JOHNS HOPKINS UNIVERSITY CIRCULARS.

Published with the approbation of the Board of Trustees.

No. 15.] BALTIMORE, MAY, 1882. [PRICE 5 CENTS.

CALENDAR, 1881–82.

September 20.	**Current Academic Year Began.**
June 5—8.	**Matriculation Examinations.**
June 9.	**Term of Instruction Closes.**
September 19.	**Next Academic Year Begins.**
September 20—23.	**Matriculation Examinations.**
September 26.	**Instructions Resumed.**

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B. I. Gilman, Secretary.

Historical and Political Science. Third Friday of each month, at 8 P. M.

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Mathematical. Third Wednesday of each month, at 8 P. M.

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PROCEEDINGS OF UNIVERSITY SOCIETIES.

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January meeting.

Smoke Ring Phenomena, by R. W. Prentiss.

These phenomena take place in what are called smoke or vortex rings, the particles of smoke showing the vortex motion of the air in which they are suspended. For the purposes of experiment and illustration they are generally made by generating smoke in a box and driving it out suddenly through a circular aperture in the side of the box; they are also familiar as the rings produced by smokers. It is well-known that when one of these rings is projected into the air it moves forward with a gradually decreasing velocity, slowly increasing in diameter until it is finally destroyed by friction and currents of air. A mathematical investigation of the motion of the air outside the ring shows that it is flowing through and about the ring in the direction of the motion of the ring. So that if two rings are formed having a common axis of projection and very nearly equal velocities, the one behind moving a little the faster, the second ring will pass through the first and then the first will pass through the second, &c., &c.; the rings changing position throughout the motion. One ring revolves about the other. It is, however, generally difficult to realize this phenomenon in experiment. When one ring is projected directly after another it slowly increases in velocity and decreases in radius until quite near the first ring when it suddenly and rapidly increases in velocity, passes through the first ring and then immediately expands to allow the first to pass through it, and this ring goes through a similar course. Produced in this way, the revolution is quite violent, and the rings are generally broken up and destroyed after only two or three revolutions. The motion, however, approaches uniformity as the velocities and circumstances under which the rings are produced are more nearly alike. One of the singular phenomena to which attention was called is the production of two rings, one smaller than the other, by the same impulse, the one revolving about the other with quite uniform motion, in accordance with the principles for the motion of two rings.

But the rings are not generally of equal density throughout, the smoke accumulates in the lower parts and the motion becomes unequal, giving rise to the second phenomenon. As the motion ceases, the dense portion of the smaller ring lags behind and falls down in a loop, while the upper portion continues its motion about the main ring, winding itself about it. The loop is drawn out into two long filaments, while the bottom of the loop separates into two strands or rolls of smoke connected by a thin sheet of smoke which finally disappears. These two strands together assume a circular shape, while the main filaments fall down in two loops through this ring, forming altogether a delicate and well defined figure, somewhat similar to the outline of a basket. This phenomenon also takes place when very dense single rings are formed; successive portions of the ring fall down in loops, each of which finally assumes the figure described above. These facts, capable of observation, seem to indicate that the whole ring is a sheet of smoke, made up of a series of still smaller successive rings. It is, in shape, like the surface generated by a plane spiral revolving about an axis in its plane. Each point of such a spiral would describe a circle which would be the circular axis of one of these constituent rings. If at any time this surface separates into two parts along one of the rings, each part keeps up the spiral motion, folding up (as a coat sleeve is folded up) into a separate ring similar to the original; and we have the case of the two rings. If the separation takes place along the lower part of one of the rings, we have the second phenomenon described above. The separated part folds up into a half ring or loop, giving rise to the *basket* outline, while the upper portion continues its motion with the main part of the ring. A quite distinct phenomenon results when a dense portion of the ring falls rapidly down. The resistance and friction of the air produces vortex motion in the outer part of the mass which gradually results in the production of a complete vortex ring.

On a Logical Problem connected with Assurances on Joint Lives, by J. J. Sylvester and F. Franklin.

Dr. Franklin stated, at the request of Professor Sylvester, some formulae obtained by the latter relating to the enumeration of objects grouped in classes, and showed how these formulae may be obtained by the rules of symbolic logic. The determination of the totient of a number, and the deduction of the values of survivorship annuities from tables of joint-life annuities, are instances of the application of Professor Sylvester's formulae.

Salpa and the Discovery of Alternations of Generations, by W. K. Brooks.

"Some years ago I became convinced, from the study of living specimens of Salpa, that the two forms, the chain salpa and the solitary salpa, are male and female, and that the single egg which each chain salpa contains at birth, passes into its body before birth, from the ovary of the solitary salpa, which is therefore the true female. I then asserted that salpa is not an example of alternation of sexual with sexless generations, and that Chamisso discovered a great natural law through an error. As my conclusion has never been accepted by other naturalists I have, for some years, watched for an opportunity to review the subject, and Professor Baird has afforded me the desired opportunity this winter, by furnishing me a supply of perfectly preserved specimens.

"The study of these specimens has afforded a complete confirmation of my view, as I have been able to procure a complete series of microscopic sections, showing the eggs at all stages, from the time when they originate in the ovary of the solitary salpa to the time of impregnation in the body of the male nurse."

February meeting.

On the New Dividing Engine, by H. A. Rowland.

On the Oxidation of Di-ethyl-benzol Sulphamide, by W. A. Noyes.

It was found that when para-di-ethyl-benzol sulphamide is oxidized with potassium pyrochromate, sulphuric acid and water, a mono-basic acid containing one less carbon atom is obtained. The second ethyl group is neither oxidized nor broken down to a methyl group.

The presumption that the unoxidized ethyl group is in the ortho position to the sulphamide group is very strong, both because, in compounds containing two methyl groups with a "negative" atom or group, it is always the ortho methyl which is protected, and because in all known cases where we have sulphamide and carboxyl groups in the ortho position they form a sulphinide. This acid is not a sulphinide.

This case is of especial interest as showing that the law, stated by Professor Remsen several years ago, with regard to the oxidation of benzol derivatives, may hold true when a larger group than methyl is to be protected.

On the Effects of a Gradual Rise of Temperature on Reflex Actions in the Frog, by W. T. Sedgwick.

An abstract of this paper is given on page 180 of *University Circular, No.* 13.

March meeting.

On the Geometrical Forms called Trees, by Prof. A. Cayley.

The object was to recall attention to the remarkable theorem (due as the author believes to Prof. Sylvester or M. Camille Jordan) that every tree has in two different senses of the word a centre or a bicentre, viz: a centre or a bicentre of length, and a centre or a bicentre of number. *First*, we consider the length of the several branches which proceed from any knot (the length between consecutive knots being taken as unity)—the theorem is that there is always a knot (and one knot only) from which proceed two or more longest branches; or else a pair (and only one pair) of adjacent knots such that from the two knots proceed two or more longest branches, one at least from each of the two knots; we have thus a centre or a bicentre of length. *Secondly*, we consider the number of knots upon the several branches which proceed from any knot; if for any knot these numbers are for instance $a, \beta, \gamma, \delta, \varepsilon$ then, 1° every sum $\beta+\gamma+\delta+\varepsilon$ of all but one of these numbers is at least $=$ remaining number a; or else, 2° there is a term a which is $=\beta+\gamma+\delta+\varepsilon+1$; or else 3° there is a term a which is greater than $\beta+\gamma+\delta+\varepsilon+1$. The theorem is that there is always one (and only one) knot for which the numbers on the several branches proceeding from it, are as in 1°, or else there are two (and only two) knots (and these a pair of adjacent knots) such that for each the numbers on the several branches proceeding from it are as in 2°. We have thus a centre or a bicentre of number.

In connexion with the reference to his name in the above Prof. Sylvester stated that to M. Camille Jordan was due the credit of being the first to discover the existence of the centre or centre-pair of each kind described in the above note. In entire ignorance of M. Jordan's work he rediscovered for himself the centre or centre-pair of the first kind, and was the first to make use of the method immediately flowing therefrom to solve the problem of finding the forms and the number of tree-graphs* corresponding to a hydro-carbon or hypothetical hydro-boron series with a given number of carbon atoms. His results, which he communicated from time to time to Prof. Tait, of Edinburgh, were however as regards the ascertainment of the number of such graphs, purely arithmetical, but giving all the different forms of the so called trees or (more properly speaking) ramifications for different values of the number of atoms up to a certain arithmetical limit. The problem was subsequently taken up from this point by Prof. Cayley, who obtained general generating-function formulæ for effecting the denumeration of the graphs. Mr. Sylvester then proceeded to explain his method of arriving at the first kind of centre or centre-pair of any given tree or ramification.

To this end he supposes all the terminal branches of the tree removed. A tree with a less number of nodes is thus brought into evidence which is subjected (if possible) to like treatment and so a third tree with still fewer nodes is arrived at. As this process cannot be indefinitely continued (for if so a finite number could be continually diminished) we must at length come to a tree or ramification whose terminal branches cannot be removed without leaving nothing in the form of a tree remaining. So long as not less than three nodes remain, since they must not form a triangle, for that would be inconsistent with their appertaining to a ramification, the process of lopping off terminals cannot be brought to a close. Eventually, therefore, this process must lead to a system of branches all radiating out from a single point, or which being removed, only an isolated point remains, or else to a sort of double-headed mop or broom consisting of two such radiating systems stuck into the two ends of an axis. This is the case of bicentric or axial, the former of a monocentric ramification. Thus every ramification may be said to belong either to a central or an axial class. He concluded with suggesting that some general chemical or physical property or set of such properties might reasonably be supposed to exist serving to distinguish between these two classes or genera in the case of the well developed series of the hydro-carbons.

On the Influence of Changes of Arterial Pressure upon the Pulse Rate in the Frog and the Terrapin, by W. H. HOWELL and M. WARFIELD.

The paper gave an account of some experiments upon the isolated heart of the frog and the terrapin, similar to those carried out by Prof. Martin upon the isolated mammalian heart.

The method used in isolating the heart was to ligature all the great veins and arteries leading to and from the heart, with the exception of the vena cava inferior and one of the aortas; the former of these was connected with Marriotte's flasks containing defibrinated calf's blood diluted with an equal bulk of sodium chloride solution 0.6 per cent., which served to nourish the heart, while the latter was connected with a manometer which recorded the heart beats, and with an outflow tube, by raising or lowering which arterial pressure was varied.

