on the water to the depth of 5 ins., show that the upper surfaces of the rod and alcohol will coincide.

22. A piece of wood of specific gravity ·8 floats partly in water and partly in a liquid lighter than water, and the part immersed in water is two-sevenths of the whole. What is the specific gravity of the other liquid?

23. A cube of bronze, whose edge is 10 cm. and specific gravity 8·5, floats in mercury, of specific gravity 13·6, with two faces horizontal. What length of the edge of the cube is immersed in the mercury?

24. If, in the last question, oil of specific gravity ·85 is now poured on the mercury until the cube is totally immersed in liquid, how far will the cube rise out of the mercury?

25. Water floats upon impure mercury whose specific gravity is 13, and a mass of platinum whose specific gravity is 21 is held suspended by a string so that $\frac{19}{24}$ of its volume is immersed in the mercury and the remainder of its volume in the water. Compare the tension of the string with the weight of the platinum.

26. A cylinder of wood floats in water with its axis vertical and having three-fourths of its length immersed. Oil whose weight is half that of water is then poured into the vessel to a sufficient depth to cover the wood. How much of the cylinder will now be immersed in water?

27. Water is poured without mixing upon glycerine whose specific gravity is 1·26, and a mass of cork whose specific gravity is 0·24 is held down by a string so that half its volume is immersed in the mercury and half in the water. Compare the tension of the string with the weight of the cork.

28. A body of uniform density floats in mercury whose specific gravity is 13·6 with one-eighth of its volume immersed. If water be poured upon the mercury so that the body is completely immersed, show that one-eighteenth part of the volume will be immersed in the mercury.

29. A piece of metal floats partly in oil of specific gravity ·9 and partly in mercury of specific gravity 13·6, and the volumes of the portions in oil and mercury are in the ratio of 3 to 4. Find its specific gravity.

30. A piece of wood of specific gravity ·84 floats partly in ether of specific gravity ·72 and partly in water. What portion of its volume is in each?

31. A vessel containing water is placed in one scale of a balance and counter-balanced by weights. A person dips his hand in without touching the sides of the vessel. Will the equilibrium be disturbed? Give your reasons.

32. A cube, each edge of which is 4 ins. long, weighs 16,244 grs. in air and 95 grs. in water. Find the weight of a cubic inch of water, having given that the specific gravity of water = 770 times the specific gravity of air.

33. Find the volume of a block of chalk (specific gravity 1·9) which weighs the same as a block of iron whose volume is 125 cub. cm. and specific gravity 7·6. What will be the volume of the chalk if the weight of the air displaced be taken into account? (Specific gravity of air = ·0013.)

34. A bucket half-full of water is suspended by a string which passes over a pulley small enough to let the other end fall into the bucket. To this end is tied a ball whose specific gravity s is greater than 2. Show that, if the ball do not touch the bottom of the bucket, and if no water overflow, equilibrium is possible if the weight of the ball lie between W and $(sW)/(s-2)$, where W is the weight of the bucket and water.

EXAMINATION PAPER VI.

1. Find the resultant vertical thrust on any area immersed in a heavy liquid.

2 A rectangular area, whose sides are 1 ft. and 2 ft., is immersed in water with its shorter side in the surface and its plane inclined at an angle of 30° to the horizon. Find the resultant vertical thrust on the area.

3. A solid hemisphere whose radius is 7 cm. is immersed in liquid of specific gravity 1·5, with its curved surface uppermost and its plane surface horizontal at a depth of 20 cm. Find the resultant vertical thrusts on both the plane and the curved surfaces.

4. A cylindrical vessel, the radius of whose base is $3\frac{1}{2}$ cm. and whose height is 16 cm., is filled with water and mercury (specific gravity = 13·6), the mercury occupying a depth of 4 cm. at the bottom of the vessel. Find the pressure per square centimetre on the base of the vessel, and the total thrust on the base.

5. In the preceding question, calculate the whole pressure and the average pressure of the contained liquid on the curved surface of the vessel.

6. Show that the resultant vertical thrust on a body wholly or partially immersed in a fluid is equal to the weight of the fluid displaced.

7. A body floats in water with half of its volume immersed. What proportion of its volume will be immersed when it is placed in sulphuric acid of density 1·8?

8. A certain body just floats in fresh water. On placing it in sea water of specific gravity 1·028, it requires the addition of 5·6 gm. to just immerse it. Find its volume.

9. A body floats with half of its volume immersed in water, and when placed in oil $\frac{9}{20}$ of its volume is immersed. What is the specific gravity of the oil?

10. A piece of cork (specific gravity = ·24), whose volume is 200 cub. cm., is kept totally immersed in water by means of a string attached to it and to the bottom of the vessel. Find the tension of the string.

PNEUMATICS.

PART III.

CHAPTER XIV.

ATMOSPHERIC AIR.—BAROMETERS.

128. **Pneumatics** is that portion of Hydrostatics which treats of gases.

Gases are distinguished from liquids by

(i.) Their **compressibility,** in virtue of which they can be compressed into any volume, however small (until they liquefy), by the application of sufficiently great pressure.

Gases can be compressed by the *condensing-pump* (which will be described in Chap. XVIII.).

(ii.) Their **elasticity,** in virtue of which they expand when the pressure is reduced, so as always to fill the whole volume, however large, of the containing vessel, and exert pressure on its sides.

Gases can be rarefied by the *air-pump* (Chap. XVIII.), until a nearly perfect *vacuum* or empty space is formed in any given vessel.

The pressure of a gas on the sides of its containing vessel is sometimes called its *elastic force*, but the term *pressure* is better.

Gases, being material substances, have weight, although their density is generally very small compared with that of most solids and liquids.

Thus, a cubic inch of water when boiled at ordinary pressure yields about a cubic foot of steam. But matter is indestructible ; hence the mass of the steam is equal to that of the water, and its density is therefore only about $\frac{1}{1728}$ of the density of water.

129. **Aristotle's Experiments.** — Aristotle (B.C. 384–322), wishing to test whether air had weight, experimented by weighing a bladder when empty, and again when inflated with air. He found that the weight was the same in both cases, and hence he was led to infer that air was without weight. This conclusion was universally accepted up to the seventeenth century, when it was disproved by the following experiment:—

130. To find the density of atmospheric air.—A large glass flask furnished with a tap or stopcock is taken; the air in it is completely exhausted by means of an air-pump, and the tap is closed. The flask is then weighed with a balance (Fig. 52). On opening the tap, air rushes into the flask and depresses the scale-pan carrying it; hence the flask is heavier than before. The difference of weight is found by again weighing, and is evidently equal to the weight of air which entered the flask; and, if the volume of the flask be determined, the density and specific gravity of the air may be found.

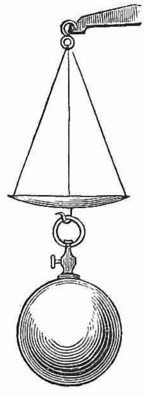
Fig. 52.

[Another method will be given in Chap. XV.]

Example.—A flask weighs 273·4 gm. when empty, 276·5001 gm. when filled with air, and 2805·1 gm. when filled with water. To find the weight of a litre of air.

Weight of air in flask = 276·5 − 273·4 = 3·1 gm.;
weight of equal volume of water = 2805·1 − 273·4 = 2531·7 gm.;
specific gravity of air = 3·1 ÷ 2531·7 = ·001224.
But a litre of water weighs 1000 gm.;
∴ weight of a litre of air = 1·224 gm.

The density of air is generally taken as about 1·3 oz. per cubic foot, so that the specific gravity, with water as the standard, is ·0013, and a litre of air weighs 1·3 gm.

But, as air is so readily compressible, its density depends on the pressure and temperature.

The specific gravities of other gases may be found in the same way. These are generally referred either to atmospheric air or to hydrogen gas as the standard substance instead of water. Hydrogen is the lightest gas known, its density being only about $\frac{1}{15}$ of that of air; hence it is convenient to take hydrogen as the standard.

131. **Effect of Buoyancy of Displaced Air.**—Since the flask is weighed in air, its apparent weight in each case is less than its true weight by the weight of air which the flask displaces. But this is *the same at both observations;* hence it does not affect the *difference* of the observed weights, which therefore still equals the weight of air inside the flask at the second observation.

Hence no allowance need be made for the buoyancy of the displaced air.

In *Aristotle's experiment*, on the other hand, the bladder expanded as it was filled, so that the more air he blew in the more air he displaced, and the buoyancy of this displaced air exactly balanced the increase of weight inside the bladder.

The buoyancy of the air displaced by a body is usefully applied in the balloon.

132. The Balloon is a large globular envelope of oiled silk or other air-proof material filled with hydrogen, coal gas, or some other gas lighter than air. Attached to it is a light car to hold the aeronauts.

The forces acting on the balloon are

(i.) The weight of the balloon and its contained gas acting downwards.

(ii.) The resultant thrust of the surrounding air which acts upwards and is equal to the weight of air displaced.

Now the gas inside the balloon weighs less than the air it displaces. Hence, if their difference is greater than the weight of the envelope and car, the balloon will ascend. By letting part of the gas escape through a valve, the volumes of the balloon and displaced air will decrease, until the balloon begins to descend.

Example.—A cubic foot of air weighs 1·29 oz., while a cubic foot of hydrogen only weighs ·09 oz. To find the volume of a hydrogen balloon which will just lift 250 lbs.

Since each cubic foot of hydrogen weighs ·09 oz. and displaces 1·29 oz. of air,

$\therefore$ 1 cub. ft. will lift 1·29 − ·09 oz., or 1·2 oz.;

∴ number of cubic feet required to lift 250 lbs.

$$= \frac{250 \times 16}{1 \cdot 2} = \frac{4000}{1 \cdot 2} = \frac{10,000}{3} = 3333\tfrac{1}{3};$$

i.e., required volume of the balloon $= 3333\tfrac{1}{3}$ cub. ft.

133. **Pressure of the Atmosphere.**—Since air has weight, the reasoning of Chap. X. shows that the atmosphere must exert a pressure on all surfaces with which it is in contact, and that with the usual density of air this pressure must increase by about 1·3 oz. per sq. ft. for every foot increase of depth.

The effects of atmospheric pressure may be illustrated by several simple experiments.

Thus, if we take a glass tumbler filled to the brim with water and lay a sheet of cardboard over the top, pressing it well down, it will be found that the glass may be inverted without the water falling out. The card is in fact held up by the thrust of the atmosphere upwards on its under side.

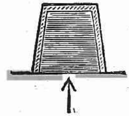

Fig. 53.

This upward thrust has to support the weight of the card and the thrust of the water on the upper side, besides pressing the card tightly against the rim of the glass.

Hence the *pressure* of the air (per square inch), acting upwards on the card, must exceed the pressure of the water downwards; otherwise the card would fall down. The atmospheric pressure is therefore greater than the pressure due to a column of water of the same height as the glass.

134. **The Magdeburg Hemispheres**, invented by Otto Guericke, of Magdeburg (1602–1686), are two hollow hemispheres whose edges fit truly when in contact. When the air is withdrawn by means of an air-pump from the spherical cavity thus formed, it will be found that the hemispheres cannot be pulled asunder except by application of considerable force. This force is required to overcome the resultant thrusts produced by the pressure of the atmosphere on the outer surfaces of the hemispheres.

135. Torricelli's Experiment.—The Barometer.—The first actual measurement of the pressure of the atmosphere is due to Torricelli (1643), and his experiment resulted in the invention of the mercurial barometer.

Fig. 54.

To perform the experiment or to construct a barometer in its simplest form, a glass tube about 33 ins. long and closed at one end is completely filled with mercury. The open end is then closed with the finger, the tube inverted into a cup of mercury, and the finger then removed, care being taken not to allow any air to get into the tube. The mercury will at once sink and leave a clear space at the top of the tube, and the height of the column of mercury above the surface in the cup will be found to be about **30 inches** or **760 millimetres.**

If the tube be furnished with a scale for reading off the height of the mercury, the apparatus constitutes a **mercurial barometer.**

The space above the mercury is practically a vacuum, and is called the **Torricellian vacuum.*** Hence there is no pressure at the top of the tube.

The atmospheric pressure at the surface of the mercury in the cup must therefore be equal to that due to the weight of the column of mercury in the tube. The height of this column is called the **height of the barometer,** or the **barometer reading.** Hence

The height of the barometer measures the pressure of the atmosphere.

Note.—If we perform Torricelli's experiment with a tube *shorter* than the column of mercury which the atmospheric pressure is capable of supporting, no vacuum will be formed.

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* Strictly, it contains a very minute quantity of the vapour of mercury; see Stewart's *Text-Book of Heat*, § 56.

Examples.—(1) If the height of the mercury be $29\frac{1}{2}$ ins., to find the atmospheric pressure in pounds per square inch, taking specific gravity of mercury = 13·6.

Weight of 1 cub. ft. of mercury = 13·6 × weight of 1 cub. ft. of water
= 13,600 oz.;

∴ pressure due to 1 ft. of mercury = 13,600 oz. per sq. ft.;

∴ ,, ,, $29\frac{1}{2}$ ins. ,, = 13,600 × 29·5/12 oz. per sq. ft.;

$$= \frac{13{,}600 \times 29\cdot5}{12 \times 16 \times 144} \text{ lbs. per sq. in.};$$

∴ pressure of atmosphere = 14·51 lbs. per square inch.

(2) If the height of the mercury be 750 mm., to find the pressure (i.) in statical, (ii.) in C.G.S. absolute, units.

(i.) The pressure due to 1 cm. of mercury = 13·6 gm. per sq. cm.;

∴ ,, ,, 75 ,, ,, = 1020 ,, ,,

i.e., required atmospheric pressure = 1020 C.G.S. statical units.

(ii.) Taking the acceleration due to gravity as 981 cm. per second per second, a gramme weighs 981 dynes;

∴ pressure of the atmosphere = 1020 × 981 dynes per square centimetre
= 1,000,620 dynes per square centimetre,

i.e., 1,000,620 C.G.S. *absolute* units.

136. **Water and Glycerine Barometers.**—Instead of performing Torricelli's experiment with mercury, we might use a column of water or any other liquid to measure the pressure of the atmosphere, provided that we took a sufficiently long tube for the purpose.

Examples.—(1) When the mercury stands at 30 ins., to find the height of the water barometer.

The density of mercury is 13·6 times that of water.

∴ pressure due to 1 ft. of mercury = pressure due to 13·6 ft. of water;

∴ ,, ,, $2\frac{1}{2}$,, ,, = ,, ,, 13·6 × $2\frac{1}{2}$,, ,,

∴ height of water barometer = 13·6 × $2\frac{1}{2}$ ft. = 34 ft.

Unless, therefore, the tube exceeded 34 ft. in height, no vacuum would be formed and the instrument would be useless.

(2) When the water barometer is at a height 34 ft., to find the height of a glycerine barometer, the specific gravity of glycerine being 1·26.

The pressure due to 1·26 ft. of water = pressure due to 1 ft. of glycerine;

∴ ,, ,, 34 ,, ,, = ,, ,, 34 ÷ 1·26 ,, ,,

∴ height of glycerine barometer = 34 ÷ 1·26 = 27 ft., nearly.

(3) To show that the heights to which liquids rise in the barometer tube are inversely proportional to their densities.

Let W, w be the specific weights of any two fluids, H, h the heights of the columns of the fluids which the atmospheric pressure is capable of supporting. Then we have

$$\text{pressure of atmosphere} = WH = wh\,;$$

$$\therefore \quad \frac{H}{h} = \frac{w}{W},$$

or

$$\frac{\text{height of first liquid}}{\text{height of second liquid}} = \frac{\text{density of second liquid}}{\text{density of first liquid}}.$$

137. The **water barometer** is much more sensitive to small changes of atmospheric pressure than a mercurial barometer.

For the column of water is *always* 13·6 times as high as the column of mercury. Thus the change of pressure which would cause the mercury to rise ·1 would cause the water to rise 13·6 times as much, or 1·36 ins.

The great objection to a water barometer is the difficulty of retaining a good vacuum at the top of the tube. Not only does water evaporate freely into the vacant space, but air gets absorbed at the surface of the cup, and is given off again at the surface of the column.

These objections are to a great extent obviated by the use of glycerine. Its specific gravity being 1·26, the **glycerine barometer** is more than ten times as sensitive as a mercurial barometer, and a much better vacuum is obtained than with water.

