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Principal Investigator: G. D. Mostow

The research performed under this contract dealt for the most part with two facets of the theory of Lie groups which involve distinctively different methods of analysis. On the one hand, problems relating to the representations of a Lie group by linear transformations of a linear space, on the other, problems relating to the operations of a Lie group as a group of topological transformations on a manifold.

Since a chronological account of work on the contract would entail constant shifting from one of these facets to the other, it is clearer to trace the progress on each facet separately.

There were several problems of the first type that were investigated. Of these investigations, the most successful ones were related to the ring of representative functions of a Lie group. These investigations were carried out jointly with G. Hochschild for the most part and are described in the following published articles:

1. "Representations and representative functions of Lie groups", Annals of Mathematics, vol. 66 (1957), pp. 495-542.
2. "Extensions of representations of Lie groups and Lie algebras I", American Journal of Mathematics, vol. 79 (1957), pp. 924-942.
3. "Extensions of representations of Lie groups II", American Journal of Mathematics, vol. 80 (1958), pp. 331-347.
4. "Representations and representative functions of Lie groups II", Annals of Mathematics, (to appear).

[Articles 1, 2, and 4 were jointly written; article 3 was written by G. D. Mostow alone].

Article 1 investigates the extent to which Tannaka's Duality Theorem is valid for general Lie groups. In order to generalize Tannaka's theorem for compact Lie groups to the non-compact case, one has first to reformulate the theorem suitably. This required introducing the notion of the <u>universal complexification</u> U(G), of a Lie group G, and in addition the group A(G) of <u>proper</u> <u>automorphisms</u> of the ring of representative functions R(G). The generalization of Tannaka's duality is the equation A(G) = U(G). The principal theorem of 1 is that Tannaka's duality holds for a Lie group with a finite number of connected components if and only if R(G) is a finitely generated ring. We investigate the condition that R(G) be finitely generated, and we find simple necessary and sufficient conditions for its validity.

In article 4 we introduce the group of <u>perfect automorphisms</u> P(G) of R(G). We prove that P(G) = U(G) if G is connected. This result is a step further towards a duality theorem for connected Lie groups.

In article 2 we use the methods introduced in 1 to obtain simple proofs of all the standard theorems on faithful representations of Lie groups and Lie algebras. In article 3 the author gives necessary and sufficient conditions that a representation of a normal analytic subgroup of an analytic group G be extendable to a representation of G. The methods in 3 lean heavily on the theory of algebraic groups.

Among the problems remaining to be investigated are:

a. What is the algebraic structure of R(G)? If G has a faithful representation as an algebraic group, the answer is given in article 4.

b. Introduce a dual object to G such that Tannaka duality becomes a genuine duality analogous to Pontriagin duality for abelian groups. One can find a "messy" solution easily enough. But some of the results in article 4 hint at the existence of an aesthetic solution.

Turning now to the second facet of the contract research, we find an area in which the problems are topological in nature rather than algebraic. The publications resulting from this work include

5. "Equivariant embeddings in Euclidean space", Annals of Mathematics, vol. 65 (1957), pp. 432-446.
6. "On a conjecture of Montgomery", Annals of Mathematics, vol. 65 (1957), pp. 513-516.
7. (with Deane Montgomery) "On the action of toroid groups on Euclidean space", Illinois Journal of Mathematics (to appear).

In addition, the principal investigator has in the last two months obtained some results on the action of compact Lie groups on Euclidean spaces which will result in publications at a later date.

Article 5 investigates the problem of linearizing the action of a compact Lie group of transformations; it gives a necessary and sufficient condition that a compact Lie group of transformations have a faithful representation as a group of orthogonal transformations operating on a subset of Eculidean space. Thus, the main result can be described as a Peter-Weyl theorem for compact transformation groups.

Article 6 solves one of the problems listed in Eilenberg's article in the Annals of Mathematics, vol. 50 (1949), "On the problems of topology". It establishes that a compact Lie group operating on a compact manifold has at most a finite number of non-conjugate isotropy subgroups.

Article 7 investigates the action of an r-dimensional toroid T^r on a space E^n which resembles Euclidean space homologically. If T^r operates effectively, then $n \leq 2r$. We investigate in greater detail the case that $n \leq 2r + 1$, proving that the action resembles linear action completely, as far as isotropy subgroups are concerned.

In addition to the two foregoing projects, there have been other investigations. One of these, on the topology of solvmanifolds, is being carried out by a research assistant and it is expected that it will form his doctoral dissertation.

Respectfully submitted,

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