All central nervous influence was removed from the heart by cutting the vagi and the sympathetics, and pithing the cervical spinal cord.

A new water manometer was described, which gives much more accurate tracings than the ordinary mercury manometer, and registers smaller variations of pressure. As the general result of the experiments, it was stated that variation of arterial pressure up to the highest point of normal blood pressure, has no direct effect whatever upon the pulse rate of the isolated heart of the frog or of the terrapin; confirming for these hearts the results obtained by Prof. Martin from the isolated mammalian heart.

On the Nature of the Molecule of Ozone, by IRA REMSEN.

Some recent experiments have shown that nascent oxygen has the power to oxidize carbon monoxide to the dioxide. Ozone has not this power. Now when ozone is transformed into oxygen by heat, free atoms of oxygen or nascent oxygen ought to be formed, if the commonly accepted ideas regarding the nature of the ozone molecule are correct. Experiments recently performed under the speaker's direction by Mr. E. H. Keiser, show that when carbon monoxide and ozone are heated together until all the latter is transformed into oxygen, the former is not oxidized. This result cannot readily be explained. A full discussion of the subject will be given in the *American Chemical Journal* for April.

April meeting.

On the 8-Square Imaginaries, by PROF. A. CAYLEY.

These are the system composed of unity and seven imaginaries (analogous to the 1, i, j, k of quaternions) and connected with the 8-square formula in the same way that the quaternion symbols are connected with the 4-square formula (or formula for the expression, as a sum of four squares, of the product of two sums each of four squares). It is shown that the new imaginaries are of necessity non-associative. As regards the quaternion symbols the definition of the relations between the symbols has reference to the only triad ijk which can be formed with the symbols—in the case in hand the definition depends upon 7 out of the 35 triads; and if for instance i, j, m belong to any one of the remaining 28 triads, then these three symbols are non-associative, but are such that $i.jm = -ij.m$. The paper will be published in the *American Journal of Mathematics.*

With reference to the above communication PROF. SYLVESTER referred to the general question of representing the product of sums of two, four or eight squares under the form of a like sum, and mentioned that Prof. Cayley had been the first to demonstrate, by an exhaustive investigation, the impossibility of extending the law applicable to 2, 4 and 8 to the case of 16 squares. The new kind of so-called imaginaries referred to by Prof. Cayley are, as far as Mr. Sylvester is aware, the first example of the introduction into Analysis of locative symbols not subject to the strict law of association, and he considers the law regulating the connexion of the two products represented by a succession of three such symbols, most interesting, inasmuch as such products are either identical, or if not identical, of the same absolute value, but with contrary signs: most persons, before this example had been brought forward, would have felt inclined to doubt the possibility of locative symbols (*vulgo* imaginary quantities)* whose multiplication table should give results inconsistent with the common associative law, being capable of forming the groundwork of any real accession to algebraical science—the results of Prof. Cayley referred to above, seem to show that such doubts are open to question. Mr. Sylvester mentioned as bearing upon the subject of so-called imaginary quantities, that in his recent researches in Multiple Algebra he had come upon a system of Nonions, the exact analogues of the Hamiltonian Quaternions and like them capable of being represented by square matrices. Mr. Charles S. Peirce, it should be stated, had to the certain knowledge of Mr. Sylvester arrived at the same result many years ago in connexion with his theory of the *logic of relatives;* but whether this result had been published by Mr. Peirce, he was unable to say.†

On Concave Gratings, by H. A. ROWLAND.

On Animal Polymorphism, by E. B. WILSON.

This paper gave a brief discussion of the nature of polymorphism in animal colonies, with especial reference to the so-called polymorphism of the Pennatulacea. In studying the development of the colony in *Renilla*, it was found that the peculiar and characteristic mode of budding shown by the sexual polyps is characteristic also of the rudimentary polyps or "zooids." There is a manifest relation between this mode of budding in the sexual polyps and the environment of the organism, and in view of the structure of other Pennatulacea, we seem to be justified in the conclusion, in accordance with the prevailing views of symmetry, that the mode of budding in the sexual polyps is directly dependent on the relation of the organism to its environment. If this conclusion is well-founded, then it follows with considerable probability that the rudimentary zooids cannot have acquired their present mode of budding in their present position, for they agree with the sexual polyps in the law of budding, but differ widely from them in their relation to the environment. And furthermore it is impossible to conceive how the zooids can ever have occupied such a position as to agree with the sexual polyps in this relation.

From these considerations it seems probable that the zooids are not degenerated polyps but are new formations which have inherited certain peculiarities from the sexual polyps. It is immaterial whether we call them organs which simulate individuals, or individuals in a state of arrested development; in either case the various members of the

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*In accordance with the nomenclature employed above, the writer uses here occasionally the word *tree*, but considers his original word *ramification* more correct. A tree is a ramification with one point fixed as a root or origin, and no such fixed origin is supposed to exist in the graphs in question.

* Using θ, h, t, u to denote thousands, hundreds, units, tens, the year of grace in which we live may be represented by $\theta + 8h + 8t + 2u$, θ, h, t, u, being locative symbols which it would be absurd to style *imaginary quantities;* but they are as much entitled to that name as the i, j, k, or any like set of symbols—the only essential difference being that the one set of symbols is limited, the other unlimited in number—and accordingly the law of combination of the one set is given by a finite and of the other set by an infinite *multiplication table.* We might mark off the specific difference between the two cases, by defining the latter set as *unlimited*, the former as *recurrent* or *periodic* locatives or locators; the *locatives* indicate out of what *basket*, so to say, the *quantities* appearing in an analytical expression are to be selected—the multiplication table determines the basket into which their product is to be thrown. Under a purely analytical point of view this is all that is wanted—but in the application of quaternions to problems in nature, it becomes necessary to give special significance to the baskets or rubrics (which would do as well) to which the quantities belong and understand them to signify that certain geometrical processes of *setting* are to be performed.

The true analytical theory of quaternions has nothing to do with this setting part of the business, and regards quaternions as matrices of the second order of a certain determinate form, and accordingly the whole analytical side of the theory of quaternions merges into a particular case of the general theory of *Multiple Algebra.*

As far as the present writer is aware, Professor Cayley in his memoir on Matrices, (Phil. Trans., 1858), was the first to recognize the parallelism between quaternions and matrices, but the idea and method of effecting their complete identification is due to the late Prof. Benjamin Peirce or to his son Mr. C. S. Peirce.

†Mr. C. S. Peirce gave a form of this Algebra in a paper "On a Notation for the Logic of Relatives," published in 1870. The class of Associative Algebras to which this belongs were termed *quadrates* by the late Professor Clifford. [*Communicated to the Editor by Mr. Peirce.*]

colony are not of morphological equivalence, that is, they are not the *direct* descendants of like individuals. This suggests that in such organisms as the Siphonophora, a similar condition may exist, some of the members being the direct descendants phylogenetically of fully-developed buds, while others have arisen *de novo* and are to be regarded morphologically as organs or as imperfectly developed buds. This view would harmonize the conflicting theories of Leuckart, Haeckel, Gegenbaur and others on the one side, and of Huxley and Metschnikoff on the other.

Philological Association.

February meeting.

On Plato's Republic, vi, 20, 21, by Professor W. W. Goodwin, of Harvard University.

The object of this paper was to show that in the illustration of the doubly bisected line Plato meant to give a division of the universe into four parts, as Plutarch assumes (*Quaest. Platon. iii*, 1). The lower main division, representing the visible world, has (*a*) a larger section containing *animals, plants, things, etc.*, and (*b*) a small section containing *images, shadows, and reflections* of these things, etc. The upper main division, representing the intellectual world, has two corresponding sections, (*a*) a larger one of *pure ideas*, and (*b*) a smaller one where certain mathematical sciences (especially geometry) appear, in which visible forms (*e. g.* triangles and spheres) are used to help the mind attain to the pure ideal forms which are the proper objects of study even in these sciences, and in which, moreover, certain assumptions are made which would be out of place in sciences which deal with pure ideas.

The shadows and reflections in the lower section of the visible world are the indirect representations of objects of sense, which ordinary men find easier to contemplate than the objects themselves, as they view the eclipsed sun by its reflection in water. They also symbolize the indirect manner in which most men look at things "in a mirror darkly" rather than "face to face." These reflections correspond to the indirect representations of mathematical ideas in visible forms in the lower section of the intellectual world, and also to the "shadows of images of things" which are first seen by the chained prisoners in the Cave. So the unchained prisoner, on first emerging from the Cave into our world (which to him is an intellectual world), while his sight is still dim, turns first to the shadows and reflections of sensible objects before he attempts to look at the objects themselves; just as a man who is striving to see the ideal forms of the intellectual world first studies those mathematical sciences (like geometry) in which he can use visible images to help his weak mental vision.

The position of the shadows and reflections in the lower section of the visible world, which has often been questioned, thus seems justified in accordance with the rest of the illustration and with the figure of the Cave.

On the Conditional Sentence in Pindar, by B. L. Gildersleeve.

After a brief statement and justification of the terminology employed in designating the four chief forms of the conditional sentence, it was shown that the *logical conditions* in Pindar far outnumber all the others put together. Occasionally generic, the logical condition even then has almost always in view a particular illustration of the principle involved. The notion is that of a type rather than that of a class. The Pindaric *anticipatory* (subjunctive) *condition* appears only in the older form εἰ and only in the generic sense. In the Pindaric *ideal condition* (εἰ w. opt.) there is a marked leaning to the wishing, day-dreaming origin. There are very few *unreal conditions* (εἰ w. ind. in protasis followed by indic. with κε). Some of the half-articulated conditional sentences illustrate the growth of the construction by parataxis. Noteworthy is the narrowing of epic license. The simple, clear, cold Pindaric condition forms a marked contrast to the manifold coloring and shading of the fluctuating Homeric hypothesis, which is all things to all circumstances. All this Homeric variety is replaced in Pindar by severe and simple lines in exact accordance with the difference of genius and sphere of work.

On *οὐ μή*, by C. D. Morris.