138. **The height of the barometer is independent of the shape and size of the tube.**

For, if h be the height of the barometer, *i.e.*, the vertical height of the surface of the mercury in the tube above its surface in the cup, and w the specific weight of mercury, then the atmospheric pressure P is given by the formula of § 95, $\quad P = wh\,;$

and it is shown in § 98 that this formula is independent of the shape and area of the tube; hence the height h depends only on the atmospheric pressure and the specific weight of the mercury.

139. *The effect of inclining the tube* will be that the mercury will rise to the same *vertical* height as before, and will therefore occupy a greater *length* of tube.

[An *inclined tube barometer* has been constructed on this principle; the upper part of the tube is inclined at a small angle to the horizon, and a small rise or fall in the vertical height therefore causes the mercury to move through a considerable length of tube.]

140. *To test if the barometer is true or faulty.*—It often happens that in an old barometer a little air has leaked into the space above the mercury, which is therefore no longer a true vacuum. This air may be detected by inclining the tube till the height of its upper end above the cup is less than the height of the barometer. If the barometer is perfect, the mercury will then fill the whole tube; if not, a bubble of air will remain.

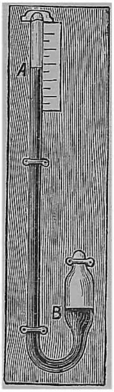

Fig. 55.

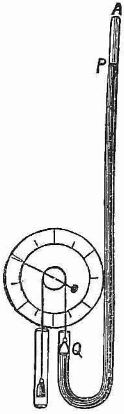

Fig. 56.

141. The Siphon Barometer consists of a **U**-tube which has branches of unequal length. The shorter branch is open to the atmosphere and corresponds to the cup of Torricelli's instrument, while the longer one is closed, and a vacuum is formed above the mercury at its upper end. When the mercury rises in one arm it falls in the other, and the height of the barometer is the difference of level of the mercury in the two branches. It is often read off on a graduated dial by means of the arrangement shown in Fig. 56.

Examples.—(1) If the sectional areas of the longer and shorter limbs are 1 sq. cm. and $1\frac{1}{2}$ sq. cm., and the height of the barometer in centimetres is read off on a scale attached to the upper limb, to find the distance between the graduations.

Let the mercury in the shorter or larger limb fall 1 cm.
Then $1\frac{1}{2}$ cub. cm. of mercury will flow into the longer limb ;

$\therefore$ the mercury in the longer limb will rise $1\frac{1}{2}$ cm. ;

.. the difference of level will increase $2\frac{1}{2}$ cm.,

i.e., barometer rises $\frac{5}{2}$ cm. when mercury in upper limb rises $\frac{3}{2}$ cm. ;

$\therefore$ barometer rises 1 cm. when mercury in upper limb rises $\frac{3}{5}$ cm.

Therefore the graduations must be $\frac{3}{5}$ cm. or 6 mm. apart to indicate centimetres of barometric height.

Similarly, if for a scale attached to the lower limb, the graduations must be 4 mm. apart, and must read *downwards*.

(2) If the sectional area of the tube of an *ordinary* barometer is $\frac{1}{4}$ sq. in., and it dips into a cistern of mercury whose superficial area is 5 sq. ins., to graduate the tube in inches of barometric height.

The area of the cistern outside the tube

$$= 5-\tfrac{1}{4} \text{ sq. in.} = \tfrac{19}{4} \text{ sq. in.}$$
$$= 19 \text{ times sectional area of tube.}$$

If, therefore, the mercury rises 1 in. in the tube, it will fall $\frac{1}{19}$ in. outside, and the change in barometer reading will be $\frac{20}{19}$ in.

Therefore 1 in. of barometer reading is measured on the scale by a length of $\frac{19}{20}$ in.

142. **The Aneroid Barometer** is a hollow metal box exhausted of air. The atmospheric pressure tends to force in the top of the box, but is resisted by the elasticity of the metal, which acts like a spring. When the pressure increases or decreases, the lid sinks or rises slightly, and moves a pointer which indicates the pressure on a dial. This dial is graduated in "inches" or "millimetres," corresponding to the readings of a mercurial barometer. The aneroid is chiefly used on account of its portability.

143. **The use of the barometer is to indicate the pressure of the atmosphere.** If the barometer rises, it indicates an increase in the atmospheric pressure, while a falling barometer indicates a decrease of pressure.

The reason why the barometer can be used to predict the weather is because experience has shown that certain changes of weather are generally accompanied by certain changes of atmospheric pressure.

Thus, when we say that "the barometer usually falls for rain," we mean that rainy weather is usually preceded by a decrease in the pressure of the atmosphere. Similarly, an improvement in the weather usually occurs simultaneously with an increase of pressure. The same changes are not indicated in the same manner in all parts of the globe. But in no case can changes of weather affect the barometer otherwise than by causing changes in the pressure of the atmosphere.

For further information on this subject, the reader is referred to treatises on meteorology.

144. **Precautions and Corrections.**—Barometers for scientific use are provided with a scale to read the lower level of the mercury in the cup or short branch, in addition to the scale on the tube. Both scales are read, the difference giving the actual height of the mercury. To this the following corrections are applied (*vide* §§ 145–147):—

(i.) Correction for capillarity.
(ii.) Correction for temperature.
(iii.) Correction for variations in intensity of gravity.
(iv.) Reduction to sea level.

145. "**Tapping the barometer.**"—When the mercury is rising or falling, a sudden jar or blow will often cause the reading to change considerably. This is due to the adhesion of the surface of the mercury to the sides of the barometer tube, which causes it to adapt itself with reluctance to changes of level. A smart blow loosens the mercury, which then at once moves to the level necessary to balance the atmospheric pressure.

This adhesion at the surface of liquids is called *capillarity*. Even when tapping has no further effect, the mercury surface will still assume a somewhat concave form, and there is therefore a further *correction for capillarity*. This correction depends on the area of the section of the tube at the surface of the mercury, being greater for small than large tubes; hence the height of the mercury is not *quite* independent of the bore of the tube at its upper end. The shape of the tube *below* the surface does not, however, affect the reading.

*146. *Corrections for Temperature and Intensity of Gravity.*

From §§ 92, 109, the absolute pressure due to a given column or head of mercury is proportional to the density of the mercury, which is affected by changes of temperature, and from § 107 this pressure is also proportional to "g," which varies slightly in different places. Hence, in order that the same barometer reading may always represent the same atmospheric pressure, it is necessary to apply corrections for temperature and intensity of gravity. The reading, when thus corrected, represents the height of the mercury column that the atmospheric pressure *would* support at a standard temperature (usually the freezing-point of water, 0° C. or 32° F.) at a standard place (usually taken at the sea level in latitude 45°). The pressure of the atmosphere is then said to be reduced to "standard inches (or millimetres) of pressure."

*147. *Reduction to sea level.*—We shall see in the next chapter that the pressure of the atmosphere depends on the altitude. In comparing the readings of the barometer for purely meteorological purposes, it is therefore necessary that the observations should all be made at the same altitude, and for this purpose the sea level is generally chosen. As this is impracticable at inland stations, the observations are corrected for altitude by being *reduced to the sea level.* They therefore represent what *would be* the corresponding reading of a barometer placed at the sea level under similar meteorological conditions.

148. **The average height of the mercurial barometer** is generally taken as **30 ins.** or **760 mm.** This corresponds to **34 ft.** height of the *water barometer*, or an *atmospheric pressure* of about **15 lbs. per sq. in.** This pressure is called **one atmosphere** (§ 70).

[N.B. It is useful to remember these numbers.]

149. **Effect of Atmospheric Pressure on Liquids.**—If the surface of a heavy liquid is exposed to atmospheric pressure, the pressure at any point will be the same as if the surface of the liquid were raised by an amount equal to the height of the barometer of that liquid and the pressure at the new surface were zero.

This theorem is obvious from the following examples :—

Examples.—(1) To find the pressure in water at a depth of 110 ft., the height of the water barometer being 34 ft.

Atmospheric pressure at surface = pressure due to 34 ft. of water
= 34,000 oz. per sq. ft.
Increase of pressure in 110 ft. = 110,000 oz. per sq. ft. ;
∴ total pressure required = 110,000 + 34,000 oz. per sq. ft.
= 144,000 oz. per sq. ft.
= 1,000 oz. or $62\frac{1}{2}$ lbs. per sq. in.

We thus see that the pressure is that due to a column of water of height 110 + 34 ft., and is therefore the same as if the depth of the water were increased by 34 ft., the height of the water barometer.

(2) To find the pressure at depth h in a heavy liquid exposed to atmospheric pressure, the height of a barometer of that liquid being H.

Let w be the specific weight of the liquid, p the atmospheric pressure. Then (by § 95)

pressure at depth $h = p + wh$.

But (by § 138) $p = wH$;

$\therefore$ required pressure $= wH + wh = w(H + h)$

pressure due to a column of height $H + h$.

NOTE.—The same result could be at once proved by supposing a barometer of the liquid to be constructed above its surface, the lower end of the tube dipping into it. The pressure at any point of the liquid would evidently be that due to the column $H + h$ extending from the surface of the "Torricellian" vacuum in the tube down to that point.

Effect on Resultant Thrusts due to Fluids.—When a vessel is filled with a heavy liquid, the atmospheric pressure acts on the outer surface of the vessel, besides being transmitted by the liquid to the inner surface. Hence the resultant thrusts on the base and sides of such a vessel are the same as if the atmospheric pressure did not exist, and are therefore found as in Chap. XII.

The pressure on the *inner surface* of the vessel is, however, increased by the atmospheric pressure.

SUMMARY.

1. *The density of air* is found by weighing a flask when exhausted and when filled with air.

2. *The pressure of the atmosphere* is measured by the height of the fluid column it supports in a *barometer*, the top of whose tube is a vacuum.

The fluid is usually mercury (specific gravity, 13·6), sometimes water or glycerine.

3. *The principal kinds of barometer* are—
   (i.) The common barometer, having cup of mercury;
   (ii.) The siphon or bent-tube barometer;
   (iii.) The aneroid barometer (not mercurial).

4. *To read the barometer accurately*, both upper and lower levels of the mercury are taken and the reading corrected for—
   (i.) Capillarity; (ii.) Temperature of mercury;
   (iii.) Intensity of gravity; (iv.) Reduction to sea level.

5. *The average height of the mercury barometer*
$= 30$ ins. $= 760$ mm.

the average height of the water barometer
$= 30$ ins. $\times 13·6 = 34$ ft.;

average pressure of atmosphere $= 34,000$ oz. per square foot
$= 15$ lbs. per square inch, roughly.

EXAMPLES XIV.

[Unless otherwise stated, the following data will be assumed :—height of mercurial barometer, 30 in., or 76 cm. ; height of water barometer, 34 ft. ; specific gravity of mercury, 13·6.]

1. A flask when empty weighs 120 gm., when full of air it weighs 121·3 gm., and when full of water, 1120 gm. Calculate the density of the air.

   Explain whether it is or is not necessary to take account of the weight of air displaced.

2. With the barometer at 760 mm., the mass of a litre of air is 1·2 gm., and of a litre of hydrogen ·089 gm. The material of a balloon weighs 50 kilog. : what must be its volume in order that it may just rise when filled with hydrogen? Explain carefully how you obtain your result.

3. If the atmospheric pressure is 15 lbs. per square inch and the diameters of a pair of Magdeburg Hemispheres are 7 ins., find the force required to pull them asunder.

4. Show that the thrust of the atmosphere on either of the Magdeburg Hemispheres is half the "whole pressure" on the hemisphere.

5. Describe an experiment to prove that the pressure of the atmosphere is measured by the height of a barometer column.

6. Explain the construction of a barometer, what it measures, and how it measures it.

   Need the bore of a barometer tube be uniform? Give reasons for your answer.

7. If the atmospheric pressure at the surface of the earth be $14\frac{1}{2}$ lbs. per square inch, find the height of the water barometer in feet.

8. Calculate the air pressures when the mercurial barometer stands at 27 ins. and at 30·5 ins., assuming that a cubic foot of water weighs 62·5 lbs.

9. If the height of the water barometer is 34 ft., and that of the mercurial barometer is 30 ins., show that the specific gravity of mercury is 13·6.

10. If the specific gravity of mercury be 13·5 and that of glycerine be 1·255, what reading of a mercurial barometer corresponds to a reading of 320 ins. on a glycerine barometer?

11. Translate pressure measured in terms of the height of a barometer mercury column (say, either 27 ins. or 60 cm.) into absolute (*i.e.*, dynamical) units of pressure.

12. Find the absolute pressure on a bottle of air immersed in sea water to a depth of 50 metres, the density of sea water being 1·027, and the value of g being 980 (cm./sec.^{2}), and compare this pressure with that of the atmosphere, the barometer standing at 76 cm.

13. If the diameters of the two branches of a siphon barometer are equal, show how to graduate (i.) the upper, (ii.) the lower, branch to indicate inches of barometric height.

14. The section of the closed limb of a siphon barometer is to that of the open limb as 3 to 17. The mercury rises 1·275 ins. in the closed branch. What change takes place in the mercury of an ordinary barometer?

15. A siphon barometer is so constructed that the long closed tube has an internal sectional area equal to $\frac{1}{4}$ sq. in., while the short open tube has an internal sectional area equal to $\frac{1}{2}$ sq. in. Find what fall will take place in the long tube of this barometer when the true pressure of the air falls 1 in.

16. What would be the height of a column of air of uniform density 1·2 oz. per cubic foot which would produce a pressure equal to that of the atmosphere?

17. A body floats in water contained in a vessel placed under an exhausted receiver with half its volume immersed. Air is then forced into the receiver till its density is 80 times that of air at atmospheric pressure. Show that the volume immersed in water will then be four-ninths of the whole volume, assuming the specific gravity of air at atmospheric pressure to be ·00125.

CHAPTER XV.

BOYLE'S LAW.

150. From the fundamental properties by which gases are defined it appears that the volume of a given quantity of gas becomes less as the pressure to which it is subjected becomes greater, and *vice versâ*. The relation which exists between the volume and pressure of a gas at any given temperature was discovered by Robert Boyle, of Lismore, Ireland (1662), and by Mariotte, in France (1679). This relation, which is known in England as Boyle's Law, and in France as Mariotte's Law, is usually stated thus :

151. BOYLE'S LAW. — The volume of a given mass of gas is inversely proportional to the pressure when the temperature is kept constant.

Thus, let p be the pressure of a gas occupying the volume v ;

Then at pressure $2p$ the volume of the gas will be $\frac{1}{2}v$,

,, ,, $3p$,, ,, ,, $\frac{1}{3}v$,

,, ,, $\frac{1}{2}p$,, ,, ,, $2v$,

,, ,, p/n ,, ,, ,, nv,

and so on. Since

$$pv = 2p \times \tfrac{1}{2}v = 3p \times \tfrac{1}{3}v = \tfrac{1}{2}p \times 2v = p/n \times nv,$$

it follows that

The product of the pressure into the volume of a given mass of gas at constant temperature is constant.

For let v be the volume of the gas when the pressure is p, V its volume when the pressure is P, the temperature being the same in both cases. Boyle's Law states that P, p are inversely proportional to V, v, that is,

$$\frac{P}{p} = \frac{\frac{1}{V}}{\frac{1}{v}} = \frac{v}{V},$$

or, clearing of fractions, we may state Boyle's Law in symbols thus,

$$PV = pv \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{1}).$$

The following alternative statement of Boyle's Law is important:

152. The pressure in a given kind of gas at a given temperature is proportional to its density.

For, if m be the mass of the gas, d, D its densities when its volumes are v, V, then, by § 14,

$$D = \frac{m}{V}, \quad d = \frac{m}{v}$$

whence, by Boyle's Law,

$$\frac{P}{p} = \frac{\frac{1}{V}}{\frac{1}{v}} = \frac{\frac{m}{V}}{\frac{m}{v}} = \frac{D}{d},$$

or

$$P : p = D : d \quad \ldots\ldots\ldots\ldots\ldots \quad (\mathbf{2}).$$

NOTE.—*This last relation is true whether the mass of gas is the same or different in the two cases, provided that it is the same kind of gas at the same temperature.*

Examples.—(1) A mass of air at atmospheric pressure occupies 44 cub. ins. To find the pressure when the volume is reduced to 24 cub. ins., taking an atmosphere as 15 lbs. per square inch.

Let p be the required pressure in pounds per square inch. Then, by Boyle's Law, $p \times 24 = 44 \times 15$,

$$\therefore \quad p = \frac{44 \times 15}{24} = \frac{11 \times 5}{2} = 27\tfrac{1}{2} \text{ lbs. per square inch.}$$

(2) To compare the weights of a cubic foot of air when the barometer stands at 29 and 30 ins.