In this paper, the origin of the use of the combined negative *οὐ μή* with the future as a form of prohibition was discussed. It was suggested that the explanation of it was probably to be found in such lines as Soph. Ai. 75, *οὐ σῖγ' ἀνέξει μηδὲ δειλίαν ἀρεῖ*; in this the former half of the line contains a positive command, expressed by the second person of the future taken interrogatively with *οὐ*: and it was shown that the mandatory sense inherent in such expressions is due to the fact that *οὐ* when used in questions assumes that an affirmative answer will be given. Since the force of *μή* thus used interrogatively is equally recognized as containing an anticipation of a negative reply, it was urged that this might just as well convey the notion of a prohibition when used with the second person of the future. Instances were quoted in which this appeared to be the effect of *μή* with such verbal forms, even when not following an affirmative command, as in the line in the Aias. But if *μή* alone with the future suffices to convey a prohibition, what is to be said of the frequent occurrences of such expressions as *οὐ μὴ λαλήσεις*; where we seem to have the combined negative in the same sense? It was suggested that the *οὐ* in such cases is to be regarded as a *free* or independent negative, not pointing forward, but indicating the attitude of the speaker's mind, as objecting to an existing state of things, as if it were *οὐ τοῦτο πρέπει, οὐ χρὴ τοῦτο ποιεῖν*. It was then shown by several examples that *οὐ μή* with the future is only used to prohibit a state of things *already existing or threatened;* and that it must not be regarded as a more emphatic equivalent of *μή* with the imperative. Though the prohibitive force of this expression is thus to be traced to the laws of the interrogative sentence, it was no doubt felt simply as a prohibition; and was then combined with other imperative expressions. in such a way as to render it inexpedient to attempt in all cases to indicate its interrogative character by punctuation or intonation. The notion that *οὐ μή* can be regarded as simply a stronger *οὐ* was rejected not only on the ground of the essential difference of the two negatives, but also because, if that were true, *οὐ μὴ λαλήσεις*; ought to be a more emphatic expression for *οὐ λαλήσεις*; whereas it does mean just the opposite.

March meeting.

On the Appendix Probi, by M. Warren.

This appendix attributed to Probus is found in a *codex Bobiensis* of saec. VII or VIII, now in Vienna. The third part devoted to orthography is the most valuable. The compilation as it stands was probably made in the fourth century, but much of the material may be referred to a still earlier period. Some of the mistakes corrected characterized the folk-spelling of the time of Cato the Censor. Cato wrote *calda* and so did Augustus, not *calida*, as Probus directs. Probus opposed the use of diminutives, the dropping out of the atonic vowel as in *oclus* for *oculus*, assimilation as *grunnio* (Varro) for *grundio*, all of which are tendencies of the folk-speech, and manifest in the Romance languages. *Scoriscus* of which Probus disapproves is really the older form of *coruscus* and contains the same root as Gk. σκαίρω. The form *infiminatus* for *effeminatus* may be due partly to the analogy of *inhumanatus*. Cod. Sangallensis 912, contains the following gloss *effeminat: in femina convertit.* Probus' injunction to write *numquit* not *mimquit*, as well as his definition of the difference between *nigro* and *migro* proves a confusion in the pronunciation of initial *n* and *m* in the folk-speech. So *mappa* became *nappe*, Fren., and *nasturtium mastuerzo*, Span. *Mim* is found, Trin. 921, in the *vetus cod.* of Plautus. (C and D have *min.*) The glosses *nim: ni, nisi, sinon* and *nimquid: non aliquid* were shown to be found in several Berne and Paris MSS. and are to be explained in the same way as Probus' *mimquit, i. e.* as due to phonetic corruption.

On Dialect Peculiarities of the Creole French, by A. M. Elliott.

In this paper the importance of a study of the outlying Romance dialects was urged in part to correct the false notions of the general philologian about their formation, and in part to show us the actual processes of language building—*das werdende* in human speech. The morphological peculiarities of the Creole French, especially in Louisiana and the Island of Mauritius, were stated and in a majority of cases found to have their exact counterparts in some one or other of the older *Langue d'oïl* idioms. The general tendency of these speech varieties is towards greater simplicity in their phonetics and a more strongly marked analytic character in their morphology and word-formation than what we find in the original sister dialects. Aphæresis and syllabic gemination—processes seldom used in the French proper—are here favorite means of arriving at euphony on the one hand and of strengthening the original stem syllable on the other. Of the various grammatical categories the article alone has no longer an independent existence, but is welded to the noun as a prosthetic and thus forming an integral part with it, has no determinative function whatever. The opinions of Egger, Coelho, and Braissac were referred to in regard to the formation of the Creole and were shown to bear rather upon the external phenomena produced by agglutinated speech elements than upon the internal causes which give rise to them.

On Nouns ending in *-tas* and *-tudo* used by Plautus and Terence, by W. C. Thayer.

Gellius, Noct. Att. XIII, 3 having been cited to prove that there was no distinction in meaning between the two forms, the relative frequency of their use in the comic poets was examined and a decided tendency of one form to supplant the other, first in Cicero and then in the later Latin, was shown. It was found that Plautus uses 72 nouns ending in *-tas* and 23 ending in *-tudo*, while Terence has 50 in *-tas* and only 9 in *-tudo*. Then having examined the sets of words used by both authors in common, and those used by one author but not by the other, it was shown that the tendency of nouns in *-tas* to multiply and supplant nouns in *-tudo*, so marked in Terence, was even more evident in Cicero and classic writers and became a prominent characteristic of the late writers, as in the IVth century Latinity of Servius. In conclusion it was found that,

1. Nouns ending in *-tudo* are old, few being coined after a certain epoch, are often *ἅπαξ λεγόμενα* in the language and very seldom become the established classical form; while
2. Nouns ending in *-tas* may be old or newly coined, but when once fairly established in the literature survive and tend to supplant all other forms of abstract nouns from the same root.

April meeting.

The Color System of Vergil, by Professor T. R. PRICE, of the University of Virginia.

This paper, after showing that color takes a far larger place in Roman poetry than in Greek, enumerates and groups, according to the divisions of the spectrum, the 42 color-terms used by Vergil, giving the number of times that each is used, 600 in all. The poet's use of the color-terms, like the painter's use of colors, is designed to express not absolute color, but color as dependent on purity, luminosity, and contrast. From this point of view, Vergil's use of his color-terms is very exact, and full of true and delicate observation. The philological method of determining the significance of any given color-term must consist of five successive processes; we have to fix the etymology of the word, the physical standard of the color, the extension on both sides of the standard, the variation in luminosity and purity, and the variation by contrast. Vergil's law of color is not absolutely impartial; he shows a marked preference for the central colors of the spectrum over the extreme colors, for the warm colors over the cold, and for the more luminous over the less luminous colors. His sense of color is far in advance of Homer's and almost up to ours. His sense of red, red-yellow, yellow, green, and blue is clearly marked by definite terms; and even for violet, although he has no one term, he shows the existence in himself of the color-sensation, and the effort to express it.

On Some New and Rare Words found in Donatus' Commentary to Terence, by M. WARREN.

An examination of the vocabulary of Donatus yielded 47 words not contained in Harper's Dictionary, 31 of which are not given by any of the best current Latin Dictionaries, viz: 10 nouns, 10 adverbs, 6 adjectives and 5 verbs. *Præsupponere*, which occurs Eun. III, 2, 34, and *respective* Eun. III, 2, 8, are given by Allgayer Krebs Antibarbarus (5th Ed. 1876) as "Neulateinisch." *Interloquium*, Eun. II, 2, 23, and *tardiloquium*, Hec. V, 1, 15, are formed after the analogy of *multiloquium*, (Plautus). *Vultuose* is used very frequently by Donatus, and *similitudinarie* occurs And. IV, 5, 19.

Thirteen words which Harper's gives as *ἅπαξ λεγόμενα* are found in Donatus, some of them occurring more than once. Especially important are *sæpiuscule*, Eun. III, 1, 21, and *semigravis*, Eun. IV, 5, 1, "*Hic semigravis inducitur vino*," which supports Livy's use in Bk. XXV, 24, 2, "Aut sopiti vino erant aut *semigraves* potabant," where Lipsius proposed *somno graves*. Finally *occidentales* used as a noun opposed to *orientales*, was cited from Eun. IV, 4, 22.

On the Gṛhyasaṁgraha-pariçiṣṭa of Gobhilaputra, by M. BLOOMFIELD.

This paper represented a notice of the Gṛhyasaṁgraha as edited by the reader of the paper in the *Zeitschrift der Deutschen Morgenländischen Gesellschaft*, Vol. XXXV, pp. 533-87. It was pointed out that the text was a pariçiṣṭa (*παραλιπόμενον*), to Gobhila's gṛhya-sūtra, the only text on house-laws and customs, belonging to the Sāma-Veda, which has hitherto been published. Certain ones of Gobhila's sūtras are explained, others are amplified by Gobhilaputra, the reputed author of the addendum. No fixed order or method is followed in the *παραλιπόμενον*, so that it was often difficult on account of the indistinct meaning of the addenda to determine the sūtras to which they severally belonged.

The material for the publication consisted of three MSS. of the India House in London; two containing the text, and the third the commentary of Dīkṣitarāmakṛṣṇa. In addition to this the Karmapradīpa, another *παραλιπόμενον* to Gobhila, and the commentary to it by Açārka, both from the Chambers collection of Sanskrit MSS. at Berlin, afforded valuable help.

Finally small portions of the text introducing some of the more interesting comments and addenda to the knowledge of Indian house-law were presented, and where possible compared with similar customs elsewhere

Historical and Political Science Association.

February meeting.

What England owes to Protection, by E. W. BEMIS.

England owes her commercial supremacy in no small degree to governmental protection of industries. Until the fourteenth century England supplied only raw materials for the Dutch manufacturers. Edward III, by promising substantial aid and governmental protection, induced Flemish weavers and wool-merchants to emigrate to England. The manufacture of glass and clocks and of many other commodities was introduced in the same way. Early English protection bears a striking resemblance to modern municipal methods of encouraging local manufactures by exemption from taxation, by free use of city-water, and other privileges. It is generally conceded by writers from the fifteenth to the end of the seventeenth century that England owed to protection the successful establishment and rapid growth of all branches of her industry. The Navigation Act of Cromwell contributed largely to the economic greatness of England. The increase in the value of land in England at the time when protection of manufactures was most oppressive, proves that the above policy was not altogether injurious even to the agriculturist. But toward the close of the military and economic struggle with Napoleon, Great Britain had acquired a monopoly of the world's trade. It was then obviously England's interest to absorb the raw materials of nations less advanced in economic development, in order to prevent competition in manufactures. But for a time she failed to do this in consequence of her corn laws and oppressive taxes upon imports, but since the introduction of free trade in 1846 English commerce has quadrupled. History, however, appears to show that in the case of a partially developed national economy, governmental protection of industries may for a long time be productive of good results. The practical point is to discover when protection ceases to protect.