Since the densities are proportional to the pressures, and these are proportional to the heights of the barometer, the required weights are in the proportion of 29 : 30.

(3) A vessel containing 2 litres of air at a pressure of $\frac{1}{2}$ atmosphere is put into communication with another vessel containing 3 litres of air at a pressure of 3 atmospheres. To find the subsequent pressure of the air in the two vessels.

If the pressure in each mass of air were changed to 1 atmosphere, the air in the first vessel would occupy $2 \times \frac{1}{2}$ litres = 1 litre, and that in the second 3×3 litres $= 9$ litres. The total mass of air would therefore occupy 10 litres at atmospheric pressure. But it has to occupy $2 + 3$ litres $= 5$ litres (the sum of the volumes of the vessels). Since the volume is thus halved; the pressure is doubled, therefore the required pressure is 2 atmospheres.

(4) A bubble of air rises from the bottom of a lake, and its diameter has doubled when it reaches the surface. To find the depth of the lake.

The volume of a sphere is proportional to the cube of its diameter [$\because$ vol. $= \frac{4}{3}\pi r^3 = \frac{1}{6}\pi$ (diam.)^3].

$\therefore$ vol. at surface $= 8$ times vol. at bottom.

Therefore, by Boyle's Law,

pressure at surface $= \frac{1}{8}$ pressure at bottom,

. pressure at bottom $= 8$ atmospheres.

Now, taking the height of the water barometer as 34 ft., the pressure increases 1 atmosphere for every 34 ft. descended. But the difference of pressure at the top and bottom is $8-1$, or 7 atmospheres.

$\therefore$ required depth of lake $= 34 \times 7 = 238$ ft.

153. To verify Boyle's Law experimentally for pressures greater than that of the atmosphere.—A piece of apparatus called **Boyle's Tube** is generally used (Fig. 57). This is a **U**-tube with very unequal branches, the longer arm being sometimes as much as 6 ft. long. A scale of inches or millimetres is attached to each branch. A little mercury is poured into the bend until it reaches the point marked zero on the two scales. The air in either branch being at atmospheric pressure, the shorter branch is now closed with a tightly fitting screw cap, the length of the enclosed column of air being measured on the scale. More mercury is then poured into the long branch, and, as its level rises, the increase of pressure diminishes the volume of the air in the closed branch. By measuring the length of the column of enclosed air and the difference of level of the mercury in the two branches, the pressure and volume of this air can be found, and, by making a

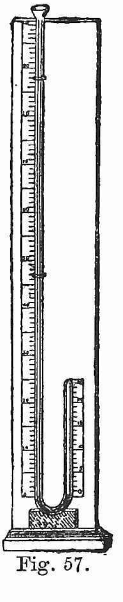

Fig. 57.

number of such experiments, the relation connecting them may be verified.

Thus, suppose the mercury poured in until the length of the air column AP (Fig. 58) is half what it was when the end A was closed. Then $O'Q - OP$, the difference of height of the mercury in the two branches, will be observed to be equal to the height of the mercurial barometer (about 30 ins.).

Now the pressure due to this column of mercury is equal to 1 atmosphere. But the surface Q is at atmospheric pressure. Hence the total pressure of the air at P is 2 atmospheres, or double what it was originally.

Hence, if the volume of the air be halved, the pressure is doubled.

Again, when the air in the short branch occupies one-third of its original volume, $O'Q - OP$, the difference of level will be observed to be *twice* the height of the mercurial barometer. This column produces a pressure of 2 atmospheres, and the surface Q is at atmospheric pressure; hence the total pressure at P is 3 atmospheres.

Hence, if the volume be reduced to one-third, the pressure is trebled, and so on.

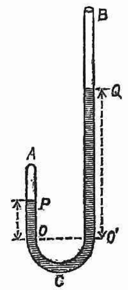

Fig. 58.

Thus Boyle's Law is verified.

Example.—The shorter branch of Boyle's tube is closed when it contains a column of air 10 cm. long. To find how much mercury must be poured into the longer branch to raise the level in the shorter branch by 2 cm., the height of the barometer being 76 cm.

The length of the air column is reduced from 10 to 8 cm. Hence its new volume is $\frac{4}{5}$ of its original volume.

$\therefore$ the pressure is $\frac{5}{4}$ atmosphere.

. . the difference of level in the two branches corresponds to a pressure of $\frac{1}{4}$ atmosphere, and is therefore $76 \times \frac{1}{4}$ cm. $=$ 19 cm.

But the level in the shorter branch has risen 2 cm.

$\therefore$ the level in the longer branch must have risen $19 + 2$ cm. $=$ 21 cm.

. . the quantity of mercury poured in must be sufficient to fill $21 + 2$ cm. or 23 cm.

154. To verify Boyle's Law for pressures less than that of the atmosphere.—The most convenient apparatus consists of a glass tube rather over 30 ins. long furnished with a screw-cap A (Fig. 59) and a cylindrical jar of the same height filled with mercury. The tube is lowered into the jar of mercury, leaving a length AO projecting, and the upper end A is closed with the cap, the air thus enclosed in the tube being at atmospheric pressure. The length AO is measured on a scale of inches or millimetres which may conveniently be engraved on the tube. On the tube being raised, the mercury rises above the outside level, as at P, but the reduction of pressure causes the air column to expand from AO to AP and occupy a greater length of tube than before. By measuring the heights AP, PQ, the volume and pressure of the enclosed air can be found and the relation connecting them verified.

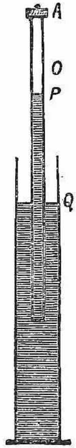

Fig. 59.

Thus, when the air column AP occupies double its original length AO, the height of the mercury column PQ is observed to be $\frac{1}{2}H$, where H is the height of the mercurial barometer. Hence the difference of the pressures at P, Q is $\frac{1}{2}$ an atmosphere, and therefore the pressure at P is $1-\frac{1}{2}$ atmosphere, or $\frac{1}{2}$ an atmosphere, or half the original pressure of the air.

Hence, if the volume be doubled, the pressure is halved, in accordance with Boyle's Law.

When the air column AP occupies three times its original length, *Boyle's Law requires* that the pressure at $P = \frac{1}{3}$ atmosphere, whence the difference of pressures at P, $Q = \frac{2}{3}$ atmosphere and therefore

$$PQ = \tfrac{2}{3}H.$$

In actual experiments the height PQ is then observed to be exactly $\frac{2}{3}H$, thus confirming Boyle's Law.

Similarly, when $AP = 4AO$, *Boyle's Law requires* that $PQ = \frac{3}{4}H$, and this also is confirmed by observation.

In like manner, Boyle's Law may be verified for *any* pressures less than that of the atmosphere.

155. To verify Boyle's Law for *any* gas it is only necessary to substitute that gas for air in the closed end of the **U**-tube of § 153, or in the barometer tube of § 154.

***156. Effect of Temperature.—Charles' Law.**—In the above experiments, *care must be taken to keep the temperature of the gas constant.* For this reason it is sometimes a good plan to keep the **U**-tube or the mercury jar immersed in a large vessel of water which is constantly stirred so as to maintain a uniform temperature.

A rise of temperature will cause a gas to expand even if its pressure is unaltered. The relation established by experiment between the volume and temperature at constant pressure is known as **Charles' Law,** and may be conveniently stated thus—

When the pressure of a gas is kept constant, the volume is proportional to 273+*t*, where *t* is the Centigrade temperature.

Combining this with Boyle's Law, it may be shown that, if the pressure, volume, and Centigrade temperature of a gas change from p, v, t to P, V, T, then

$$\frac{pv}{273+t}=\frac{PV}{273+T} \quad \text{or} \quad \frac{pv}{PV}=\frac{273+t}{273+T}.$$

The quantity $273+t$ is called the *absolute temperature*; the temperature -273° C. being called the *absolute zero.**

Hence the product of the pressure and volume of a given mass of gas is proportional to the absolute temperature.

***157. Limits of Boyle's Law.**—From experiments such as those described above, Boyle's Law may be proved to be a *very approximate* statement of the relation between the pressure and the volume of air or other gases when the pressure is not very great. But more accurate observations show that in no gas is the pressure *exactly* proportional to the density.

The divergence from Boyle's Law in most gases is too small to be of any practical importance, and is therefore commonly neglected *except* when the pressure approaches the amount required to cause liquefaction. When, however, air is *saturated* with the vapour of water, an increase of pressure produces condensation, and Boyle's Law no longer holds good.

A *perfect gas* is defined as an ideal substance which always obeys Boyle's Law. Like a perfect fluid, no such substance really exists.

In numerical calculations and problems, it is always assumed that gases obey Boyle's Law and that their temperatures remain constant unless the contrary is specified.

* For further details, see Stewart's *Text-Book of Heat*, Chap. V.

158. Application of Boyle's Law to the faulty Barometer. — When a little air has got into the end of a barometer tube it expands and depresses the mercury. As the mercury rises and falls this air obeys Boyle's Law, and hence the relation between the readings and those of a perfect barometer may be found, as in the following example, which may be taken as typical.

Example.—A faulty barometer reads 28 ins. and 30 ins. when a true barometer reads $28\frac{1}{2}$ ins. and 31 ins. respectively. To find (i.) the whole length of the tube of the faulty barometer, (ii.) the true reading when the faulty barometer stands at 29 ins.

(i.) Let l be the length of the tube. Then at the first observation, the air in the upper end occupies a space $l-28$ ins., and is under a pressure $28\frac{1}{2}-28$ ins. of mercury (the difference of height of the two barometers). At the second the air occupies $l-30$ ins., under pressure $31-30$ ins. Therefore, by Boyle's Law ($PV=pv$),

$$(l-28)\times\tfrac{1}{2}=(l-30)\times 1,\ \text{ whence } l=32 \text{ ins.}$$

(ii.) Hence at the first observation, the air occupied $32-28$ ins. at a pressure of $\frac{1}{2}$ in. When the faulty barometer reads 29 ins., this air occupies $32-29$ ins.; therefore, if p be its pressure in inches, Boyle's Law gives

$$p\times 3=\tfrac{1}{2}\times 4,\quad \text{whence } p=\tfrac{2}{3}.$$

Therefore the true barometer reading is $\frac{2}{3}$ in. higher, or $29\frac{2}{3}$ ins.

159. Determination of Heights by the Barometer.— Since the pressure of the atmosphere is due to the weight of the superincumbent air, it increases with the depth, as is evident from Chap. X. When therefore we ascend a few hundred feet, the weight of the column of air traversed makes a perceptible difference in the pressure and the barometer reading is perceptibly lower at the top than at the bottom of the column. If, then, we know the difference of atmospheric pressure at the top and bottom of a mountain, and also the density of the air, we can find the height of the mountain.

Conversely, to find the density of atmospheric air, it is only necessary to observe the difference of pressure at the top and bottom of a tower or hill whose height is known. [This is an alternative to the method of § 130.]

Example.—To find the height of a hill if the barometer readings at the top and bottom differ by $\frac{1}{10}$ in., the densities of air and mercury being ·001 and 13·6.

Here density of mercury $= 13,600 \times$ density of air.

Hence difference of pressure due to ·1 in. of mercury = pressure due to a column of air of $13,600 \times \cdot 1$ ins.

$\therefore$ height of hill = 1360 ins. = 113 ft.

160. Since, by Boyle's Law, the density of air is proportional to its pressure, this density decreases as we ascend, and hence the reduction of pressure is not strictly proportional to the height risen unless this height be small.

Thus, when we ascend 1000 ft., the pressure, and therefore the density, of the air decreases. The density of the second 1000 ft. is therefore less than that of the first; therefore the reduction of pressure in the second 1000 ft. is also less than in the first 1000 ft. Hence, if a barometer is carried up 2000 ft., the mercury falls *less* than twice as much as in the first 1000 ft. Similarly for each rise of 100 ft., the fall of the barometer is rather less than for the preceding 100 ft.

[Since the density of the air depends also on the *temperature*, this also must be observed in determining heights by the barometer; unless the temperature of the air column is the same throughout, the calculation is one of considerable difficulty.

*161. **Specific Gravities of Gases.**—The specific gravity of a gas is independent of the pressure, provided that the standard substance is another gas *at the same pressure*. Thus, if a cubic foot of air and a cubic foot of hydrogen at the same pressure are weighed, and if the pressure is then doubled, the volumes of air and hydrogen will each be $\frac{1}{2}$ cub. ft., and will therefore be equal. Hence the ratio of the weights of equal volumes of two gases is independent of the pressure provided that this is the same for both gases.

[By Charles' Law, the ratio of the weights of equal volumes of two gases at the same *temperature* and pressure may be shown to be independent also of the *temperature*.]

SUMMARY.

1. *Boyle's Law (or Mariotte's Law).*—The volume of a given mass of a given gas at constant temperature is inversely proportional to the pressure.

Hence the density of a given gas at constant temperature is proportional to the pressure.

Taking the kind of gas and the temperature to be the same in the two cases considered, we have

$$PV = pv \,\ldots\ldots\ldots\, (1) \quad \text{and} \quad P/p = D/d \,\ldots\ldots\ldots\, (2),$$

where the *mass* of gas must be kept the same in (1), but not necessarily in (2).

[* The relations are also written thus (where $\propto$ means "*varies as*"):—

$$pv = \text{const.}, \quad p \propto \frac{1}{v}, \quad p = d \times \text{const.}, \quad p \propto d.$$

2. If the temperature varies, *Charles' Law* asserts that v is proportional to $273+t$, with p constant, and therefore generally

$$pv/PV = (273+t)/(273+T) \quad \text{or} \quad pv \propto 273+t,$$

where t, T are Centigrade temperatures,
$273+t$, $273+T$ are *absolute* temperatures.]

3. *For pressures greater than an atmosphere*, Boyle's Law is verified by means of *Boyle's Tube* of § 153 (**U**-tube with air or gas in shorter closed branch).

4. *For pressures less than an atmosphere*, a barometer tube and jar of mercury are used (see § 154).

5. *The air in a faulty barometer* obeys Boyle's Law.

6. *Heights may be measured by the barometer* if the densities of the intervening strata of air be known.

EXAMPLES XV.

[The data given on page 149 are assumed.]

1. A wide-mouthed bottle full of air is closed with a well-ground glass stopper, 5 cm. in diameter, when the barometer stands at 772 mm. What weight must the stopper have in order that it may be just blown out if the barometer goes down to 730, the temperature remaining the same?

2. In a tube of uniform bore a quantity of air is enclosed. What will be the length of this column of air under a pressure of 3 atmospheres, and what under a pressure of $\frac{1}{3}$ atmosphere, its length under the pressure of a single atmosphere being 12 ins.?

3. If a vessel of 3 cub. ft. capacity, containing air at a pressure of 2 atmospheres, is put into communication with a vessel of

18 cub. ft. capacity, containing air at a pressure of $\frac{1}{4}$ atmosphere, what will be the pressure of the air in the two vessels? State the principle or law on which the solution of this question depends.

4. What do you know about the density of gases in relation to temperature and pressure? Describe experiments which show that the density of a gas at constant temperature is proportional to its pressure.

5. A Mariotte's tube has a uniform section of 1 sq. in., and is graduated in inches; 6 cub. ins. are enclosed in the shorter (closed) limb, when the mercury is at the same level in both tubes. What volume of mercury must be poured into the longer limb in order to compress the air into 2 ins.?

6. Mercury is poured into a uniform bent tube, open at both ends, and having its two branches vertical. One end is closed, its height above the mercury being 4 ins. How much mercury must be poured into the open end so that the mercury may rise 1 in. in the closed branch?

7. The height of the column of mercury in the open branch of an eudiometer is 12 ins. above that of the column in the closed branch, and the air in the closed branch occupies a length of 4 ins. How much mercury must be poured into the open branch in order to compress the air to half its volume?

8. A uniform tube closed at top, open at bottom, is plunged into mercury, so that it contains 25 c.c. of gas at atmospheric pressure of 76 cm.; the tube is now raised until the gas occupies 50 c.c. How much has it been raised?

9. A straight uniform tube closed at one end, whose length is $2h$, has the open end just immersed in a basin of mercury. If the tube contain a quantity of air which under atmospheric pressure would occupy a length of the tube equal to $\frac{4}{9}h$, show that the mercury will rise in the tube to a height equal to $\frac{2}{3}h$, h being the height of the mercurial barometer at the time of the experiment.

10. A cylindrical vessel, closed at one end only, is 20 cm. tall, and its open end is immersed in mercury until the interior level is 5 cm. below that of the general level of the liquid outside. The barometric height being 75 cm., calculate how far the mercury has risen into the vessel, or how deep the lip of the vessel has been submerged.