Free Schools in Early Maryland, by L. W. WILHELM.

The idea of free schools was inherited from the mother country and is the product of no one section or individual colony in America. Free schools were established in Maryland before the close of the seventeenth century. The first mention of them is in a bill dated April 13, 1671, which passed one branch of the legislature, but failed to become a law. The first bill for free schools approved by both houses was passed October 18, 1694, by the aid of Governor Francis Nicholson. It provided for the erection of one free school at Annapolis and contemplated the erection of others in various parts of the province. The first school was erected entirely by the aid of voluntary subscriptions, Governor Nicholson contributing liberally and nearly every member of the legislature following his example. The school was continued by revenues arising from special fines and taxes appropriated for its support. In 1696, the colony sought the patronage of King William for a college to be erected at Annapolis, which foundation, called King William's School, was made by authority of the legislature in 1704. In 1723 legislative provision was made for a free school in every county, to be governed by a board of seven visitors. Among the eighty committee-men thus appointed we find Governor Charles Calvert, twelve colonels, and thirteen ministers, the others being planters and merchants of acknowledged integrity and public spirit. These schools were supported by fines and imposts, including duties upon the importation of negroes and the exportation of tobacco. Donations from private individuals were not infrequent. By a statute passed in 1728, provision was made for the education of poor children in each county. The frequent mention of free schools in the statutory law of Maryland attests their popularity and importance in the minds of legislators, who fairly represented the better sense of the colony. Considering how slowly Maryland grew, how isolated her people were (their manor-houses being widely separated), it is remarkable that free schools were so early established. That these schools exercised considerable influence upon the culture of the people is beyond question. Mr. Eddis, in his letters from America in the last century, remarks upon the purity and even elegance of language in Maryland. Of Annapolis, where the College was located, he said, few towns in the British dominions can boast a more polished society. The present prosperity of the school system in Maryland is in no small degree the outgrowth of that first planting two centuries ago by wise legislators and philanthropic citizens.

Free Schools in South Carolina, by B. J. RAMAGE.

The free school system in South Carolina is coeval with the parish and the district. The early legislators of the colony were deeply impressed with the importance of popular education. So dominant was this idea that it had much to do with the division of the counties into parishes, in order to obtain aid from the society, in the mother country, for propagating the Gospel in foreign parts. This society lent its aid in supplying parish ministers and teachers. Parochial libraries were founded and parish schools were established, at first under the immediate control of vestrymen. The history of popular education in South Carolina dates from these parish beginnings. Mayor Courtenay, of Charleston, in his recent address on education in South Carolina, leaves the erroneous impression that free schools were not instituted in that State until 1811, whereas they were really erected in many parishes and districts at a much earlier period. As far back as 1710, a free school was established in Charleston. Similar institutions were planted at Dorchester, Childsbury, Beaufort, Ninety-Six, St. Thomas' Parish, St. James, Santee, and elsewhere. Many of these schools owe their origin to legacies bequeathed them by generous parishioners, like Beresford, Ludlam, Childs, and others. Dorchester was settled by a colony of Congregationalists from Dorchester, Massachusetts. This little colony, the offshoot of the best type of a parish town in New England, emigrated to South Carolina about the beginning of the last century and settled in a place which they called after their old parish-home. These democratic Puritans brought with them their love of popular education, and, through legislative coöperation, established a free school in Dorchester, S. C., in the year 1724, which school was long maintained by its liberal founders. With the growth of primary education, the demand for the higher education increased, resulting in the establishment of no less than five colleges and several academies before the expiration of the eighteenth century. Many charitable societies also maintained schools, usually for the education of the poor. In some cases even slaves were taught to read. The State constantly encouraged education by donations, immunities, and by vesting in school-boards escheated property in villages or parishes. In 1801, the South Carolina College was founded. By the year 1811, the demand for popular education had so increased and the country was being so rapidly settled that a general system of free schools was inaugurated. From this time on there was a steady increase in educational facilities until the outbreak of the civil war.

Early English Military Institutions, by E. R. L. Gould.

Blackstone says Alfred was the first to create a national militia in England. No record remains of the laws of this king which justify such a statement. Indeed there is a great dearth of martial legislation throughout the early English period. The military organization of the Saxons in England can be understood only through a study of their civil institutions, which, in many cases, are evidently the outgrowth of military beginnings. There is a very close connection between the organization of early England and of ancient Germany, as regards civil and military matters. The tithing, hundred, and shire correspond, in certain respects, to the *vicus*, *pagus*, and *civitas*, described in the *Germania* of Tacitus. The *comitatus* of the German local *principes* is seen in the company of thegns attached to a petty English king, ealdorman, or bishop. The body of personal followers was a kind of standing army, independent of the local militia. There are many striking resemblances between the warlike institutions of the early Teutons and of the Vedic peoples of ancient India. The military constitution of early England probably sprang from the martial customs of the Aryan race. There is a perfect historic continuity in the main idea of the militia system, namely, liability of the entire mass of able-bodied men to serve as a *Landsturm*. Occasional military service appears to have been a kind of tax paid by the citizen to the commonwealth, whether national or tribal. In ancient India and Germany, as well as in early England, the leaders of the martial host, which was gathered from local precincts, were also the governing authorities of those precincts in time of peace.

Review of the Situation in Dalmatia, with extracts from MS. letters by Arthur Evans, Ragusa Correspondent of the *Manchester Guardian*, and by Edward A. Freeman, D. C. L., read by H. B. Adams.

March meeting.

Local Government in New York, by E. W. Bemis.

The management of local affairs in New Netherlands was in the hands of village magistrates nominated by the people and confirmed by the Governor. With the influx of immigrants from New England, after the conquest of New Netherlands by the English, in 1664, came important modifications of the Dutch system of local government. In 1684, New York was divided into counties. This English unit of administration still retains an important place in the local autonomy of the above State. The county is composed of towns from thirty to sixty square miles in area. Powers of county legislation are invested in county boards consisting of the supervisors or heads of the several towns forming the larger unit. The county boards issue warrants for the collection of all taxes in support of roads, bridges, the erection of court houses, and other county purposes. Town assessors report to the county authorities. All purely local officers are elected by the towns in town meeting assembled. The towns can make by-laws and establish penalties for local offences; they can regulate their own common schools, common lands, fences, and town property; they can raise money by local taxation for public uses, such as roads and bridges, if more than the county allowance is needed. The distinguishing feature of the New York system of local government, as compared with the New England system, is the great power of the county in control of its towns. New York has a compromise system, combining the best features of the New England town and of the Southern county. From the New York system has developed that of Illinois, Michigan, and Wisconsin.

Local Self-Government in Pennsylvania, by E. R. L. Gould.

In early colonial times, under the administration of the Duke of York, the town or parish was the centre of local life. County government did not practically exist. All town officers were under the management of a constable and a board of overseers, who promulgated constitutions and by-laws and adjudicated petty cases in the "Towne Court." This body also looked after the ecclesiastical interests of the parish. The local administration of this period was essentially a reproduction of the English system. In the early stages of Proprietary rule the town lost its former position and the county became most prominent. The County Court of Quarter Sessions was the centre of authority, and rates were levied, roads opened, bridges constructed, and the poor cared for under county direction. Subsequently the charge of paupers and highways was transferred to the township, on which the overseers and supervisors laid rates to cover expenditure entailed by these burdens.

The Pennsylvania system at the present time is a compromise between the town polity of New England and the county administration of the Southern States. The system aims at a partition of powers. There is no town meeting where the people may come together and frame by-laws for their own government. Neither is the township represented by a supervisor upon the County Board as in New York, Michigan, Illinois, and other of the Northern and North-western States. The township officers have the power to levy a road tax, but only on the basis of the last adjusted county valuation. County assessments are made to defray the ordinary expenses of local government. The poor are usually, though not always, a county charge, and when such is the case are supported from the general county fund. The county is the leading local unit and, under the commonwealth, may be said to wield the largest share of political power. It regulates affairs directly and its officers are responsible to the people for the exercise of administrative control. (This paper was read before the Pennsylvania Historical Society, May 1, 1882, and is now in press).

Local Self-Government in Illinois, by Albert Shaw.

The local institutions of the West are not novelties nor experiments. In main features they reproduce those social arrangements under which the Anglo-Saxon insists upon living, wherever he may be transplanted. Illinois may be taken as the model western State in respect to local government. The precise form which its institutions have assumed has been largely shaped by the State's history. Illinois illustrates the tendency of migration to follow the parallels of latitude. Its southern half was peopled from Southern States, especially Virginia, while men from New England and New York occupied the northern half. The settlers brought their ideas and habits with them. The New England men came from self-governing agricultural communities, where the people made laws, chose officers and voted local taxes in the democratic "town meeting." The New England county was little more than a name; the township was the real political unit. The Virginia settlers, on the other hand, had not been accustomed to organized neighborhood life or to local self-government. In the States of the South, county officers appointed by the State government administered State laws.

Illinois passed from French to English hands in 1763, and was in 1778 made a Virginia county. A decade later it passed under the government established by the Ordinance of 1787, which confirmed forever to the people many great principles. In 1818 it was admitted as a State. Its early connection with Virginia, as well as the temporary government erected under the Ordinance, had already given a southern cast to its administrative forms. The southern half of the State alone being settled at the time of its admission to the Union, the Constitution of 1818, naturally, provided for a county system similar to Virginia's. Southern migration to Illinois declined after the Missouri Compromise of 1820, and Northern people filled up the State. The new Constitution of 1848 compromised with the two systems of local government by providing that counties might, on popular vote, be placed under township organization. The northern counties at once adopted the township system, southern counties remaining under the county system. At the present time about four-fifths of the 102 counties in the State are under township government. This result was materially furthered by the U. S. Land Survey system, which divided the public domain into townships, and allotted land in each township for school purposes.