11. A bubble of air whose volume is ·004 c.c. is dislodged from the bottom of a lake 51 ft. deep, and rises to the surface. What is its volume when it reaches the surface?

12. A bubble of air whose volume is ·0004 cub. in. is formed at the bottom of a pond 17 ft. deep. What will be its volume when it reaches the surface?

13. A bubble of air $\frac{1}{10}$ in. in diameter starts from the bottom of the Atlantic at a depth of 2 miles. Find its size on reaching the surface.

14. A bubble of air whose volume is ·0028 cub. in. is formed at the bottom of a pond; on reaching the surface its volume is ·004 cub. in. What is the depth of the pond?

15. Why does a small weight of air introduced into the upper part of the tube depress the mercury considerably, whereas a small piece of iron floating on the mercury does not depress it?

16. A mercury barometer, whose cross-section is 1 sq. cm., stands at 76 cm., and the length of the vacuum above the column of mercury is 3 cm. How much air at ordinary atmospheric pressure must be introduced into the tube in order to depress the mercury 16 cm.?

17. The readings of a true barometer and of a barometer which contains a small quantity of air in the upper portion of the tube are respectively 30 and 28 ins. When both barometers are placed under the receiver of an air-pump from which the air is partially exhausted, the readings are observed to be 15 and 14·6 ins. respectively. Show that the length of the tube of the faulty barometer, measured from the surface of the mercury in the basin, is 31·35 ins.

18. When the reading of the true barometer is 30 ins., the reading of a barometer the tube of which contains a small quantity of air, and whose height above the surface of the mercury in which it is immersed is $31\frac{1}{3}$ ins., is 28 ins. If the reading of the true barometer fall to 29 ins., show that the reading of the faulty barometer will be $27\frac{1}{3}$ ins.

19. A barometer reads 30 ins. at the base of a tower, and 29·8 ins. at the top, 180 ft. above. Find the average mass of a cubic foot of air in the tower, taking the specific gravity of mercury as 13·5, and the mass of a cubic foot of water as 62·4 lbs.

20. Find the height between two stations, having given the following data :—

Density of mercury, 13·6 gm. per cubic centimetre ;

Mean density of air between the two stations, ·00121 gm. per cubic centimentre ;

Height of barometer at lower station, 785 mm. ;

Height of barometer at upper station, 630 mm.

21. A balloon is filled with a gas whose specific gravity is one-tenth of that of air at the pressure of 760 mm. of mercury at 0° C. Compare the lifting power of the balloon in air when the height of the barometer is 750 mm. with its lifting power when the barometer stands at 760 mm. The temperature in both cases is 0° C., and the volume of the balloon is supposed to remain unaltered.

22. If a body be suspended by an elastic string, explain how the length of the string will be affected by a rise in the barometer.

23. A cube floats in distilled water under the pressure of the atmosphere with four-fifths of its volume immersed and with two of its faces horizontal. When it is placed under a condenser where the pressure is that of 10 atmospheres, find the alteration in the depth of immersion (the specific gravity of air at the atmospheric pressure being ·0013).

*24. State the law connecting the pressure, volume, and absolute temperature of a gas.

A mass of air under a given pressure occupies 24 cub. ins. at the temperature of 39° C. If the pressure be diminished in the ratio of 3 : 4, and the temperature raised to 78° C., show that the volume of the air will be 36 cub. ins.

*25. A mass of air under given pressure occupies 44 cub. ins. at a temperature of 13° C. If the volume of the air be reduced to 24 cub. ins., and the temperature raised to 39° C., show that the pressure will be doubled.

26. A retort of 3 litres capacity, and with its open end submerged 3·4 cm. below the surface of water in a trough, is seen to be completely full of air on a certain day. Next day the mercury barometer is observed to have fallen from 76 to 74 cm., without any change of temperature. How much of the air originally in the retort has by that time bubbled out?

EXAMINATION PAPER VII.

1. Describe the mercurial barometer, and show that it measures accurately the pressure of the atmosphere.

2. A bottle when full of air weighs 3·544 gm., when full of water it weighs 103·425 gm., and when full of alcohol (specific gravity = ·835) it weighs 86·925 gm. Calculate the specific gravity of air.

3. A barometer reads 761 mm. at the base of a tower, and 754 mm. at the top, 75 metres above. Find the average mass of a cubic metre of air in the tower, taking the specific gravity of mercury as 13·5.

4. Enunciate Boyle's Law.

5. If the specific gravity of air is ·001, calculate the weight of the air that escapes from a room 20 ft. long, 25 ft. wide, and 10 ft. high, on the barometer falling from 31 ins. to 30 ins.

6. 100 cub. ins. of air, at a pressure of 15 lbs. to the square inch, are pumped into a chamber already containing 50 cub. ins. of air at a pressure of 10 lbs. to the square inch. What is the pressure of the mixture?

7. Explain the use and action of the vent-peg.

8. A bubble of air, $\frac{1}{20}$ cub. in., rises from the bottom of a lake at a point where it is 200 ft. deep. On reaching the surface, its volume is ·35147 cub. in. Find the specific gravity of the water of the lake.

9. A mercurial barometer 34 ins. long stands at 30 ins.; $\frac{1}{10}$ cub. in. of external air is introduced, and the mercury drops 4 ins. What is the sectional area of the barometer?

10. In a siphon barometer the sectional area of the open end is $3\frac{1}{3}$ times that of the closed end. A fall of $\frac{1}{2}$ in. takes place in it. What fall occurs at the same time in an ordinary barometer?

CHAPTER XVI.

SIMPLER PNEUMATIC APPLIANCES.

We shall now describe certain simple apparatus depending on the principles proved in the foregoing chapters, leaving more complicated contrivances, such as pumps, to be treated in the two following chapters.

162. **Hare's Hydrometer** is a kind of inverted **U** tube for comparing the specific gravities of two liquids. The lower ends of the two branches are immersed in the liquids, and part of the air is drawn out of the upper part of the tube by means of an air pump or otherwise. The atmospheric pressure outside the tubes causes the liquids to rise to heights which are inversely proportional to their densities.

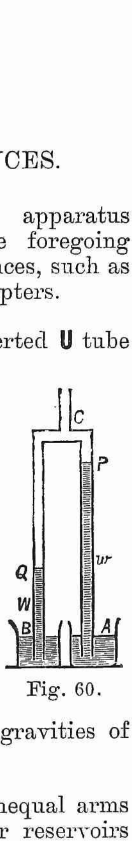

Fig. 60.

For, if w, W be the specific weights of the liquid columns AP, BQ, we have

$$\text{pressure of atmosphere} - \text{pressure in tube } PCQ = w \, . \, AP = W \, . \, BQ\,;$$

$$\therefore \quad AP : BQ = W : w.$$

Hence by measuring AP, BQ the specific gravities of the liquids may be compared.

163. The Siphon is a bent tube with unequal arms used for drawing off liquid from vessels or reservoirs which have no outlet at the bottom.

To explain its action, suppose that the siphon has been filled with liquid, both ends A, D having been temporarily closed with plugs, and that the shorter arm has been lowered into a vessel of the same liquid as in Fig. 61. Now let the end A be opened, the end D being still closed.

Then, if the height PB is less than the height to which the liquid would ascend in a barometer, the pressure of the atmosphere on the surface P will prevent a vacuum from forming in the tube, which will therefore remain filled with liquid.

And if Q be taken on the longer arm on the level of the surface at P, then (by connecting Q with P by a zigzag or horizontal and vertical lines) we may show that the pressure at Q is equal to that at P, *i.e.*, to the atmospheric pressure. The pressure inside the tube at D is therefore greater than outside by the amount due to the column QD, and this excess of pressure tends to force the plug out.

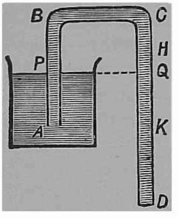

Fig. 61.

If, therefore, the plug be removed, the liquid will flow out at D. And, since no vacuum is formed in the tube, the pressure of the atmosphere at P will cause fresh liquid to rise in the tube at A, thus producing a continuous stream.

Example.—To examine the effect of making holes in the siphon at different points.

When the end D is closed, the pressure in the tube at any point H in the part $PBCQ$ *above* the horizontal line PQ is less than the atmospheric pressure. Hence, if a hole be made at H, air will enter and fill the bend and will stop the working of the siphon.

If, however, a hole be made at K *below* Q, the remaining portion above K will still form a siphon through which liquid will continue to flow, just as it would do if the portion DK were altogether removed.

[NOTE.—Experiment shows that in this case bubbles of air are sucked in at K and carried down the tube KD with the liquid. If the arm QD is sufficiently long, it is found that the same thing *may* happen if a *sufficiently small* hole is made above Q, *provided that the siphon is in full working* at the time

164. **Open-tube Manometers.**—Any instrument used for measuring pressures of gases or vapours is called a **manometer** or **pressure-gauge.**

Such a gauge may be used to measure the pressure of the steam in the boiler of a steam engine, the pressure of the air still left in the receiver of an air pump, and so on.

In the gauges now to be described, differences of pressure are measured by the height of a column of mercury (or other fluid), just in the same way that the mercury column of the barometer measures the pressure of the atmosphere.

165. **The barometer gauge** (Fig. 62) is a flask filled with mercury from the bottom of which rises a long glass tube having a scale of inches or millimetres attached.

To measure pressures greater than an atmosphere, the upper end of the tube is left open to the air, and the flask communicates with the air or steam whose pressure is required. This pressure forces up a column of mercury into the tube whose height QP measures the amount by which the required pressure *exceeds* that of the atmosphere.

To measure pressures less than an atmosphere, the flask is open to the air and the upper end of the tube communicates with the receiver. The height QP to which the mercury rises in the tube now measures the amount by which the pressure in the receiver *is less than* that of the atmosphere. For a perfect vacuum the height of the mercury column is equal to that of the barometer.

Example.—If the barometer stands at 30 ins. when the barometer gauge is at 24 ins.,* to find the pressure in the receiver.

The pressure is that due to $30-24$, or 6 ins. of mercury, and is therefore $\frac{6}{30}$ or $\cdot 2$ of an atmosphere ;

i.e., $15 \times \cdot 2$, or 3 lbs. per square inch approximately.

166. **The siphon gauge** is a glass **U**-tube about half full with mercury or any other convenient liquid. If one branch be connected with a receiver or vessel containing gas, the other being left open to the air, the mercury will fall in the branch having the greater pressure and rise in the other, the difference of level QP measuring the difference of pressure in the branches (Fig. 63).

* This is sometimes expressed by saying that the receiver has "a vacuum of 24 ins."

If the arms are of equal section, and the mercury rises $\frac{1}{2}$ in. in one it will fall $\frac{1}{2}$ in. in the other, indicating a difference of pressure of 1 in.

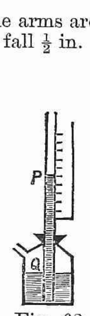

Fig. 62.

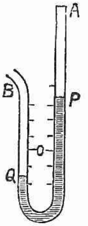

Fig. 63.

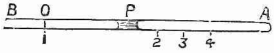

Fig. 64.

167. **The vacuum gauge** of an air pump differs from the siphon gauge just described ((Fig. 63) in having no air in the end *A* of the tube which is closed. At a certain stage of the exhaustion the mercury falls and leaves a vacuum in this end, and its difference of level in the two branches measures the pressure of the residual air in the receiver. When the vacuum is perfect, the mercury stands at the same level in both branches.

168. **Compressed-air manometers.**—The **condenser gauge** is a narrow glass tube *AB* (Fig. 64) whose closed end contains some air separated off by the drop of mercury *P*. By Boyle's Law, the length *AP* is inversely proportional to the pressure; hence by measuring *AP* the pressure of any gas connected with the end *B* can be found.

Thus, if *O* be the position of the drop when the air is at atmospheric pressure, then, under pressures of 2, 3, 4 atmospheres, the distances of the drop from *A* are $\frac{1}{2}AO$, $\frac{1}{3}AO$, $\frac{1}{4}AO$, respectively.

169. Another form of **closed-tube manometer** is a siphon manometer like that represented in Fig. 63, but with the end *A* closed and containing air instead of being open.

When the end *B* is exposed to pressure greater than that of the atmosphere, the mercury falls at *Q* and rises at *P*, and by reading off its height on a scale the required pressure at *Q* may be found.

By Boyle's Law the pressure at *P* is inversely proportional to *AP*, and the difference of pressures at *P*, *Q* is proportional to the height *QP*. The sum of these quantities gives the pressure at *Q*.

170. The Diving-bell is a large bell-shaped, or nearly cylindrical, vessel, of iron, open at the bottom, and containing a platform and seats for the persons inside. It is lowered into the water by a chain, and is of sufficient weight to sink even when filled with air. As the bell descends, the pressure of the water increases and compresses the air in the interior. Hence, to prevent water from rising into the bell, and also to enable the workmen to breathe, a constant supply of atmospheric air is pumped into the bell through a tube from the surface by means of a condensing pump (Chap. XVIII.), the superfluous air overflowing and bubbling out round the bottom.

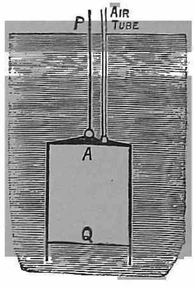

Fig. 65.

The pressure of the air inside the bell exceeds the atmospheric pressure by the amount due to a column of water whose height is the depth of the surface of the water in the bell below the surface of the water outside.

The pull on the chain is the excess of the weight of the bell and its contained air over the weight of the water displaced; the weight of the air may generally be neglected.

Examples. — (1) An iron diving-bell weighs 6 tons, and holds 200 cub. ft. of air. To find the tension on the supporting chain when the bell is completely immersed in sea-water and kept full of air (specific gravity of iron = 7·2, of sea water 1·024),

Weight of a cubic foot of sea-water = 1024 oz. = 64 lbs.
∴ weight of water displaced by air inside = 64 × 200 = 12800 lbs.
,, ,, ,, iron of bell = 6 × 1024 ÷ 7·2 tons,
= 1911 lbs. (to nearest lb.);
∴ total weight of water displaced = 14,711 lbs.
But weight of bell = 6 × 2240 = 13,440 lbs.
tension of chain = 1271 lbs.

(2) If a bell whose internal capacity is 200 cub. ft. is lowered in a river till its base is 20 ft. below the surface, to find how many cubic feet of air at atmospheric pressure must be pumped in to prevent the water from rising inside.

Let v be the volume which the air filling the bell would occupy when at atmospheric pressure.

The pressure of atmosphere = that due to 34 ft. of water,

" " inside bell = " 34 + 20 ft. "

The volume actually occupied by the air = 200 cub. ft.

Therefore by Boyle's Law,

$$v \times 34 = 200 \times 54;$$

$$\therefore \quad v = 200 \times 54 \div 34 = 318 \text{ cub. ft. nearly.}$$

Hence 318 − 200, or 118 cub. ft. of air at atmospheric pressure must be pumped in.

(3) A bottle full of air is inverted and lowered in water to a depth of 51 ft. To find how much water has entered the bottle.

Here the pressure increases 1 atmosphere for 34 ft., or $1\frac{1}{2}$ atmospheres for 51 ft. descended. Therefore the pressure at 51 ft. depth is $2\frac{1}{2}$ or $\frac{5}{2}$ times that at the surface. Hence, by Boyle's Law, the volume of the air is $\frac{2}{5}$ its volume at the surface. Therefore the water enters till it fills the remaining $\frac{3}{5}$ of the volume of the bottle.

(4) A cylindrical diving-bell 9 ft. high is lowered into a lake until the top of the bell is 11 ft. below the surface. If no air is pumped in, to find how high the water rises in the interior.

Let x ft. be the height still occupied by air (AQ, Fig. 65).

Then the depth $PQ = (11 + x)$ ft.

The pressure at Q is therefore that due to a head of water of

$$(34 + 11 + x) \text{ ft.} = (45 + x) \text{ ft.}$$

But the air originally occupied a length of 9 ft. under a pressure of 34 ft. head of water. Therefore, by Boyle's Law,

$$34 \times 9 = (45 + x) \times x;$$

$$\therefore \quad x^2 + 45x - 216 = 0.$$

Solving this quadratic equation by factorizing or otherwise, we have

$$(x + 51)(x - 6) = 0;$$

$$\therefore \quad x = -51 \text{ or } 6.$$

Now the length occupied by air cannot be a *minus* quantity; $x = 6$; and the water rises in the bell through 9 − 6 or 3 ft.

(5) To find the effect of making a hole in the side of a diving-bell.