The annual "town meeting" of the whole voting population is the central fact in township government. At this meeting the people elect local officers, discuss neighborhood interests, vote taxes and enact by-laws and rules covering a wide range of local topics. The Supervisor is the business officer of the township, and the Supervisors of the several townships constitute a County Board which manages county affairs. Highways and schools are provided for by the townships, while the care of the poor is entrusted either to the county or the townships as the people may determine. Each township elects two justices of the peace. The county maintains a judicial establishment and controls matters which concern the townships in common. As in the South, so in Illinois and throughout the West the county is the political unit within the State. General laws provide for the incorporation and municipal government of "villages" and "cities," requiring a minimum population of 300 for the former, and of 1000 for the latter. The compass of this abstract precludes details of county, township, and municipal governments as provided for in the statutes of Illinois.

Administration of the City of Berlin, by RICHARD T. ELY.

Berlin, like other German cities, occupies somewhat the same position with regard to the state government exercising control over it, that the German states do with reference to the Empire or our states with reference to the Federal Government at Washington. But the government of Berlin resembles an American State more closely than it does a German one. It is republican with an aristocratic tendency. The citizens are divided into three classes according to the taxes they pay. The first class includes the highest tax payers. It begins with the citizen paying the highest tax and ends when enough of the largest tax payers are included, so that the sum total of the taxes they pay is equal to one-third of the taxes paid by the citizens of the municipality. The second class comprises in similar manner those who pay the second third of the entire tax. The third class embraces those who are left—the poorest citizens. Each class has the same number of representatives in the city assembly, which is a body corresponding to a state legislature. It consists of 108 members, which ought not to be considered too large to represent properly a city containing over one million inhabitants. The representatives in the municipal assembly are elected for six years, the term of one-third expiring every two years. They receive no pay, as their office is what the Germans call an *Ehrenamt*—a place of honor. One-half of them must possess houses.

The executive branch of the city government is the magistracy, a collegiate body, consisting of thirty-four members. The *Oberbürgermeister* (chief mayor) and *Bürgermeister* (mayor) are at the head of the magistracy. The chief mayor, mayor, and other paid magistrates, whose whole time is consumed by their official duties, are elected for twelve years. The other magistrates are elected for six years, one-half retiring at the expiration of each three years. The members of the magistracy must be confirmed by the King.

The higher officers in Berlin, as in other Prussian cities, must be appointed for life. This is one of the provisions of the municipal law of Prussia. This avoids rings and machine politics. The life appointees have proved faithful and honest in Berlin as in Vienna. This is a better device for getting rid of rings than the one a number of American cities appear disposed to try, namely, that of making the mayor absolute. We have already too many forms of despotism in this country.

The net expenditures of the city government of Berlin in 1876 were considerably less than $7,000,000. One single item in the cost of the municipal administration of New York city for 1881 exceeded this by over $1,000,000. This was the interest on the city debt.

The paper, of which the above is a brief abstract, appeared in *The Nation*, New York, March 23 and 30, 1882.

The Institutions of North Carolina, by HENRY E. SHEPHERD.

No one of the original thirteen States preserved, with more devoted tenacity, the form and the spirit of English institutions than North Carolina. In some instances antiquated usages were retained after their final abolition in England; as for example, benefit of clergy, abolished in England in 1827, but in force in North Carolina until 1855. The North Carolina Bill of Rights, and the Halifax Constitution, Dec., 1776, were largely drawn in language, as well as in spirit, from Magna Charta and the English Bill of Rights. This can be ascertained by the most casual examination of the several documents. The strictest property qualifications were requisite to render eligible to membership in either branch of the Legislature, or to the right of voting for members of the Senate. In accordance with a genuine English tradition, property in land was looked upon as the basis of civic freedom. The popular branch of the general assembly was known as the House of Commons, a simple transfer of the English designation.

British statutes have been reënacted in North Carolina, and the common law repeatedly declared in force. Modes and forms of procedure have been introduced from England, and the decisions of the Supreme Court largely controlled by a strict regard to English precedent. Sir Edward Coke has ever been revered as the oracle of the common law, and American commentators accepted with reserve if not with distrust.

The character of legislation in the State has been in great measure affected by English precedents; as for example, the revolution of 1688. Thus the XXXII Article of the North Carolina Constitution of 1776, imposing disabilities upon Roman Catholics, was inspired by the bitter memories of James II's time. In this same Constitution the sixth section of the Act of Settlement is substantially reënacted. Borough representation prevailed in North Carolina until the adoption of the amended Constitution, 1835, three years later than the passage of the memorable English Reform Bill, which destroyed so many "rotten" boroughs.

Until 1835, the General Assembly met annually, and the Governor was elected annually by the General Assembly. The morbid dread of executive usurpation or encroachment which animated the leading spirits of '76, impelled them to reduce the office of Governor to a mere phantom, destitute of authority or dignity. The reformed Constitutions, adopted in accordance with the Reconstruction Acts, have destroyed much of what was antiquated and venerable in the legal and political system of North Carolina.

Review of Professor C. K. Adams's "Manual of Historical Literature," by H. B. ADAMS.

April meeting.

Markets, Fairs, and the Atlanta Exposition, by B. J. RAMAGE.

The national fair at Atlanta forms an interesting chapter in institutional history. It must be looked upon as the historic outgrowth of old English fairs and markets, which were reproduced in this country at a very early date. In all countries where inter-communication is slow, fairs are a very necessary institution. They are held several times a year at specified places, and are the distributing centres of all slightly advanced societies. "What we know as a fair," says Herbert Spencer, "is the commercial wave in its first form." In ancient times neighboring tribes, in order to avoid any disputes in regard to boundary lines, always left an open, neutral tract, *e. g.*, the *mearc* or *march*, between their different village communities. Here it was safe for merchants to come with their wares, and here also treaties were made. Hence Hermes, who was the god of boundaries, was also the guardian of merchants and ambassadors. The intimate connection between fairs and markets, and the growth of international ideas is thus clearly seen. The Roman *jus gentium*, according to Sir Henry Maine, was the outcome of old market law. The English fairs and markets are of remote origin. A study of ancient Anglo-Saxon law will show the high estimate which early English kings put upon these institutions. No fair or market could exist without a grant or prescriptive right. Two courts of an inferior nature were attached to these fairs and markets. One was the clerk of the market's court, and the other was the court of *pie poudre*. The former, presided over by the clerk of the market, the old Saxon portreeve, had cognizance of various violations of market regulations, such as forestalling, regrating, the use of false weights and measures, etc. The court of *pie poudre*, so called from the dusty feet of the suitors, who usually attended the fairs on foot, was established for securing summary justice, and was presided over by the steward of the lord of the fair. Each lord of the fair had the right to collect toll, which was defined by Edward the Confessor to be "*libertatem emendi et vendendi in terra sua.*" This old toll, which dates far back in early English history, seems also to have at times partaken of the nature of an *earnest* or *partial* payment, and still lives on in many rural districts, where persons entering on a contract are accustomed to exchange a button or a piece of metal as a pledge of good faith. The "Mirrour of Justice," in describing the abhorrence of the Anglo-Saxons for all secret contracts, defines the toll as an expedient for compelling open contracts. The universality of fairs held in all lands, from ancient India to the Aztec cities of Mexico, shows that it is a mistaken view to regard them as of Church origin, though they were considerably influenced by the Church. The latter, however, built itself and its various fairs and festivals upon institutions already existing. These old English fairs and markets, with all their peculiar historic characteristics, were transplanted to the English colonies in America, and were encouraged by statutory law in New York, Massachusetts, Maryland, Virginia, the Carolinas, and elsewhere. With the increased growth of commercial centres and improved means of inter-communication, these old institutions, while losing almost entirely their commercial importance, still live in the various county and state fairs and exhibitions. The same causes and influences which have resulted in the world fairs of Europe, at Paris, London, Dublin, Amsterdam, and other cities, with all the attendant influences upon the growth of international ideas, and the higher development of the commercial *jus gentium*, have also slowly been at work in America. New York and Philadelphia have each had an International Exposition, and Atlanta, the *market* town of the New South, the chosen *march*, where Northern and Southern ideas are interchanging, has just closed a great national fair, which has had many wholesome influences upon all sections of the Union.

Note of the Discovery of a Court Leet upon St. Clement's Manor, in St. Mary's, by JOHN JOHNSON.

No authentic instance of a Court Leet can be found in the written history of Maryland. Even the existence of the institution, either in this province or under the proprietary government of Pennsylvania, is denied by some writers. Recent investigations concerning the manorial system of early Maryland, undertaken in connection with the Seminary of Institutional History by Mr. J. Johnson, a graduate of the University, and now a teacher at McDonogh Institute, who was guided in his researches by Mr. Lee, Librarian of the Maryland Historical Society, and by Mr. G. Johnston, author of the lately published history of Cecil County, have led to the careful examination of certain manorial records, preserved by the above Society, and to the interesting discovery, not only of the existence, but of the actual proceedings of a Court Leet in Maryland, about the middle of the seventeenth century, upon the old Manor of St. Clement's, in the oldest of Maryland Counties, St. Mary's, where the Pilgrims of *Terra Mariae* landed. Like Northern towns and Southern parishes, like constables and tithingmen, the Court Leet (German *Leute*), is a Saxon survival of popular self-government. The Leet represents the popular court of the ancient Hundred, brought under the personal jurisdiction of the lord of a Manor. Indeed, there is some evidence from early Maryland records that the terms Manor and Hundred were used synonymously. For example, in St. Mary's, the "Hundred of Mattapanient" for the "Manor or division of Mattapanient" (Bozman, ii., 98.) "St. Clement's Hundred" was perhaps not very different, institutionally considered, from St. Clement's Manor. The subject of Maryland and Delaware Hundreds is now under investigation by H. B. Adams. Mr. Johnson's paper upon the Manorial Institutions of Maryland, when completed, will be presented to the Maryland Historical Society, in connection with a transcript of the Records of St. Clement, which has lately been made by Mr. Lee.

Montauk and the Common Lands of Easthampton, Long Island, by J. F. Jameson.