If the hole is above the surface of the water in the bell, the pressure inside the bell will be rather greater than the pressure of the water outside the hole. Therefore air will escape through the hole and water will rise in the bell until it reaches the level of the hole.

(6) To examine whether the tension of the chain increases or decreases as the bell descends.

(i.) If no air is pumped in, the air inside will become compressed and will displace less water ; hence the tension will increase.

(ii.) If more air is pumped in to keep the bell full, the weight of this air will increase, and the tension will increase somewhat, but much less than before.

171. **Caissons.**—Where masonry has to be built under water (as, for example, in laying the foundations for the Forth Bridge), a great portion of the work has to be carried on in **caissons**, or large cylindrical cases of metal, sunk to the bottom of the water and filled with compressed air at the same pressure as the water outside.

In entering or leaving a caisson, the workmen have to pass through an "**air-lock,**" a small chamber with a door at each end opening towards the caisson. Without such a lock the air would all escape from the caisson.

Example.—An empty bottle is uncorked and again corked inside a caisson, and then removed from the caisson. What happens?

Since the bottle originally contained air at atmospheric pressure, on uncorking in the caisson air rushes in till its pressure is the same as in the caisson. When the bottle is removed from the caisson, the pressure of the enclosed air is greater than the atmospheric pressure, and therefore it tends to blow the cork out.

SUMMARY.

1. *Hare's Hydrometer* is an inverted **U**-tube.
2. *The siphon* will draw liquid from a vessel provided that—
   (i.) The outlet is below the liquid surface in the vessel;
   (ii.) The greatest height above the surface in the vessel < the barometric height of the liquid.
3. The principal kinds of *manometer* are—
   (i.) The barometer gauges ;
   (ii.) The siphon, open-tube, vacuum, and compressed-air gauges;
   (iii.) The condenser gauge.
4. *The diving-bell.*—Problems on this generally depend on applying Boyle's Law to the air inside the bell, and noting that the total pressure is that due to a head of water extending from the surface of the water inside the bell to a point above the surface equal to the height of the water barometer.

EXAMPLES XVI.

[Height of water barometer = 34 ft. Specific gravity of mercury = 13·6.]

1. What is the limit to the height over which a water siphon can act when the barometer stands at 30·25 ins.?

2. A bubble of air is inserted into a siphon while it is working. What effect does it have?

3. A siphon is filled with water and inverted into a vessel of liquid of specific gravity 1·6. What is the condition that the liquid may flow through the siphon?

4. If the mercury in a siphon manometer be of specific gravity 13·5, find in lbs. per sq. in. the difference of pressure which will give a difference of level of 8 ins. in the two branches.

5. What would have to be the height of a mercurial open-tube manometer adapted for measuring pressures up to 10 atmospheres?

6. A barometer in a diving-bell indicates a pressure of $38\frac{1}{2}$ ins., whereas at the surface of the water it indicates a pressure of 30 ins., of mercury. What is the depth of the diving-bell?

7. A diving-bell whose capacity is 500 cub. ft. is lowered in water until its mouth is at a depth of 51 ft. below the surface. How much air at ordinary atmospheric pressure must be pumped in so that all the water may be expelled?

8. A diving-bell of 200 cub. ft. capacity is lowered in fresh water, and air is pumped in so as to keep the water completely out. What depth has it reached when 600 cub. ft. of air has been pumped in?

9. What depth is reached in Question 5 if the bell is lowered in the sea instead of fresh water?

10. The top of a cylindrical diving-bell, whose volume is 200 cub. ft. and height 8 ft., is at a depth of 60 ft. below the surface of the water. How much air at ordinary atmospheric pressure must be pumped in to keep the bell full of air?

11. A small bottle, the capacity of which is 10 c.c., is carried mouth downwards to the bottom of a pond $8\frac{1}{2}$ ft. deep. How much water will have entered the bottle when it reaches the bottom?

12. If the density of the air in a closed vessel be double that of atmospheric air, and the vessel be lowered into a lake, explain what will happen if a hole be made in the bottom of the vessel when its depth is (i.) less than, (ii.) equal to, (iii.) greater than 34 ft.

13. A diving-bell is lowered into the sea until the surface of the water inside is at a depth of 20 ft. What proportion of its volume is occupied by air, the specific gravity of sea-water being 1·025?

14. A deep-sea sounding apparatus has been invented, consisting of a glass tube 3 ft. long, open at the bottom and closed at the top, and weighted so that it sinks in a vertical position. It is let down to the bottom, and the length of the inside of the tube, which has been wetted, is afterwards measured. If this length is 35 ins., find the depth of the sea, the (sea-) water barometer standing at 32 ft.

15. A diving-bell 8 ft. high is lowered in water until its top is 60 ft. below the surface. What depth of water will have entered the bell?

16. A diving-bell is lowered in a lake until two-thirds of it is filled with water. Show that, if d be the depth of the top of the bell below the surface, the height of the bell is $3(2h-d)$, where h is the height of the water barometer.

17. A diving-bell is lowered first in water and afterwards to the same depth as before in a fluid of less specific gravity than water. Does the water or the other fluid rise higher in the bell? In which case is the tension of the chain greater? Give your reasons in each case.

18. Describe an arrangement by means of which people could pass in and out of a caisson filled with compressed air without allowing more than a small fraction of the air to escape. Why would it be necessary to have small air valves which could be opened at either end of the air-lock besides the large doors?

CHAPTER XVII.

WATER PUMPS.

In the present chapter we shall describe the action of different kinds of pumps used for raising water. Most of these pumps depend on the principle that the pressure of the atmosphere is capable of supporting any column of water whose height does not exceed the height of the water barometer.

172. The Common Pump consists of a barrel or cylinder connected with the well or source of water by a pipe which opens into its lower end, and is covered by a **valve** or lid *U* opening upwards.

In the barrel is a closely fitting **piston** or plug *P* which can be raised or lowered by means of the rod. This piston also contains an opening which is covered by a valve *V* opening upwards.

The top of the barrel is generally furnished with a spout *S*, and the piston rod is worked by the lever or "pump handle" *L*.

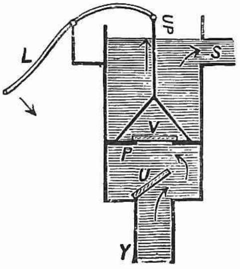

Fig. 66.

TO EXPLAIN THE ACTION OF THE PUMP, let us start with the barrel full of water and the piston at the bottom of the cylinder.

In the up-stroke (Fig. 66) the valve V remains closed, and the pressure below the piston is reduced, and the atmospheric pressure acting on the surface of the water in the well forces water up the pipe which lifts the valve U and enters the barrel. At the same time the water above the piston is raised to the level of the spout, and runs out.

In the down-stroke (Fig. 67) the valve U closes, and the water lifts the valve V and passes from the lower to the upper side of the piston P.

In the next up-stroke this water is raised to the spout, while a fresh supply of water runs into the barrel through the valve U.

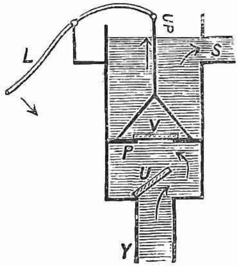

Fig. 66.

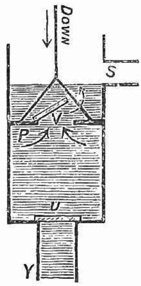

Fig. 67.

Examples.—(1) To find the force required to lift the piston (neglecting the weight of the piston), if its sectional area is 100 sq. cm. and the spout is 10 metres above the water-surface in the well.

Let x cm. be the depth of the piston below the spout, h cm. the height of the water barometer. Then the pressures above and below the piston are due to heads of water of heights

$$(h+x) \text{ and } \{h-(1000-x)\} \text{ cm.},$$

respectively. Therefore their difference is that due to a head of 1000 cm. (the total height of the column, as we should expect). Hence

diff. of pressures on two sides of the piston = 1000 gm. per sq. cm.

Also area of piston = 100 sq. cm.;

$\therefore$ resultant force on piston $= 1000 \times 100$ gm. $= 100$ kilog.

[Notice that the force depends only on the total height of the column to be raised and the area of the piston, and not on the position of the piston in the stroke.]

(2) If the spout is 10 ft. above the water surface, and 5 lbs. of water are delivered at each stroke, to find the work done in the up-stroke.

Let the length of the stroke be l ft., and let the sectional area of the piston be A sq. ft.

The difference of pressures on the two sides of the piston

$$\begin{aligned} &= \text{that due to a head of 10 ft. of water} \\ &= 10{,}000 \text{ oz. per square foot} \\ &= 10{,}000/16 \text{ lbs. per square foot;} \end{aligned}$$

$\therefore$ resultant thrust on piston $= 10{,}000 \times A/16$ lbs.;

$\therefore$ work done in up-stroke $= 10{,}000 \times Al/16$ ft.-lbs.

Now Al = volume of water raised to spout in cubic feet;

$\therefore$ $1000Al$ = weight of water raised in ounces,

and $1000Al/16$ = weight of water raised in pounds

= 5 lbs. (by data);

$\therefore$ work done in up-stroke $= 5 \times 10$ ft.-lbs. $= 50$ ft.-lbs.

This is the work required to raise the 5 lbs. of water through the total height of 10 ft.

Hence the work done by the pump is the same as if the water were lifted directly up from the bottom of the well to the spout. This is in accordance with the Principle of Conservation of Energy.

173. Limits to the action of the common pump.—Since the water below the piston is raised from below by the pressure of the atmosphere, it follows that *the height of the piston above the surface of the water must never exceed the height of the water barometer* (about 34 ft.) Otherwise a vacuum will be formed in the barrel, and water will cease to flow in.*

If the weight of the lower valve U be taken into account, the limit to the height of the piston will have to be rather *less* than 34 ft. in order that the water may *lift* this valve.

If the pump is used for raising any other liquid, the greatest height is, of course, the height of a barometer of *that* liquid; *e.g.*, mercury could only be drawn up 30 ins. with a pump.

* If during a *portion* of the stroke the piston is less than 34 ft. above the water level, water will then enter the barrel; but the portion of the stroke in which the piston rises *above* that height will be useless.

174. Action of the pump at first starting.—When a pump is first placed in water, the pipe and barrel are full of air, which must be pumped out before the water will rise into the barrel.

Suppose the piston at the lowest point of the cylinder.

In the first up-stroke, the air in the pipe expands and part of it rushes through the valve *U* into the barrel, while the reduction of pressure allows a column of water to rise up into the pipe.

In the first down-stroke, the valve *U* closes, and as soon as the air in the barrel has got compressed to atmospheric pressure it begins to escape through *V*.

In the next up-stroke, the air in the pipe again expands through the valve *U* into the cylinder, and the reduction of pressure allows the water to rise still further in the pipe. This process continues till the water at last reaches the barrel, when the continuous action as a water-pump begins, and a volume of water equal to that of the barrel is raised at each stroke.

Examples.—(1) If the lower valve is 17 ft. above the water, to find the volume of the barrel if the water just reaches it in the first stroke.

The water barometer being 34 ft. high, the pressure of the air inside the pump when the water reaches the valve is that due to $34-17$ ft., or $\frac{1}{2}$ atmosphere. Therefore the volume of the air is double of what it was at the beginning of the stroke, and the volume of the barrel must be double that of the pipe.

(2) If the barrel is 11 ins. long, and its bottom 21 ft. above the surface of the water, and if the section of the pipe is $\frac{3}{14}$ of that of the barrel, to find the height of the water at the end of the first stroke, given the height of the water barometer $= 32$ ft.

Let x ft. be the required height of the water.

Before the up-stroke, the air occupies 21 ft. of pipe under a pressure of 32 ft. of water.

After the up-stroke, the air occupies $(21-x)$ ft. of pipe plus the volume of the barrel, under a pressure of $(32-x)$ ft. of water.

Also the volume of the barrel is $\frac{14}{3}$ times the volume of an equal length of pipe, and is therefore equal to that of $\frac{14}{3}\times\frac{3}{2}$ ft. of pipe, *i.e.*, 7 ft. of pipe. Hence the air occupies a total volume equal to $(21-x+7)$ ft., or $(28-x)$ ft. of pipe. Therefore, by Boyle's Law,

$$201\times 32=(28-x)(32-x)\,; \qquad x^2-60x+224=0\,;$$

$$\therefore\quad (x-56)(x-4)=0\,; \qquad \therefore\quad x=56,\text{ or }4.$$

Now the water evidently cannot rise 56 ft., therefore $x=4$, and the water rises 4 ft.

175. **Clearance.**—When the piston does not descend quite to the bottom of the barrel, the space left below it is called the *clearance*.

Example.—(1) If the length of the stroke is 12 ins. and the clearance is 5 ins., to find the greatest height to which the water will rise.

If the valve *U* remains closed, the air, which at the beginning of the up-stroke occupied 5 ins., at atmospheric pressure, will at the end of the stroke occupy 17 ins., and its pressure will therefore be $\frac{5}{17}$ atmosphere. Hence, in order to lift the lower valve, the pressure on the underside must exceed $\frac{5}{17}$ atmosphere, *i.e.*, that due to 10 ft. head of water. Therefore the water cannot rise more than $34-10$ ft., or 24 ft. If then the height of the lower valve exceed 24 ft., the pump will never fill with water, although if once started it would work continuously.

[In such cases, the proper way to start the pump is to pour water into the clearance, and this is called *priming the pump.*]

176. **The Lift Pump** is a modification of the common pump, adapted for raising water to a cistern at any desired height above the barrel.

The top of the barrel, instead of being open, is covered with a lid in which the piston-rod passes through a tight-fitting collar *C*. From this lid rises a pipe *K* conducting the water to the required height. The bottom of this pipe is sometimes furnished with a third valve *W* opening upwards.

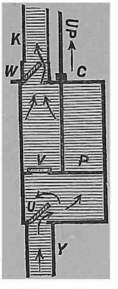

Fig. 68.

In the up-stroke (Fig. 68), the water above the piston is *lifted* up into the pipe *K* through the valve *W*, and the atmospheric pressure in the well forces water through the valve *U* into the barrel below the piston.

In the down-stroke, the water in the barrel passes through the valve *V*, just as in the common pump.

The valve *W* is unnecessary, for the lower valve *U* is sufficient to keep the water from flowing back.

There is no limit to the height to which water can be lifted above the piston, but, as in the common pump, the column *below* the piston cannot exceed the height of a barometer of the liquid that is being pumped.

The force on the piston-rod in the up-stroke may be found as in § 172, Ex. 1. The difference of pressures on the two sides is that due to the head of water extending from the surface in the well up to the outlet.

177. **The Forcing Pump** has already been mentioned in connexion with the Bramah Press. It differs from the common pump in having no aperture in the piston P, but instead of this a pipe K containing a valve F opening outwards conducts the water from the barrel to any desired height.

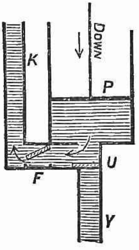

Fig. 69.

In the up-stroke (see right-hand barrel in Fig. 70) the valve F closes, and water enters the barrel through the valve U, as in the common pump.

In the down-stroke (Fig. 69) this water is forced out again through the valve F and up the pipe K.

As in the lift pump, water may be *forced up* to any height above the piston, but it cannot be raised from a greater depth below the piston than about 34 ft.

Example.—The area of the piston of the forcing pump being 90 sq. in., find the force on the piston-rod necessary to raise water from a well 20 ft. deep and force it up to a cistern 30 ft. high.

In the up-stroke the piston supports the column leading from the well, and in the down-stroke it supports the column leading up to the cistern. Therefore the difference of pressure on the two faces of the piston (the upper face being under atmospheric pressure) is 20×1000 oz. per sq. ft. in the up-stroke and 30×1000 oz. per sq. ft. in the down-stroke;

$$\therefore \quad \text{force required to raise piston} = 20 \times 1000 \times 90/144 \text{ oz.}$$
$$= 12{,}500 \text{ oz.} = 781\tfrac{1}{4} \text{ lbs.};$$
$$\text{force required to lower piston} = 30 \times 1000 \times 90/144 \text{ oz.}$$
$$= 18{,}750 \text{ oz.} = 1171 \text{ lbs. } 14 \text{ oz.}$$

178. The "Manual" Fire Engine consists of two forcing pumps worked by alternately raising and lowering the two handles of a double lever *HL* (Fig. 70), so that as one piston descends the other ascends, and water is forced out at each stroke.

179. Air-Vessel. — The action of the pumps is not perfectly continuous, because the pistons momentarily stop when their motions are reversed. In order to produce a continuous jet of water from the hose, the pumps communicate with an **air-vessel** *A* (Fig. 70). This is

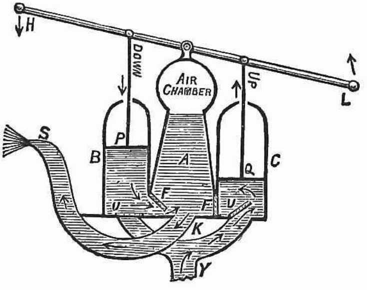

Fig. 70.

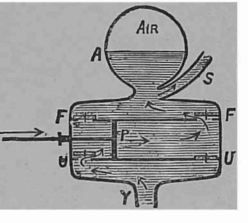

Fig. 71.

a large metal dome partly filled with air. When the pistons are moving most rapidly, water is delivered into the air-vessel faster than it can escape; hence it rises in the dome and compresses the air. When the action of the pumps stops for an instant, the air again expands and forces water out of the hose *S*.

180. **The Steam Fire Engine** is a *double-action forcing pump* furnished with an *air-vessel A* (Fig. 71). The piston is driven backwards and forwards by steam-power, and water enters the barrel on the two sides of the piston, alternately. Each end is furnished with separate valves, and forms a complete pump, one or other of these pumps producing a discharge at each stroke of the piston.

The arrangement of Fig. 71 is used in most steam pumps. The barrel is usually horizontal.

Summary.

1. *The different forms of water pumps* are—

(i.) The common pump } with valve in piston;
(ii.) The lift pump }
(iii.) The forcing pump, with second valve at side of barrel.

2. *The fire engine* consists of two forcing pumps, or a double forcing pump, with an air-vessel to produce continuous stream.

3. *The condition that the pump may work continuously* is that
height of piston above water surface $<$ height of water barometer.
But water may be lifted or forced to any height.

4. *The force on the piston rod* $= wAh$, where $A =$ area of piston, $h =$ height of column raised, $w =$ specific weight of water.

EXAMPLES XVII.

1. Describe and explain the action of the common suction pump. Why will it not work equally well at the top of a high mountain?

2. One foot of the length of the barrel of a suction pump holds 8 lbs. of water. At each stroke the piston works through 3 ins. The spout is 24 ft. above the surface of the water in the well. How many ft.-lbs. of work are done per stroke?

3. What is the greatest length of the suction tube of a pump used for raising sea-water, the height of the mercury barometer being 30 ins.? (Specific gravity of sea-water = 1·028.)

4. A tank on the sea-shore is filled by the tide with sea-water whose specific gravity is 1·025. It is desired to empty it at low tide by means of a common pump whose lower valve is on the same level as the top of the tank. Find the greatest depth which the tank can have, so that this may be possible, when the water barometer stands at 34 ft. 2 ins.

5. If the water barometer stand at 33 ft. 8 ins., and if a common pump is to be used to raise petroleum from an oil-well, find the greatest height at which the lower valve of the pump can be placed above the surface of the oil in the well. (The specific gravity of petroleum is ·8.)

6. In the common pump, if the barrel is 18 ins. in length and its bottom 21 ft. above the surface of the water, and if the section of

the pipe is $\frac{3}{14}$ of that of the barrel, find the height of the water in the pipe at the end of the first stroke; given the height of the water-barometer = 32 ft.

7. The height of the lower valve of a common pump above the surface of the water to be raised is 10 ft., and the cross-section of the barrel is five times that of the pipe. What must be the length of the stroke in order that the water may rise to the lower valve at the end of the first stroke (the water barometer standing at 34 ft.)?

8. If the pump in the last question be used for raising sea-water of specific gravity 1·025, will the stroke be shorter or longer, and by how much?

9. If the fixed valve of the common pump be 29 ft. above the surface of the water, and the piston, the entire length of whose stroke is 6 ins., is, when at the lowest point of its stroke, 4 ins. from the fixed valve, find whether the water will reach the pump-barrel, the height of the water barometer being 32 ft.

10. If the plunger of the force-pump has a cross-section of 8 sq. ins. and works 50 ft. below the cistern, what thrust is required to force it down?

11. In the common pump, why is the lower tube narrower than the upper? What are the forces acting on the piston when the pump is in action?

12. How would you arrange a pump so that the work done in lifting the weight of the piston and its connecting rod should not be wasted?

13. What additional apparatus is necessary to make the supply of water continuous instead of being intermittent?

14. If the piston only traverses the upper half of the pump-barrel, find the greatest height to which the water will rise, the pump being originally full of air.

EXAMINATION PAPER VIII.

1. Explain the action of the siphon. Why cannot it be used to carry water over a mountain?

2. What will be the effect on the working of a siphon if a hole be made (i.) at the highest point, (ii.) at a point above the surface in the longer branch?

3. A vessel containing water is to be emptied by means of a siphon which is filled with a liquid of specific gravity ·8. Find the minimum length of the longer arm when the length of the shorter is 5 ft., in order that the siphon may work.

4. Describe some simple form of gauge which would enable you to measure the pressure at which gas is supplied, and explain the principle upon which it is constructed.

5. Describe the construction and use of the diving-bell, and show how to find the tension of the supporting rope when the bell is full of air at any given depth.

6. What will happen if, when the bell is totally immersed, a small aperture is made in the vertical side of the bell above the surface of the water inside?

7. What volume of air must be introduced into a cylindrical diving-bell to keep the water outside from entering it if the bell has an internal section of 15 sq. ft. and an internal height of 10 ft., and the top of the bell is immersed to a depth of 90 ft. in fresh water?

8. What will be the effect of inverting a siphon full of air and placing it under the rim of a diving-bell with the shorter arm projecting upwards into the air in the bell?

9. Describe and explain the action of the common pump. What is meant by the term "clearance"?

10. One foot length of the barrel of a pump holds 15 lbs. of water; at each stroke the piston works through 3 ins., and the spout is 20 ft. above the water in the well. How much work is done per stroke?

CHAPTER XVIII.

AIR PUMPS.

181. The pumps used for compressing or rarefying air are almost identical in construction with the water pumps described in the last chapter, which they also closely resemble in principle.

Any of these pumps may be (and often are) called air pumps. But, in general, the term *air pump* means a pump for exhausting air (§ 186). A pump for compressing air is called a *condenser*, and has many important uses, such as for supplying air to a diving-bell or caisson (§§ 170, 171), inflating the pneumatic tires of a bicycle, filling the reservoirs and pipes of the Westinghouse Brake (§ 195), making aerated waters, &c.

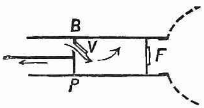

Fig. 72.

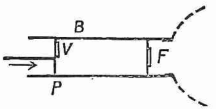

Fig. 73.

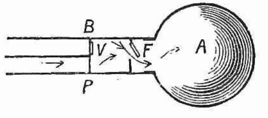

Fig. 74.

182. The Condenser or **Condensing Pump** consists of a barrel *B*, traversed by a piston *P*, and communicating at one end with the vessel *A*, into which air is to be compressed.

This vessel is called the *receiver*, and is shown only in Fig. 74.

Both the piston and the end of the barrel contain valves *V*, *F* opening *from* the outside air *towards* the receiver.

In the backward stroke (*i.e.*, when the piston *P* is being pulled back, Fig. 72), the valve *F* is closed by the pressure in the receiver, while air at atmospheric pressure passes through the valve *V* to the front of the piston and fills the barrel.

In the beginning of the forward stroke (Fig. 73) both valves V, F remain closed, and the air inside the barrel is compressed until its pressure just equals that in the receiver.*

In the remainder of the forward stroke (Fig. 74), the valve F opens, and air is forced through it into the receiver.

In what follows, the backward and forward strokes of the piston of a pump are together considered as constituting **one complete stroke** of the pump.

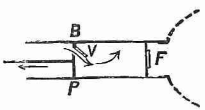

Fig. 72.

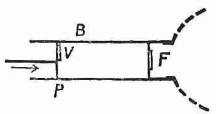

Fig. 73.

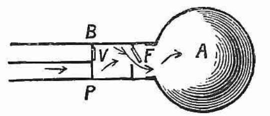

Fig. 74.

Examples.—(1) The volume of the receiver is 80 cub. ins., and that of the barrel 20 cub. ins. Find how many strokes must be made before the pressure of the air in the receiver is 3 atmospheres.

By Boyle's Law, the density in the receiver is three times the density of atmospheric air. Hence the air in the receiver would occupy 240 cub. ins. at atmospheric pressure;

$\therefore$ 160 cub. ins. of air have been forced in.

But at each back-stroke 20 cub. ins. of air enter the barrel, and are forced into the receiver at the forward stroke;

$\therefore$ number of complete strokes $= 160/20 = 8$.

(2) To find when the valve in the barrel opens in the next forward stroke (see Ex. 1).

The valve F opens when the air in the barrel has a pressure of 3 atmospheres, that is, when it occupies one-third its original volume, or the piston has traversed two-thirds the length of the barrel.

* Except in the first stroke, when the air in the receiver is at atmospheric pressure and F opens at once

183. To find the density and pressure in the receiver after n complete strokes.

Let A be the volume of the receiver, B that of the barrel, D the density of atmospheric air, d the density in the receiver after n strokes.

Then the receiver originally contained a mass of air AD

At each backward stroke a volume B of air at atmospheric density D enters the barrel. At the forward stroke this air enters the receiver. Hence, after n *complete* strokes,

$$\text{mass of air in receiver} = (A+nB)\,D.$$

But its volume $= A$;

$$\therefore \quad \textbf{its density } d = \frac{A+nB}{A}D = \left(1+n\frac{B}{A}\right)D \;\ldots\; (1).$$

This relation is independent of the law connecting the pressure and density. If, however, these follow Boyle's Law, we have also

$$\textbf{pressure in receiver} = \left(1+n\frac{B}{A}\right) \textbf{ atmospheres.}$$

184. **Limits to the action.**—In obtaining (1), we have supposed that *all* the air which enters the barrel is forced into the receiver in the forward stroke. In such cases, there is no limit to the pressure which can be produced in the receiver.

In actual pumps, however, the action is limited by the existence of a **clearance,** or residual space, left between the valve F and the piston, after the latter has been pushed as far forward as it will go.

Example.—The volume of the barrel is 20 cub. ins., and the clearance $\frac{1}{2}$ cub. in.; to find the greatest pressure that can be produced.

If the air in the barrel is *all forced down into the clearance*, its greatest pressure will be $20 \div \frac{1}{2}$ or 40 atmospheres. Hence the pressure in the receiver can never be greater than 40 atmospheres, for otherwise the valve F would not open.

185. **Difference between the condensing and the air pump.**—In the condensing pump, a quantity of air whose volume is that of the barrel is forced into the receiver at each stroke, and *the density of this air is always that of the outside air.* Consequently, the *mass* of the air forced in at each stroke is *constant.* But in the air pump, though the same volume of air is extracted at each stroke, *its density diminishes with each stroke*, and therefore the *mass* of the air extracted also *diminishes.*

186. The Air Pump.—If we suppose a common pump (§ 172) used for pumping out air instead of water, we shall have an **air pump.** The vessel to be exhausted of air is called the **receiver** (A, Figs. 76, 77), and the pump itself consists essentially of a cylinder B traversed by a piston P, both containing valves opening outwards from the receiver.

These valves must be light enough to yield to a very slight excess of pressure on their lower side; hence the valves used in a water pump would be far too heavy.

To describe its action, suppose the piston at the bottom of the barrel.

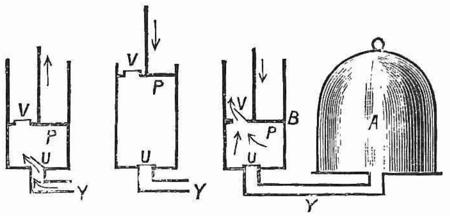

Fig. 74. Fig. 75. Fig. 76.

In the up-stroke (Fig. 74) the valve V closes, and the air in the receiver and tube lifts the valve U, and part of it passes into the barrel. At the end of the up-stroke the barrel is therefore filled with air at the same pressure, and therefore also at the same density, as the air left in the receiver.

In the first part of the down-stroke (Fig. 75) the valve U closes, *and the valve V also remains closed, while the air beneath the piston is compressed until its pressure equals that of the atmosphere.**

In the remainder of the down-stroke (Fig. 76) the piston-valve V opens and allows the air to escape from beneath the piston.

* In consequence of the compressibility of the air, the piston-valve V *does not open at once*, as it would do if the barrel contained water. The present action also takes place in a water pump before the water reaches the barrel.

187. Hawksbee's Air Pump (sometimes called the *double-barrelled air pump*) is provided with two barrels instead of one, and the pistons are worked up and down by means of what is called a **rack and pinion** (Fig. 77), so that, as the handle H is moved to and fro, one piston rises as the other falls. G is a mercurial vacuum gauge (see § 167).

ADVANTAGES.—This arrangement possesses two advantages :

1st.—The air is exhausted twice as quickly as with a single barrel.

2nd.—During the up-stroke and the first part of the down-stroke, the pressure in the barrel is less than the pressure of the atmosphere. This excess of pressure on the upper side of the piston makes the single-barrelled pump hard to work. In the double-barrelled pump the resultant thrust of the air on the descending piston assists in pulling the other piston up.

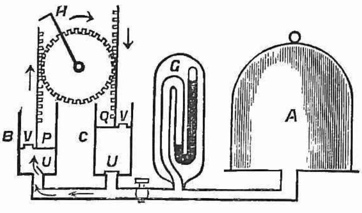

Fig. 77.

Example.—If the volumes of the barrel and receiver are equal, to find the pressure left in the receiver after 5 complete strokes.

In the first up-stroke, half the air from the receiver enters the barrel and half is left behind ; therefore, by Boyle's Law,

pressure in receiver after the stroke $= \frac{1}{2}$ atmosphere.

In the second stroke, half the remaining air passes into the barrel ;

$\therefore$ pressure in receiver after 2 strokes $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ atmosphere.

Similarly, at each complete stroke, the quantity of air in the receiver, and therefore also the pressure, is reduced by one-half.

Hence, evidently, pressure of air left after 5 complete strokes

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = (\tfrac{1}{2})^5 = \tfrac{1}{32} \text{ atmosphere.}$$

Taking an atmosphere as 15 lbs. per square inch,

the required pressure $= 7\frac{1}{2}$ oz. per sq. in.

188. To find the density and pressure of the air left in the receiver after n strokes.

Let A be the volume of the receiver and connecting-pipe, B that of the barrel. Let D be the density of the atmospheric air originally in the receiver. Let $d_1, d_2, \dots d_n$ be the densities of the air left after 1, 2, ... n strokes respectively.

After the first up-stroke, the air originally in the receiver expands from volume A to volume $A+B$. Hence, since its mass is unaltered, its densities are connected by the relation

$$d_1(A+B) = DA;$$

$$\therefore \quad d_1 = D\frac{A}{A+B}.$$

During the down-stroke the air left in the receiver remains at the same density d_1 unaltered, but in the next up-stroke it again expands in volume from A to $A+B$. Hence, for its subsequent density, we have

$$d_2(A+B) = d_1 A;$$

$$\therefore \quad d_2 = d_1\frac{A}{A+B} = D\left(\frac{A}{A+B}\right)^2.$$

At the third stroke the air left in the receiver again expands in volume from A to $A+B$, and therefore

$$d_3(A+B) = d_2 A;$$

$$\therefore \quad d_3 = d_2\frac{A}{A+B} = D\left(\frac{A}{A+B}\right)^3.$$

Proceeding in this way, it is obvious that the density of the air is reduced at each up-stroke in the ratio of A to $A+B$, and therefore after n strokes it is given by

$$\boldsymbol{d_n = D\left(\frac{A}{A+B}\right)^n} \text{ .... (2).}$$

This is true independently of the law connecting the pressure and density. If, however, Boyle's Law be assumed, we have

pressure in receiver after n strokes

$$= \left(\frac{A}{A+B}\right)^n \textbf{ atmospheres.}$$

[This result might be obtained without first finding the density by following the method of Ex. 1 below.]

Examples.—(1) The volumes of the barrel and receiver are 25 and 75 cub. ins.; to find the pressure of the air left after 3 strokes.