This paper related the history of the common lands of the town of Easthampton, at the east end of Long Island, and of the pasture-lands of the peninsula of Montauk, owned by certain proprietors in Easthampton as tenants in common till 1879. An introductory account of the common-land system of the early Germans (embodying some of the results of recent German labors in agrarian history), and of the open-field system in England, whence the custom was transplanted to Easthampton and other towns of New England. The remainder of the paper was founded upon the MS. records of the town of Easthampton and upon MS. papers examined at Montauk. The probable date of the settlement of Easthampton is 1650. At first, each settler received a house lot and an allotment in the great fields near the village. The latter were not owned in common after allotment, but seem to have been "open fields," with perhaps common cultivation. Soon further allotments of large tracts were made, and henceforth to a certain lot in the village belonged a certain parcel in each of these scattered tracts. Lands not yet divided were used for common pasturage. This almost exactly resembles the Germanic *hide*. Gradually, almost all the common lands have been allotted or sold, but some traces of the system still exist. Shares in town commons were reckoned in terms of the nominal "acre," and were owned by the descendants of the first settlers. By successive purchases in 1662, 1670, and 1687, certain of the inhabitants of Easthampton came into possession of the peninsula of Montauk, which for two hundred years was used by the "proprietors of Montauk" as a common pasture, subject to certain curious rights of the Montauk Indians. After the shares in the three purchases were thrown together, owners had rights of pasturage proportioned to their interests, which were reckoned in nominal pounds, shillings, and pence to the last. Many customs followed are curiously like those of Germany, Switzerland, and the Rhine Provinces. The management was generally confided by the body of proprietors to the trustees of the town; after 1852, to a committee, incorporated as the Trustees of Montauk. In 1879 Montauk was sold, and the old institution brought to an end.

An Account, by H. B. Adams, of Governor Bradford's Manuscript History of "Plimoth Plantation," now retained in the Fulham Library, England, with a Review of Justin Winsor's Paper, read before the Massachusetts Historical Society, November 10, 1881.

A Review of the St. Clair Papers and of the Ordinance of 1787 for the Government of the North-western Territory (published in *The Nation*, New York, May 4, 1882), by H. B. Adams.

Metaphysical Club.

February and March meetings.

On the Algebra of Logic, by O. H. Mitchell.

I. Let A and B be two terms, and A^{-1}, B^{-1} their negatives. Let
AB denote "A and B have something in common,"
$(AB)^{-1}$ denote "A and B have nothing in common."

Then I, E, O, A and the four complementary propositions of De Morgan become, respectively,

$$AB,\ (AB)^{-1},\ AB^{-1},\ (AB^{-1})^{-1},\ A^{-1}B^{-1},\ (A^{-1}B^{-1})^{-1},\ A^{-1}B,\ (A^{-1}B)^{-1}.$$

In like manner $ABC\ldots$, $(ABC:\ldots)^{-1}$, etc. denote propositions of more than two terms. If A, B, etc. be propositional terms, the only "something in common" which they can have is time. The two formulae of inference from premises like the above are

$$(AB\ldots Q)^{-1}(Q^{-1}R\ldots Z)^{-1} \prec (AB\ldots PR\ldots Z)^{-1}$$
$$(AB\ldots LM\ldots Q)(M\ldots QR\ldots Z)^{-1} \prec AB\ldots L\,(R\ldots Z)^{-1}$$

where the conclusion is in each case the *algebraic* product of the premises. But by an obvious substitution these reduce to

$$(SM)^{-1}(M^{-1}P)^{-1} \prec (SP)^{-1},$$
$$(SM)(MP)^{-1} \prec SP^{-1}.$$

When M is not eliminated by the process of multiplication, there is no inference from S to P. For instance, the premises of Darapti, Felapton, and Fesapo are, respectively,

$$(MP^{-1})^{-1}(MS^{-1})^{-1},\ (MP)^{-1}(MS^{-1})^{-1},\ (PM)^{-1}(MS^{-1})^{-1},$$

and this notation is seen to accord with the now recognized invalidity of any inference in either case. The premises of Bramantip are $(PM^{-1})^{-1}(MS^{-1})^{-1}$ from which the inference is $(PS^{-1})^{-1}$.

II. Let U denote the Universe of Discourse, and V the Universe of Time, *i. e.* a quantity of time limited or unlimited. Let P be any logical function of class terms, and let

$(P)_{1,1}$ denote "All of U, during all of V, remains P,"
$(P)_{u,1}$ " "Some of U, during all of V, is P,"
$(P)_{u'1}$ " "The same U, during all V, is P,"
$(P)_{1,v}$ " "All of U, during some of V, is P,"
$(P)_{1v'}$ " "All of U, during the same V, is P,"
$(P)_{u,v}$ " "Some of U, during some of V, is P."

From this point of view "A is always B," becomes $(\bar{A}+B)_{1,1}$, *i. e.* "All of U, during all of V, is $\bar{A}+B$," where the negative of a term is now denoted with greater convenience in the ordinary way. "Some A is always B, becomes now $(AB)_{u,1}$." The most general proposition from this point of view is, then, the product of propositions of the form

$$\Sigma\,(P)_{1,1} + \Sigma\,(Q)_{u,1} + \Sigma\,(R)_{1,v} + \Sigma\,(S)_{u,v} + \Sigma\,(T)_{u'1} + \Sigma\,(W)_{1v'},$$

where P, Q, etc., are logical functions of class terms. From such premises inference is obtained by *taking their logical product and erasing the terms to be eliminated.* The predicate of any term (simple or complex) is then found by multiplying the product of the premises by the term. Such propositions are multiple logical quantities. The multiplication table of the different kinds of units is

	1 1	u 1	1 v	u v	u′ 1	1 v′
1 1	1 1	u 1	1 v	u v	u′ 1	1 v′
u 1	u 1	∞	u v	∞	∞	u v
1 v	1 v	u v	∞	∞	u v	∞
u v	u v	∞	∞	∞	∞	∞
u′ 1	u′ 1	∞	u v	∞	∞	u v
1 v′	1 v′	u v	∞	∞	u v	∞

where ∞ is the symbol for "anything." Thus

$$(P)_{11}(P')_{11} = (PP')_{11},\ (P)_{11}(Q)_{u1} \prec (PQ)_{u1},\ (Q)_{u1}(Q')_{u1} \prec \infty,\ \text{etc.}$$

The first is the only relation of equality; the others are implications, denoted by $\prec$.

The logic of two universes suggests immediately the introduction of "hyper" universes, in analogy with the "hyper" dimensions of space.

For further development of the subject, reference is made to Mr. Peirce's forthcoming volume of contributions to logic.

Review of certain points in Murphy's "Habit and Intelligence," by E. B. Wilson.

This paper was an examination of the evidence, brought forward by Mr. Murphy, in support of the proposition that there are many cases in the organic world where structure has been laid down as a preparation for function, before the function could be brought into action. If such structures can be shown to exist, natural selection or the survival of the fittest will not explain them, and they can only be accounted for under the assumption of a directive force or agency; this power is believed by the author to be an organizing intelligence.

Murphy's argument is defective in two essential points: in the first place, the imperfection of our knowledge in regard to the functions of certain structures in the lower animals is taken as equivalent to a demonstration that such structures perform no functions; and in the second place, the conditions resulting from the cessation or transformation of function are not considered. It is absurd to maintain that a structure performs no function because we cannot perceive it, for it has often happened that structures which upon superficial examination appeared useless, have been found to be in reality of vital importance to the organism. On the other hand, there are countless cases of structures which were once functionally active but have now lost their usefulness.

Looking into the particular biological evidence introduced by the author of "Habit and Intelligence" we find that some of it is clearly opposed to known facts. The statement, for example, that the abdomen of the Zoea and the *chorda* of the Ascidian larva are structures devoid of function simply shows that the author cannot have studied the uses of these structures in the living Zoea or Appendicularia. In other cases, as in considering the metamorphoses of Crustacea, Mr. Murphy has overlooked the fact that the present structure of an organism has been determined in accordance not only with the present, but also with all past conditions of life, and that uniformity of conditions at present does not imply uniformity in past time.

The cases of "structure in anticipation of function" brought forward by the author as proof of a formative agency in evolution are in every instance open to a quite different interpretation, and the validity of the entire argument based on this evidence must therefore be seriously questioned.

Wundt's Theory of Volition, by B. E. SMITH.

Wundt's doctrine that will is identical with psychical "spontaneity" as such is not admissible. It identifies two essentially distinct forms of psychical activity, viz: *reactions* upon external stimuli which are wholly determined by grounds lying outside of consciousness, and acts which are at least partially determined by the consciousness of an *aim* as such (motive). The term *volition* can be applied only to the latter. Nor can it be granted that volition—both inner and outer—is identical with "apperception" or attention. It is on the contrary to be assumed that attention which is truly voluntary is the effect of volition rather than volition itself. The ultimate psychical "spontaneity" is to be found not in "apperception," but in feeling (pleasure and pain) which is the ground of attention and—when guided by the knowledge gained through attention—of volition. Lastly, this conception of volition does not, as Wundt affirms, conflict with the principles of the continuous development of consciousness.

A Review of Bowne's "Metaphysics," by B. E. SMITH.

Mathematical Seminary.

February meeting.

On a Problem of Analytical Geometry, by PROFESSOR CAYLEY.