In the first up-stroke, 75 cub. ins. of air at atmospheric pressure expand till they fill the receiver and barrel, *i.e.*, 100 cub. ins.;

$$\therefore \text{ pressure after the stroke } = \tfrac{75}{100} = \tfrac{3}{4} \text{ atmosphere.}$$

In each succeeding up-stroke, the air remaining in the receiver expands from 75 to 100 cub. ins., and its pressure is therefore reduced to $\frac{3}{4}$ what it was before;

$$\therefore \text{ pressure after 3 strokes } = \tfrac{3}{4} \times \tfrac{3}{4} \times \tfrac{3}{4} = (\tfrac{3}{4})^3 = \tfrac{27}{64} \text{ atmosphere.}$$

(2) The volume of the barrel being two-fifths that of the receiver, to find how many strokes are required to reduce the density to less than one-third the original density.

$$\text{Here} \qquad B = \frac{2}{5}A; \qquad \therefore \quad \frac{A}{A+B} = \frac{5}{5+2} = \frac{5}{7}.$$

$$\text{Now } \left(\frac{5}{7}\right)^2 = \frac{25}{49} > \frac{1}{3}, \quad \left(\frac{5}{7}\right)^3 = \frac{125}{343} > \frac{1}{3}, \quad \left(\frac{5}{7}\right)^4 = \frac{625}{2401} < \frac{1}{3}.$$

Hence 4 strokes are required.

189. **Limits to Exhaustion.**—The fraction $\{A/(A+B)\}^n$ can, by taking n sufficiently large, be made as small as we please; hence, *theoretically*, we could attain any required degree of exhaustion short of a perfect vacuum if we were only to work the pump long enough. But in an actual pump the degree of exhaustion falls short of that given by (1), owing to the following causes:—

(i.) *The clearance.*—Even when the piston is pushed "full home," there must be a little space or **clearance** between the two valves. If we go on pumping long enough, we shall at last arrive at a limit beyond which the valves never open, and the air between them alternately expands into the barrel and is forced back into the clearance.

(ii.) *The weight of the valves.*—The pressure in the receiver can never become less than the amount necessary to lift the lower valve; when this is attained, further exhaustion is impossible.

In order to reduce the weight of the valves as much as possible, they are sometimes made of a very thin film of gutta percha or oiled silk overlying very small air holes in the piston and cylinder, respectively.

190. **Smeaton's Air Pump** (Fig. 78) is identical in the arrangement of its parts with the lift pump of Chap. XVII. It differs from the common air pump in having the barrel closed by a lid in which the piston-rod passes through a tight-fitting collar, and this lid has a small hole covered by a valve *W* opening into the air.

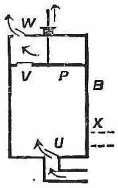

Fig. 78.

THE ACTION is as follows :—

In the up-stroke, air from the receiver enters through the valve *V*. *At first*, the valve *W* remains closed, until the air above the piston is compressed to atmospheric pressure; *subsequently*, *W* opens and this air escapes.

In the down-stroke, the piston-valve *V* opens at *once* and air passes from the under to the upper side of the piston.

THE ADVANTAGES of Smeaton's Pump are as follows :—

(i.) The difficulty of working is far less than in an ordinary single-barrelled pump, because in the greater portion of the complete stroke the pressure on top of the piston is less than atmospheric pressure.

(ii.) The action is much less limited by the clearance at the bottom of the barrel, the valve *V* opening more readily owing to the reduction of pressure above it.

In a modified form, the air from the receiver enters the side of the barrel at *X*, and, as the piston descends below *X*, this air flows straight on to its upper side without having to lift the weight of any valves.

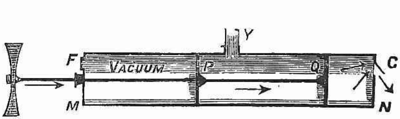

Fig. 79.

191. **Tate's Air Pump** (Fig. 79) has two pistons *P*, *Q* attached to the piston-rod at a distance apart of rather less than half the length of the barrel. The air from the receiver enters at the middle of the barrel at *Y*, and valves *F*, *G* open outwards at both ends.

THE ACTION is as follows :—

In the forward stroke, represented in Fig. 79, the air in QG is first compressed to atmospheric pressure and then forced out through G. At the same time a vacuum is formed in FP, and when the piston P has just passed beyond Y, air from the receiver rushes into this vacuum.

In the backward stroke this air is forced out through F, and a vacuum is formed in QG which receives air when the piston Q has passed beyond Y.

THE ADVANTAGES are as follows :—

(i.) Double action with a single barrel.

(ii.) No valves have to be lifted by the pressure of the air in the receiver ; consequently a much better vacuum is obtainable.

[It is only *after the air has been compressed* at the ends of the barrel that its pressure has to lift the valves F, G.]

*192. **Sprengel's Air Pump** (Fig. 80), although called a "pump," has no pistons or valves. The funnel A contains mercury, and as this falls down the tube AB air from the receiver enters at P and is carried down in bubbles alternating with columns of mercury.

The air bubbles escape into the atmosphere at the surface of the cup B.

On turning off the tap H, mercury again rises from B and prevents the reflux of air, its height measuring the degree of exhaustion as in a barometer gauge.

There is no limit to the exhaustion, short of a perfect vacuum, provided that the tube PB exceeds the height of the mercurial barometer.

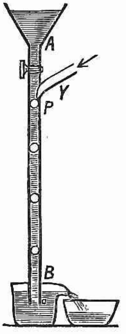

Fig. 80.

193. **The ejector of a Vacuum Brake** is similar in principle to Sprengel's Pump, but a powerful jet of steam from the locomotive boiler replaces the mercury column. This jet rushing through the tube, as in Fig. 81 (p. 192), carries with it the air from the brakes, producing a very fair vacuum.

194. **The Vacuum Brake.**—By means of this apparatus the pressure of the atmosphere is made to apply the brakes simultaneously to the wheels of all the carriages in a railway train. The brakes on each carriage are connected by levers with a piston P working in a large cylinder (Fig. 81). A pipe running along the whole train connects these cylinders with the engine.

When the train is running, the ejector on the engine exhausts the air

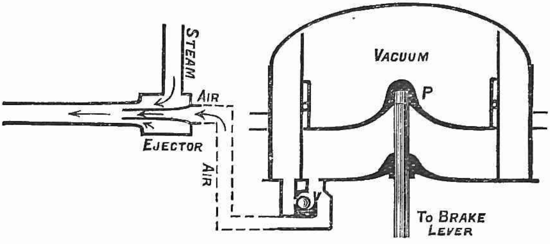

Fig. 81.

on *both* sides of the piston P by means of the ball-valve V. The piston P remains at the bottom of the cylinder, and the brakes are off.

To stop the train, air is readmitted by the train pipe to the *under* side of the piston P, but the ball-valve V closes and prevents its passing to the upper side. Hence the pressure of the air lifts the piston and applies the brakes.

195. **The Westinghouse Brake** is worked by compressed air. Each carriage is provided with a receiver R (Figs. 82, 83), a brake cylinder B and large piston H, and a "*triple valve*" F consisting of a small piston Q and slide-valve S.

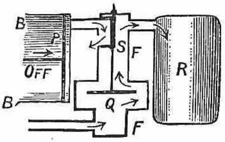

Fig. 82.

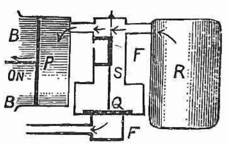

Fig. 83.

When the train is running (Fig. 82), air is forced into the train pipe by a condensing pump on the engine, and it lifts the small piston Q and enters the receiver R. Any air in the brake cylinder B can escape through the slide-valve S, and the brakes are off.

To stop the train (Fig. 83), air is allowed to escape from the train pipe. The excess of pressure in R depresses the piston Q and valve S. Compressed air now rushes from R into the brake cylinder B, pushes out the piston P, and applies the brakes.

[The actual working apparatus contains many additional complications.]

SUMMARY.

1. *With the condensing pump*, the density after n strokes

$$= \frac{A+nB}{A} \times \text{(density of atmospheric air)}.$$

2. *With the air pump*, the density after n strokes

$$= \left(\frac{A}{A+B}\right)^n \times \text{(density of atmospheric air)}.$$

In each case, A = volume of barrel, B = volume of receiver.

3. *The different air pumps* can be classified thus:

(*a*) Mechanical air pumps, with pistons and valves—
   (i.) The common air pump, single barrelled;
   (ii.) Hawksbee's, double barrelled;
   (iii.) Smeaton's;
   (iv.) Tate's, double acting, with single barrel.

(*b*) Air pumps worked by a stream of fluid—
   (i.) Sprengel's mercurial pump;
   (ii.) The ejector (steam).

*4. The *Vacuum Brake* is applied by atmospheric pressure on piston in a vacuum chamber, while the *Westinghouse Brake* is applied by compressed air entering brake cylinder on reduction of pressure.

EXAMPLES XVIII.

1. The volume of the receiver in a condensing air pump being 8 times that of the barrel, after how many strokes will the density of the air in the receiver be twice that of the external air?

2. The volumes of the receiver and barrel of a condenser are in the ratio of 5 to 1; find the density of the air in the receiver after 3 complete strokes.

3. The receiver of a condenser is 9 times as large as the barrel; how many strokes must be made before the density of the air in the receiver is 4 times that of the external air?

4. Describe the common air pump, and state the principal causes which limit the action of a pump of this construction.

5. The volume of the receiver in an exhausting air pump being 8 times that of the barrel, after how many strokes will the density of the air in the receiver be half that of the external air?

6. The volumes of the receiver and barrel of an exhausting air pump are in the ratio of 6 to 1; find the density of the air in the receiver after 4 complete strokes.

7. The volume of the receiver of an exhausting air pump being 9 times that of the barrel, how many strokes must be made before the density of the air in the receiver is one-third that of the external air?

8. If the receiver of a Tate's air pump holds 90 grs. of air at the ordinary pressure, and the piston-barrel 10 grs., what weight of air will be left in the receiver after 4 complete strokes of the piston?

9. In one exhausting air pump the volume of the barrel is one-tenth of that of the receiver, and in another it is one-fifth of it. Show that the densities of the air in the two receivers after 3 ascents of the pistons are as $12^3 : 11^3$.

10. The contents of the receiver of an exhausting air pump is 6 times that of the barrel. Find the elastic force of the air in the receiver at the end of the eighth stroke of the piston, when the atmospheric pressure is 15 lbs. to the square inch.

11. Supposing the receiver of an air pump to be made of such a form that a mercury barometer can be placed inside, and its volume to be 8 times that of the barrel, how far will the mercury have fallen at the end of the second and third strokes, the height of the mercury being originally 729 mm.?

12. If the volume of the space between the bottom of the pump-barrel and the lower surface of the piston when the latter is at the end of its downward stroke be ·01 cub. in., and the volume of the pump-barrel be 15 cub. ins., find the pressure of the air in the receiver when the greatest exhaustion has taken place, the height of the barometer being 30 ins., and the pump being supposed in other respects perfect.

13. If the volume of the barrel of an air pump is 4 cub. ins., and there is a clearance of $\frac{1}{50}$ cub. in. at the bottom, find the pressure in the receiver when the pump ceases to act.

14. If the receiver of an air pump is connected with both a barometer gauge and a siphon gauge whose closed end is empty, show that the sum of the heights of the mercury columns in the two gauges is equal to the height of the barometer.

15. Why is Hawksbee's air pump made with two barrels, and Smeaton's with only one?

Show that the expression for the density after n strokes is the same whether the common air pump or Smeaton's is used.

16. A Cartesian diver consists of an indiarubber figure containing air, and loaded so as to just rise to the surface in water. When placed in the receiver of a condenser, the diver sinks. Why is this?

17. The area of the piston of a vacuum brake is 200 sq. ins. Find the maximum force which it is capable of exerting when the barometer stands at $29\frac{1}{2}$ ins. Is it easier (*theoretically*) to stop the train when the barometer is high or low?

18. Explain what happens when some of the carriages of a train fitted with the Westinghouse brake become detached owing to the couplings breaking.

EXAMINATION PAPER IX.

1. Describe an apparatus suitable for inflating the pneumatic tires of a bicycle.

2. The volume of a receiver of an air condenser is six times that of the barrel. After how many strokes will the density of the air in the receiver be five times that of the external air?

3. Describe Hawksbee's air pump, and explain the advantage gained by the use of two pistons.

4. Find the pressure of the air in the receiver of an air pump after n strokes.

5. If the pressure is reduced to $\frac{1}{4}$ of the atmospheric pressure in 6 strokes, to what will it be reduced in 9 strokes?

6. How may the degree of exhaustion of the receiver of an air pump be determined by a body floating in water within the receiver?

7. If the barometer stands at 29·6 ins., what will the mercurial gauge of an air pump read when the quantity of air withdrawn is 6 times as much as the quantity left in the receiver?

8. Describe the action of an ordinary pair of bellows. How can a continuous blast of air be obtained as in the forge bellows?

9. A siphon is made to transfer mercury from one vessel to another, the whole being under a bell jar. When the air is exhausted to one-third of its original density, the siphon ceases to act. Find the height of its highest point above the mercury in the lower vessel when this occurs.

10. Describe and explain the action of the Vacuum Brake.

RESULTS IN MENSURATION.

The following facts in Solid Geometry and Mensuration are assumed. The references given below are to the articles in Briggs and Edmondson's *Mensuration*, where the reader will find the properties in question fully proved. Proofs of them are also given in most elementary treatises on Solid Geometry. The *results* alone need be remembered:—

(1) **The area of a triangle**

$$= \frac{1}{2}\,(base) \times (altitude). \qquad (\S\ 45.)$$

(2) **The area of a trapezoid** (*i.e.* a quadrilateral with two sides parallel) $= (its\ height) \times (\frac{1}{2}\ sum\ of\ parallel\ sides)$. (§ 49.)

(3) **The length of the circumference of a circle** of radius r

$$= \pi \times (diameter)$$
$$= 2\pi r; \qquad (\S\ 57.)$$

where the Greek letter π ("**pi**") stands for a certain "incommensurable" number (that is, a number which cannot be expressed as an exact arithmetical fraction), whose value lies between 3·141592 and 3·141593. The following approximate values should be remembered and used, unless otherwise stated.

$$\pi = \frac{22}{7}, \text{ for all } \textit{rough} \text{ calculations};$$

$$\pi = 3{\cdot}1416, \text{ more approximately.}$$

(4) **The area of the circle**

$$= \frac{1}{2}\,(radius) \times (circumference)$$
$$= \pi r^2. \qquad (\S\ 58.)$$

(5) **The volume of a pyramid**

$$= \frac{1}{3} (\textit{height}) \times (\textit{area of base})$$

$$= \frac{1}{3} hA, \qquad (\S\ 105.)$$

the height h being the perpendicular from the vertex on the plane of the base, and A the area of the base.

(6) **The area of the curved surface of a cylinder,** whose height is h and the radius of whose base is r,

$$= (\textit{height}) \times (\textit{circumference of base})$$

$$= 2\pi rh. \qquad (\S\ 115.)$$

(7) **The volume of the cylinder**

$$= (\textit{height}) \times (\textit{area of base})$$

$$= \pi r^2 h. \qquad (\S\ 116.)$$

(8) **The area of the curved surface of a right circular cone,** whose height is h and the radius of whose base is r,

$$= \frac{1}{2} (\textit{circumference of base}) \times (\textit{length of slant side})$$

$$= \pi r \sqrt{(h^2+r^2)}; \qquad (\S\ 117.)$$

a *slant side* being a line drawn from the vertex to a point in the circumference of the base.

(9) **The volume of the cone**

$$= \frac{1}{3} (\textit{vol. of cylinder of same base and height})$$

$$= \frac{1}{3} \pi r^2 h. \qquad (\S\ 118.)$$

(10) **The area of the surface of a sphere** of radius r

$$= 4 \textit{ times area of circle of same radius}$$

$$= 4\pi r^2. \qquad (\S\ 126.)$$

(11) **The volume of the sphere**

$$= \frac{1}{3} (\textit{radius}) \times (\textit{surface})$$

$$= \frac{4}{3} \pi r^3. \qquad (\S\S\ 127,\ 128.)$$

ANSWERS.

EXAMPLES I. (PAGE 14.)

3 $6\frac{1}{4}$. **4.** 30·3

EXAMPLES II. (PAGES 21, 22.)