The object of the present note is only to call attention to a problem of Analytical Geometry presenting itself in connexion with the reduction of an algebraical integral, and which is solved, pp. 21–22 of Clebsch and Gordan's *Theorie der Abelschen Functionen* (Leipzig, 1866), viz., the problem is, considering a line drawn through two given points of a curve $f=0$ of the order n, to find the equation of a curve $\Omega=0$ of the order $n-2$ passing through the remaining $n-2$ points of intersection of the line with the curve f, through the double points of f, and through as many other given points as are required for the determination of the curve. If, for instance, f is a quartic curve without double points, then Ω is the quadric curve which passes through the remaining two intersections of the line with Ω, and through three given points. Take (ξ_1, η_1, ζ_1), (ξ_2, η_2, ζ_2) the coordinates of the two given points on the curve f; (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) for the coordinates of the three given points: and write $\Omega, =\big(a, b, c, f, g, h \between x, y, z\big)^2=0$ for the equation of the required curve. In the equation $f, =(x, y, z)^4=0$, write $x, y, z=\lambda\xi_1+\mu\xi_2$, $\lambda\eta_1+\mu\eta_2$, $\lambda\zeta_1+\mu\zeta_2$: we obtain an equation originally of the fourth order in (λ, μ), but which divides by $\lambda\mu$, and which when this factor is thrown out becomes

$$a\lambda^2+\beta\lambda\mu+\gamma\mu^2=0;$$

where
$$\begin{aligned} a &= (\xi_1, \eta_1, \zeta_1)^3(\xi_2, \eta_2, \zeta_2), \ =(\xi_2\partial_{\xi_1}+\eta_2\partial_{\eta_1}+\zeta_2\partial_{\zeta_1})f_1 \\ \beta &= (\xi_1, \eta_1, \zeta_1)^2(\xi_2, \eta_2, \zeta_2)^2, \\ \gamma &= (\xi_1, \eta_1, \zeta_1)\ (\xi_2, \eta_2, \zeta_2)^3, =(\xi_1\partial_{\xi_2}+\eta_1\partial_{\eta_2}+\zeta_1\partial_{\zeta_2})f_2, \end{aligned}$$

where for shortness f_1, f_2 are written to denote $f(\xi_1, \eta_1, \zeta_1), f(\xi_2, \eta_2, \zeta_2)$ respectively.

The condition as to the two points obviously is that making the same substitution $x, y, z=\lambda\xi_1+\mu\xi_2, \lambda\eta_1+\mu\eta_2, \lambda\zeta_1+\mu\zeta_2$ in the equation $\Omega= \big(a, \ldots \between x, y, z\big)^2, =0$, we must obtain the same quadric equation in (λ, μ). We have thus two conditions, which, introducing an indeterminate multiplier θ, are expressed by the three equations

$$\begin{aligned} \big(a, \ldots \between \xi_1, \eta_1, \zeta_1\big)^2 &= \theta a, \\ \big(a, \ldots \between \xi_1, \eta_1, \zeta_1\big)\big(\xi_2, \eta_2, \zeta_2\big) &= \theta\beta, \\ \big(a, \ldots \between \xi_2, \eta_2, \zeta_2\big)^2 &= \theta\gamma. \end{aligned}$$

The conditions as to the three points are obviously

$$\begin{aligned} \big(a, \ldots \between x_1, y_1, z_1\big)^2 &= 0, \\ \big(a, \ldots \between x_2, y_2, z_2\big)^2 &= 0, \\ \big(a, \ldots \between x_3, y_3, z_3\big)^2 &= 0, \end{aligned}$$

and these equations determine the ratios of a, b, c, f, g, h. But to complete the solution the convenient course is to regard the function $\Omega, = \big(a, \ldots \between x, y, z\big)^2$ as a quantity to be determined, and consequently to join to the foregoing the equation

$$\big(a, \ldots \between x, y, z\big)^2=\Omega;$$

we have thus seven equations from which (a, b, c, f, g, h) may be eliminated, the result being expressed by means of a determinant of the seventh order

$$\begin{vmatrix} \big(x, \ \ y, \ \ z\big)^2, & \Omega \\ \big(\xi_1, \eta_1, \zeta_1\big)^2, & \theta a \\ \big(\xi_1, \eta_1, \zeta_1 \between \xi_2, \eta_2, \zeta_2\big), & \theta\beta \\ \big(\xi_2, \eta_2, \zeta_2\big)^2, & \theta\gamma \\ \big(x_1, y_1, z_1\big)^2, & 0 \\ \big(x_2, y_2, z_2\big)^2, & 0 \\ \big(x_3, y_3, z_3\big)^2, & 0 \end{vmatrix}=0,$$

viz., this is an equation of the form $A\Omega=\theta\nabla$, where A is a constant determinant of the sixth order (*i. e.* a determinant not involving x, y, z), ∇ a determinant of the seventh order, a quadric function of (x, y, z), obtained from the foregoing determinant by writing therein $\Omega=0$ and $\theta=1$: the multiplier θ is and remains arbitrary: but it is convenient to take it to be $=1$, viz., we thus not only find the equation $\Omega=0$, of the required conic, but we put a determinate value on the quadric function Ω itself. And this being so, it is to be remarked that for $(x, y, z)=(\xi_1, \eta_1, \zeta_1)$, we have $\Omega=a, =(\xi_2\partial_{\xi_1}+\eta_2\partial_{\eta_1}+\zeta_2\partial_{\zeta_1})f_1$: and so for $(x, y, z)=(\xi_2, \eta_2, \zeta_2)$, we have $\Omega=\gamma, =(\xi_1\partial_{\xi_2}+\eta_1\partial_{\eta_2}+\zeta_1\partial_{\zeta_2})f_2$.

On a Geometrical Treatment of a Theorem in Numbers, by J. J. SYLVESTER.

The author made some remarks additional to those made on the same subject at the preceding meeting of the seminarium. In a plane reticulation four cases present themselves, viz, a line may be drawn through a line of nodes, or through a solitary node, or parallel to a line of nodes, or so as neither to pass through any node nor to be parallel to a line of nodes. In the third case the distance of the nodes of nearest approach is constant: in the second and fourth cases it approximates continually to zero. So in a solid reticulation eight cases present themselves, viz, four in addition to those last detailed: for without lying in a nodal plane, the line of flight may (a) pass through a single node, or (β) it may be parallel to a line of nodes, or (γ) it may be parallel to a nodal plane but not to a nodal line, or (δ) it may not pass through any node. In case (β) the distance of the nodes of nearest approach is constant; in case (γ) it approximates to a constant finite limit: in cases (a) and (δ) it approximates to zero.

There are thus four cases in all for which the distance from the nodes of nearest approach is a continually decreasing infinitesimal, viz: two for which the line of flight does not pass through any node, and two for which it does pass through a node—these latter two being those which serve to establish the theorem relating to the non-existence of trebly periodic functions.

The author further drew attention to the singular metamorphosis undergone by the geometrical setting forth of this theorem. It may be put under the form of asserting that a trilateral whose three sides are conditioned to be exact multiples of, and parallel to, three given straight lines lying in a plane may either be made to form a closed triangle or else such that the line closing the trilateral shall be less than any assigned quantity. On the other hand, the very same fact lends itself to, and is absolutely equivalent in substance to the statement that an arrow let fly from a node of a solid reticulation whether it speed along a nodal plane or be shot miscellaneously at the stars must (the law of gravity being supposed to be suspended) pass *indefinitely near* an infinite number of nodes in the course of its flight. The corresponding theorem for space of five dimensions serves to show that Quaternion Functions cannot have a higher than a quadruple periodicity.

Note on Areas of Corresponding Surfaces, by T. CRAIG.

Let two surfaces be connected by the relations

$$\xi=F_1(xyz), \qquad \eta=F_2(xyz), \qquad \zeta=F_3(xyz),$$

(xyz) being a point on one surface and $(\xi\eta\zeta)$ the corresponding point on the other. Denote by a, b, c the direction cosines of the normal to the first surface at the point (xyz); also let $\alpha\beta\gamma$ denote the direction cosines of the normal to the second surface at the corresponding point $(\xi\eta\zeta)$. We have then

$$a,\, b,\, c = \frac{dy}{du}\frac{dz}{dv} - \frac{dz}{dv}\frac{dy}{du} \div \sqrt{EG - F^2},$$
$$\frac{dz}{du}\frac{dx}{dv} - \frac{dz}{dv}\frac{dx}{du} \div \qquad ``$$
$$\frac{dx}{du}\frac{dy}{dv} - \frac{dx}{du}\frac{dy}{dv} \div \qquad ``$$

similar values for α, β, γ. Write these as

$$a,\, b,\, c = \frac{A}{V}, \frac{B}{V}, \frac{C}{V},$$

and

$$\alpha,\, \beta,\, \gamma = \frac{A'}{V'}, \frac{B'}{V'}, \frac{C'}{V'}.$$

Then we have at once

$$V = aA + bB + cC,$$
$$V' = \alpha A' + \beta B' + \gamma C',$$

and

$$dS = V\, du\, dv,$$
$$d\Sigma = V'\, du\, dv,$$

then

$$\frac{d\Sigma}{dS} = \frac{V'}{V}.$$

Now obviously

$$V = \begin{vmatrix} a & b & c \\ \frac{dx}{du} & \frac{dy}{du} & \frac{dz}{du} \\ \frac{dx}{dv} & \frac{dy}{dv} & \frac{dz}{dv} \end{vmatrix}$$

$$V' = \begin{vmatrix} \alpha & \beta & \gamma \\ \frac{d\xi}{dx}\frac{dx}{du} + \ldots, & \frac{d\eta}{dx}\frac{dx}{du} + \ldots, & \frac{d\zeta}{dx}\frac{dx}{du} + \ldots \\ \frac{d\xi}{dx}\frac{dx}{dv} + \ldots, & \frac{d\eta}{dx}\frac{dx}{dv} + \ldots, & \frac{d\zeta}{dx}\frac{dx}{dv} + \ldots \end{vmatrix}.$$

Border this by

$$0,\ a\frac{dx}{du} + b\frac{dy}{du} + c\frac{dz}{du},\ a\frac{dx}{dv} + b\frac{dy}{dv} + c\frac{dz}{dv},\ a^2 + b^2 + c^2$$

as the last vertical, *i. e.*

$$0 \quad 0 \quad 0 \quad 1$$

Further border now with

$$a\frac{d\xi}{dx} + \ldots,\ a\frac{d\eta}{dx} + \ldots,\ a\frac{d\zeta}{dx} + \ldots,\ 1.$$

This last determinant then breaks up into the product of two determinants. The first of the two is V bordered by 1 0 0 0 horizontally and 1 0 0 0 vertically. The final result is readily found to be

$$\frac{d\Sigma}{dS} = \frac{V'}{V} = \begin{vmatrix} \alpha & \frac{d\xi}{dx} & \frac{d\xi}{dy} & \frac{d\xi}{dz} \\ \beta & \frac{d\eta}{dx} & \frac{d\eta}{dy} & \frac{d\eta}{dz} \\ \gamma & \frac{d\zeta}{dx} & \frac{d\zeta}{dy} & \frac{d\zeta}{dz} \\ 0 & a & b & c \end{vmatrix}$$

By writing α, β, $\gamma = a$, b, $c = \xi$, η, ζ we have at once Gauss's expression for the measure of curvature expressed as a determinant of the fourth order.*

Note on Partitions, by O. H. Mitchell.