1. (i.) 5000 oz., 5120 oz. (ii.) 120,000 oz., 122,880 oz.
(iii.) 40·92 oz., 41·9 oz.
2. (i.) 20 gm., 272 gm. (ii.) $59\frac{5}{7}$ gm., $812\frac{4}{35}$ gm.
(iii.) $2095\frac{5}{21}$ gm., $28495\frac{5}{21}$ gm.
3. 101·28. **4.** 100. **5.** 49 : 26, or 1·88 : 1. **6.** 450.
8. (i.) $556\frac{1}{4}$ lbs. (ii.) 6·57 oz. (iii.) 32·4 oz. (iv.) 8·35 lbs.
(v.) 19·35 gm. (vi.) 920 kilog. (vii.) 13,600 gm. (viii.) 102·4 kilog.
9. (i.) 1·728. (ii.) $1{\cdot}32\dot{7}4\dot{0}$. (iii.) ·8. (iv.) $1\frac{10}{11}$. (v.) $4{\cdot}1\dot{6}$. (vi.) 1·6.
10. 2·5. **11.** 96 : 1.
16. Unit of wt. $= \frac{1}{1000}$ wt. of unit vol. of standard substance. **18.** Yes.

EXAMPLES III. (PAGES 30, 31.)

1. $\frac{25}{26}$. **2.** 8·96 oz. **3.** ·8. **5.** 1·0689. **6.** 3 : 1 by vol.
7. $\frac{1}{2}d_3 + \frac{1}{4}(d_1 + d_2)$. **8.** ·154 c.c. **9.** 12. **10.** 72 to 17 by vol.
11. 20·45 gm. zinc, 79·55 gm. copper. **12.** 3 of heavier, 5 of lighter.
13. 1·24 and 1. **14.** $\{v_1(s_1 - s) + v_2(s_2 - s)\} / s$.
15. 6 and 2. **17.** The volumes are equal.

EXAMINATION PAPER I. (PAGE 32.)

1. See §§ 3, 5, 7. **2.** See §§ 12, 16. **3.** 1000. **4.** 4000 lbs.
5. See § 22. **6.** 20. **7.** 4·264.
8. $(W_1 + W_2) \Big/ r\left(\dfrac{W_1}{s_1} + \dfrac{W_2}{s_2}\right)$. **9.** 8·2. **10.** 27·24 oz., nearly.

EXAMPLES IV. (PAGES 37, 38.)

1. 2·84. **3.** 2·046. **4.** $7{\cdot}03\dot{1}$. **5.** ·04.
6. $1{\cdot}3\dot{8}$. **7.** 7·692. **8.** 2·5. **9.** 4·08 gm.
10. ·028 sq. cm. **11.** 154 ft. 4 in., nearly. **12.** $(B-w)/(A-w)$.

EXAMPLES V. (PAGES 44, 45.)

3. 2·56231. **4.** $115\frac{20}{27}$ oz. **5.** 21·6 in. side. **6.** ·8. **7.** 1·8.
8. 7·31. **9.** $2\frac{1}{2}$ cub. ft. **10.** 2·975, 4·76 gm. **11.** ·25.
12. $\frac{3}{5}$. **13.** 36·42 c.c., 7·55. **14.** $1{\cdot}\dot{7}2\dot{9}$. **16.** ·85 sec.

EXAMINATION PAPER II. (PAGE 46.)

1. 5·6. **2.** 2700 oz. per cu. ft. **3.** 3 cm. **4.** See §§ 30–32. **5.** See § 35.
6. ·72. **7.** $2{\cdot}\dot{6}$. **8.** 2. **9.** ·001293, 14·46. **10.** See §§ 38, 39.

EXAMPLES VI. (PAGES 54–56.)

1. 21. **2.** 11·36. **3.** $108\frac{4}{7}$ oz. **4.** 2·4.
5. 8, 34·56 cub. in. **6.** 260 gm. **7.** 9·6 gm. **8.** ·1935.
9. ·1935. **10.** 111·6 gm. **11.** 2 cub. in.; 7·523 oz.
12. ·7846. **13.** ·803. **14.** 6·158, ·842. **15.** 1·841.
16. 8·5, ·85. **17.** 1·5. **18.** ·94. **19.** 19·2, ·72. **20.** ·848.
21. 50 gm. **22.** 4·53125. **23.** 2080 gr. **24.** ·865. **25.** 7·5.
26. $19\frac{1}{5}$ lbs. **27.** $\frac{5}{18}$. **29.** 1·00352. **30.** 100 c.c.

EXAMPLES VII. (PAGES 65–67.)

1. 18 : 19. **2.** 10 oz. **3.** $\frac{3}{4}$ oz. **4.** 2·5. **5.** 14 gm.
6. 2·84. **7.** $2\frac{6}{7}$. **9.** 1·03. **10.** $7\frac{17}{44}$ or $7{\cdot}3\dot{8}6\dot{3}$ c.c.
11. 190 : 191. **12.** $\frac{20}{21}$. **13.** 3·456, 3·142, 2·88 cub. in. **14.** $1\frac{3}{23}$.
15. $\dfrac{12b+5a}{17}$, $\dfrac{15a-4b}{11}$, where a and b are the readings corresponding to specific gravities 1 and ·8.
16. 1·728. **17.** $\frac{7}{25}$. **18.** $6\frac{2}{3}$ oz. **19.** 1·4 cm. **20.** $3\frac{1}{2}$ ft.; $\frac{4}{7}$.

EXAMINATION PAPER III. (PAGE 68.)

1. See § 44. **2.** 7·5. **3.** ·6. **4.** ·96. **5.** See § 48.
6. 3·6. **7.** See § 59. **8.** See §§ 54–58. **9.** 3·5. **10.** ·9.

Examples VIII. (Page 80.)

1. 864 : 25 or 34·56 : 1. **2.** (i.) 81 : 80. (ii.) 1 : 9. (iii.) 400 : 23.
3. (i.) 34560. (ii.) 69120. (iii.) $8\frac{19}{28}$. (iv.) $729\frac{1}{6}$.
4. (i.) 1030. (ii.) 1,030,000. (iii.) 1030. (iv.) 101,043.
5. 983,430. **6.** 172,800 lbs. per sq. in. **7.** 64 lbs.
8. 100 gm., 100 gm., 120 gm., 48 gm.; ·0048, ·0048, ·004, ·01.

Examples IX. (Pages 88, 89.)

1. $127\frac{3}{11}$ lbs. per sq. cm., $17\frac{6}{7}$ tons, $571\frac{3}{7}$. **2.** 14,400 lbs.
3. $12\frac{1}{2}$ lbs. **4.** 7 : 1. **5.** 10 sq. in. **7.** 268·8 lbs. per sq. in.
8. 38,880. **9.** 44 lbs. **10.** $150\frac{1}{16}$ tons weight.

Examination Paper IV. (Page 90.)

1. See §§ 68, 69. **2.** 7185 dynes per sq. cm., nearly.
3. See §§ 73, 79. **4.** $9\frac{3}{5}$ oz. **5.** See § 87. **6.** 192 lbs.
7. See § 82. **8.** 5 tons. **9.** 100 lbs. **10.** 80 : 1.

Examples X. (Pages 105, 106.)

1. $113\frac{8}{9}$. **2.** ·434.
3. (i.) 13·021 lbs. per sq. in. (ii.) $2\frac{7}{24}$ lbs. per sq. in.
(iii.) 102·4 kilog. per sq. cm. (iv.) 1·0336 kilog. per sq. cm.
4. On the sp. weight of the fluid. **5.** 250 lbs. per sq. ft., 1250 lbs.
6. The thrusts on the bases are $8437\frac{1}{2}$ lbs., 13,000 lbs., $21,437\frac{1}{2}$ lbs.
7. $\frac{125}{144}$ lbs. per sq. in. **8.** $7\frac{1}{2}$ ft. **9.** 4·4 in., nearly.
10. $22\frac{1}{2}$ fathoms, allowing for atm. pressure. **11.** 320 lbs. **12.** 2 : 1.
15. $P+zsw$, where w is weight of unit volume of standard substance.
16. 260 cwt. **17.** 56·7 lbs. per sq. in.
18. 1188·48 lbs. per sq. in. **20.** 98 ft.
21. 14·7 lbs. per sq. in. **22.** 15 lbs. per sq. in.

Examples XI. (Pages 111, 112.)

2. $10\frac{5}{12}$ lbs. per sq. in. **3.** $31\frac{1}{4}$ lbs. **4.** 2·52 lbs.
5. "Whole pressure" = 65·12 kilog.; thrust on base = 23·408 kilog.
6. 4 ins. water, 5 ins. oil. **7.** See § 112. **8.** 6 ins. **9.** $8\frac{71}{108}$ oz.

EXAMPLES XII. (PAGES 121–123.)

1. 1250 lbs., $1312\frac{1}{2}$ lbs. **2.** 1 : 4. **3.** 3 : 2.
4. $\frac{22}{21}$ kilog. **5.** No. **6.** 703,125. **7.** 5·441 tons.
8. 447·3 tons, nearly. **9.** 2500 oz., 3000 oz., 3500 oz.
10. $32{,}266\frac{2}{3}$ tons, 484,000 tons. **11.** $104\frac{1}{6}$ oz., $166\frac{2}{3}$ oz., $229\frac{1}{6}$ oz.
12. $31\frac{1}{4}$ lbs.
13. $h : h - \frac{b}{2} : h + \frac{b}{2}$, where h is the given depth and b the length of the other edge.
14. $\frac{1}{2}$ and $\frac{7}{22}$ of the wt. of water in hemisphere, respectively. **15.** 21·82 oz.
16. 21·82 oz. **17.** $w(a_1 z_1 + a_2 z_2 + \ldots)$. **19.** $118\frac{3}{14}$ lbs.
20. $93\frac{3}{4}$ tons. **21.** $43\frac{41}{63}$ lbs., $9\frac{397}{567}$ lbs. **22.** Half-way down.

EXAMINATION PAPER V. (PAGE 124.)

1. See §§ 78, 91. **2.** See § 92. **3.** 7500, 3,584,000, approx.
4. See § 97. **5.** See §§ 116, 117.
6. $407\frac{11}{27}$ oz., $425\frac{25}{27}$ oz. **7.** 27,623 tons, nearly.
8. Thrust on top face = $159\frac{3}{8}$ lbs., on bottom face = $265\frac{5}{8}$ lbs., on each side face = $212\frac{1}{2}$ lbs.
9. $25\frac{25}{54}$ lbs. **10.** See § 101.

EXAMPLES XIII. (PAGES 131–135.)

2. $V(1-s)$; increased. **3.** $V(s-1)$; increased.
4. 5 : 52; lead; $\frac{47}{57}$ of weight of lead. **5.** 31 gm.
6. 125 gm. **7.** 46 lbs. **8.** 216 cub. in., 108 cub. in.
9. 137 : 134. **10.** 2131·3 gm. **11.** Edge = 28·8 in.
13. $\frac{7}{10}$ area immersed. **14.** 7·4 oz. nearly. **15.** $26W$.
17. The wood will rise, as it now displaces oil instead of air.
18. Volumes 10 : 3; weights 440 : 171.
19. 3 parts in oil, 1 part in mercury. **22.** ·72.
23. 6·25 cm. **24.** ·25 cm. **25.** 1 : 2. **26.** $\frac{1}{2}$ vol.
27. 3·71. **29.** 8·15 nearly. **30.** $\frac{4}{7}$ in ether, $\frac{3}{7}$ in water.
31. The scale-pan on which the vessel is placed will go down, for the level of the water is raised, and consequently the pressure on the base is increased.
32. 252·65 grs. **33.** 500 c.c., 500·25 c.c., nearly.

Examination Paper VI. (Page 136.)

1. See § 115. **2.** $500\sqrt{3}$ oz. **3.** 4·62 kilog., 3·542 kilog.
4. 66·4 gm., 2556·4 gm. **5.** 5033·6 gm.; 14·3 gm. per sq. cm.
6. See § 120. **7.** $\frac{5}{18}$. **8.** 200 c.c. **9.** ·9. **10.** 152 gm.

Examples XIV. (Pages 149, 150.)

1. ·0013; No. **2.** 45,004·5 litres. **3.** 1155 lbs. **6.** No.
7. 33·408 ft. **8.** 13·281 and 15·003 lbs. per sq. in.
10. $29\cdot7\dot{4}8\dot{1}$ in.
11. 425 poundals per sq. in., 800,496 dynes per sq. cm.
12. 6,045,228 dynes per sq. cm.; 6 : 1 nearly.
14. $1\frac{1}{2}$ in. rise. **15.** $\frac{2}{3}$ in. **16.** 5·366 miles.

Examples XV. (Pages 159–162.)

1. 1·122 kilog. **2.** 4 in., 36 in. **3.** $\frac{1}{2}$ atmosphere.
5. 68 in. **6.** Enough to fill 12 in. of tube.
7. Enough to fill 46 in. of tube. **8.** 63 cm. if section be 1 sq. cm.
10. The mercury rises $1\frac{1}{4}$ cm. **11.** ·01 c.c.
12. ·0006 cub. in. **13.** ·163 cub. in. **14.** $14\frac{4}{7}$ ft. **16.** 4 c.c.
19. ·078 lb. **20.** 1742 metres, nearly. **21.** 337 : 342.
22. Density of air increases, weight of the body in air decreases, the string contracts.
23. ·0024 less of its edge immersed. **26.** $\frac{8}{305}$.

Examination Paper VII. (Page 163.)

1. See § 135. **2.** ·00119. **3.** 1260 gm.
4. See § 151. **5.** 10 lbs. $1\frac{9}{31}$ oz. **6.** 40 lbs. to the sq. in.
8. 1·025. **9.** $\frac{3}{32}$ sq. in. **10.** ·65 in.

Examples XVI. (Pages 171, 172.)

1. $34\cdot28\dot{3}$ ft.
3. The highest point must be less than $21\frac{1}{4}$ ft. above the level of the liquid in the vessel.
4. $3\frac{29}{32}$ lbs. per sq. in. **5.** 25 ft. **6.** $9\cdot6\dot{3}$ ft.
7. 750 cub. ft. **8.** 102 ft. **9.** 99·61 ft.
10. 400 cub. ft. **11.** 2 c.c.
12. (i.) Air rushes out. (ii.) No change. (iii.) Water rushes in.
13. $\frac{68}{109}$. **14.** 1120 ft. **15.** 5·19 ft.
17. In water; in the latter case.

EXAMPLES XVII. (PAGES 180, 181).

1. The atmospheric pressure, forcing the water up, is less.
2. 48 ft.-lbs. **3.** 33·074 ft. **4.** 33 ft. 4 in. **5.** 42 ft. 1 in.
6. 4 ft. **7.** 2 ft. 10 in. **8.** $2\frac{82}{85}$ ft. **9.** No.
10. 173·61 lbs. **13.** An air-vessel; see § 179. **14.** 17 ft.

EXAMINATION PAPER VIII. (PAGE 182.)

1. See § 163; the vertical height of the highest point of the siphon above the level of the water must be less than the height of the water barometer.
2. (i.) and (ii.) The liquid in the two branches flows in opposite directions from the point at which the hole is made; the siphon empties itself and ceases to work.
3. 5 ft. **4.** See §§ 164–169. **5.** See § 170.
6. Air will escape through the hole, and water will rise in the bell to the level of the hole. **7.** $441\frac{3}{17}$ cub. ft.
8. Air will escape through the siphon and water will rise in the bell.
9. See §§ 172–175. **10.** 75 ft.-lbs.

EXAMPLES XVIII. (PAGES 193–195.)

1. 8. **2.** 1·6. **3.** 27. **5.** 6. **6.** ·5398. **7.** 11.
8. About 59 grs. **10.** 4·37 lbs. per sq. in. **11.** 153 mm.; 217 mm.
12. ·02 in. of mercury. **13.** ·15 in. of mercury.
16. The increase of pressure diminishes the volume of the air in the diver, and therefore the weight of the fluid displaced diminishes; the diver therefore sinks.
17. 2902 lbs. When the barometer is high.

EXAMINATION PAPER IX. (PAGE 196.)

1. See § 182. **2.** 24. **3.** See § 187. **4.** See § 188. **5.** $\frac{1}{8}$.
6. The body sinks in the water as the air is exhausted; if the volume of the body (*e.g.*, a vertical cylinder) be graduated, the density of the air it displaces can be calculated from the respective volumes immersed in water and air.
7. 4·23 in. **9.** 10 in. **10.** See § 194.

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4. *Hints and Answers to Students reading for London University.*
5. *Answers to Correspondents on all University Matters.*
6. *Papers set at London Examinations.*
7. *Full Solutions to Matriculation Papers.*
8. *Pass Lists of London University Examinations.*
9. *Calendar of London University Events.*
10. *Latest University News.*
11. *Test Papers (with Answers) for London Matriculation.*
12. *Articles on Special Subjects for London University Examinations.*
13. *A Series of Articles on the Universities of the United Kingdom.*
14. *Ladies' Column.*
15. *Reviews of Current Educational Literature.*
16. *List of Educational Books published during the month.*