If the number of ways of dividing an integer w into j or fewer parts, none greater than i, be denoted by the symbol $(w : i, j)$, and if

$$\phi_j(w) = w - E\left(\frac{w + j - 1}{j}\right) = E\left(\frac{(j-1)w}{j}\right),$$

where E means "the number of integers in," then

$$(w : i, j) = \sum_{x = w - i}^{x = \phi_j(w)} (x : w - x, j - 1);$$

by a successive application of this formula j may be reduced to unity, whence

$$(w : i, j) = \sum_{x_1 = w - i}^{x_1 = \phi_j(w)} \ \sum_{x_2 = 2x_1 - w_1}^{x_2 = \phi_{j-1}(x_1)} \ \sum_{x_3 = 2x_2 - x_1}^{x_3 = \phi_{j-2}(x_2)} \cdots\cdots \sum_{x_{j-1} = 2x_{j-2} - x_{j-3}}^{x_{j-1} = \phi_2(x_{j-2})} \qquad (1).$$

*Some weeks after the above note had been read I discovered that the Theorem had been announced, without proof, by Neumann, in Vol. XI of the *Mathematische Annalen*.
T. C.

This is written symbolically for what the expression would become if the last sign of summation were replaced by $1 + \phi_2(x_{j-2}) - [2x_{j-2} - x_{j-3}]$, the range of its variable. The last two signs of summation may be replaced by

$$\tfrac{1}{2}\left\{\phi_2[2x_{j-3} - x_{j-4} + \varepsilon] + \phi_2\phi_3(x_{j-3}) + 2\right\}\left\{\phi_3(x_{j-3}) - [2x_{j-3} - x_{j-4}] + 1 - \varepsilon\right\} + \varepsilon\left\{\phi_2[2x_{j-3} - x_{j-4}] + 1\right\} - \left\{3x_{j-3} - 2x_{j-4} + \phi_2[2x_{j-4} - 3x_{j-3} + 2] + \phi_3(x_{j-3}) - [2x_{j-3} - x_{j-4}]\right\}\left\{1 + \phi_3(x_{j-3}) - [2x_{j-3} - x_{j-4}] - \phi_2[2x_{j-4} - 3x_{j-3} + 2]\right\},$$

where ε is the residue, modulus 2, of $\phi_3(x_{j-3}) - [2x_{j-3} - x_{j-4}] + 1$, and where negative values of the quantities inclosed in [ ] are to be neglected.

As a special case of the above

$$(w : i, 3) = \tfrac{1}{2}\left\{\phi_2(w - i + \varepsilon) + \phi_2\phi_3(w) + 2\right\}\left\{\phi_3(w) - w + i + 1 - \varepsilon\right\} + \varepsilon\left\{\phi_2(w - i) + 1\right\} - \left\{\phi_3(w) - i + \phi_2[2i - w + 2]\right\}\left\{\phi_3(w) - w + i + 1 - \phi_2[2i - w + 2]\right\},$$

where ε = residue mod. 2 of $\phi_3(w) - w + i + 1$.

Thus $(100 : 85, 3) = \tfrac{1}{2}\left\{7 + 33 + 2\right\}\left\{66 - 14\right\} - \left\{66 - 85 + 36\right\}\left\{66 - 14 - 36\right\} = 42.26 - 17.16 = 820.$

By use of the formula it is easy to obtain Prof. Sylvester's formula* for $(w : i, j) - (w - 1 : i, j)$ in terms of partitions of lower orders, viz.:

$$(w : i, j) - (w - 1 : i, j) = \sum_{\delta = 0}^{\delta = \phi_j(w) - [w - i]} (w - 2i + 2\delta : i - \delta, j - 2) - (w - i - 1 : i, j - 1).$$

March meeting.

On the Geometrical Representation of an Equation between Two Variables, by Professor Cayley.

An equation between two variables cannot be represented in a satisfactory manner by a curve, for this serves only to represent the corresponding *real* values of the variables: to represent the imaginary values the natural course is to represent each variable by a point in a plane, viz: the variable z, $= x + iy$, will be represented by a point the coordinates of which are the components x and y of the variable, and similarly the variable z', $= x' + iy'$, by a point the coordinates of which are the components x', y' of the variable: the equation between the two variables then establishes a correspondence between the two variable points, or say a correspondence between the planes which contain the two points respectively: and it is this correspondence of two planes which is the proper geometrical representation of the equation between the two variables: to exhibit the correspondence we may in either of the planes draw a network of curves at pleasure, and then draw in the other plane the network of corresponding curves. This well known theory was illustrated for the case $z' = \sqrt{z^4 - 1}$; taking in the infinite half-plane $y = +$ about the origin as centre a system of semicircles, to these correspond in the infinite plane of $x'y'$ a series of lemniscate-shaped curves: and by means of these it is easy to show in the second plane the path corresponding to a given path of the point z, $= x + iy$, in the first half-plane.

On the Properties of a Split Matrix, by J. J. Sylvester.

Suppose a square matrix split into two sets of lines which need not be contiguous and may be called ranges, say *ABC*, *DEFG*. Let the sum of the products of the corresponding elements of any two lines be called their product. It is well known (see Salmon's Higher Alg., 3d Ed., p. 82) that if the product of each line in the first range by every line in the other is zero, the opposite complete minors of the two ranges will be in a constant ratio, say in the ratio $l : \lambda$. Call the content of the matrix Δ: then it follows, if S, Σ denote the sums of the squares of the complete minors in the two ranges respectively, that

$$\frac{\lambda}{l} S = \frac{l}{\lambda}\Sigma = \Delta.$$

But by a theorem of Cauchy concerning rectangular matrices S is equal to the determinant $(A, B, C)^2$, *i. e.* to the determinant

$$\begin{matrix} AA & AB & AC \\ BA & BB & BC \\ CA & CB & CC \end{matrix}$$

and similarly $\Sigma = (D, E, F, G)^2$

so that $\lambda^2 : l^2 :: (D, E, F, G)^2 : (A, B, C)^2$

and $S\Sigma = \Delta^2$.

Suppose now that the product of *every* two lines in the entire matrix is zero. Then into whatever two ranges the matrix be divided the ratio

* See *Messenger of Mathematics*, May, 1878.

$\lambda^2 : l^2$ (since all but the diagonal terms in the matrices which express the ratio $l^2 : \lambda^2$ vanish) will be expressed by the ratio of one simple product to another: thus *ex. gr.* for the ranges $ABC : DEFG$

$\lambda^2 : l^2 :: D^2.E^2.F^2.G^2 : A^2.B^2.C^2$; also $\Delta^2 = A^2.B^2.C^2.D^2.E^2.F^2.G^2$.

If we now further suppose that the sum of the squares of the elements in each line is unity, *i. e.* that $A^2 = B^2 = C^2 = D^2 = E^2 = F^2 = G^2 = 1$, it will follow that every minor whatever divided by its opposite will be equal to Δ (for on the hypothesis made, $\frac{\lambda}{l} = \frac{\Delta}{S} = \Delta$).

Also Δ will be plus or minus unity since $\Delta^2 = 1$. Thus it is seen that we may pass by a natural transition from the theory of a split to that of an orthogonal or self-reciprocal matrix—to show which was the principal motive to the present communication. It is by aid of the theorem of the *split matrix* that I prove a remarkable theorem in Multiple Algebra, viz: that if the product of two matrices of the same order is a complete null, the sum of the nullities of the two factors must be at least equal to the order of the matrix—the nullity of a matrix of the order ω being regarded as unity, when its determinant simply is zero, as 2 when each first minor simply is zero, as 3 when each second minor is zero . . . as $(\omega - 1)$ when each quadratic minor is zero and as ω (or absolute) when every element is zero. This theorem again is included in the more general and precise one following—*If any number of matrices of the same order be multiplied together, the nullity of their product is not less than the nullity of any single factor and not greater than the sum of the nullities of all the several factors.*

In Prof. Cayley's memoir on Matrices (Phil. Trans., 1858) the very important proposition is stated that if

$$\begin{matrix} a & b & c & d \\ a' & b' & c' & d' \\ a'' & b'' & c'' & d'' \\ a''' & b''' & c''' & d''' \end{matrix}$$

be any matrix of substitution, say m (here taken by way of illustration of the order 4) the determinant $\begin{vmatrix} a-m & b & c & d \\ a' & b'-m & c' & d' \\ a'' & b'' & c''-m & d'' \\ a''' & b''' & c''' & d'''-m \end{vmatrix}$ is identically zero; or in other words, its nullity is complete. By means of the above theorem it may be shown that the nullity of any i distinct algebraical factors of such matrix is equal to i, i having any value from unity up to the number which expresses the order of the matrix, inclusive.

Note on the Non-Euclidean Trigonometry, by W. E. STORY.

Using Dr. Klein's generalization of Prof. Cayley's projective measurement, it is proved that the formulae of spherical trigonometry apply also to any non-Euclidean space of two dimensions, provided that angles and sides (lines) are replaced by certain constant multiples of them. The deduction will be given in detail in the *American Journal of Mathematics*.

Note on Partitions, by G. S. ELY.

A table of partitions may be constructed in the following manner. Fill the first line and the first two columns of a rectangle with 1's, and any other element is obtained in this way. For the i^{th} element in the j^{th} column, add to the $(i-1)^{th}$ element in the j^{th} column the i^{th} element in the $(j-i)^{th}$ column. Thus we get the table

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8
3	1	1	2	3	4	5	7	8	10	12	14	16	19	21	24	27
4	1	1	2	3	5	6	9	11	15	18	23	27	34	39	47	54
5	1	1	2	3	5	7	10	13	18	23	30	37	47	57	70	84
6	1	1	2	3	5	7	11	14	20	26	35	44	58	71	90	110
7	1	1	2	3	5	7	11	15	21	28	38	49	65	82	105	131

Number the columns commencing with 0 and the lines commencing with 1. This is the same table Euler gave in the Com. Arith. Coll., Vol. I, p. 97, and is of course capable of indefinite extension. Euler constructed his table by lines instead of by columns, and he shows that the number of partitions of w things into j or fewer parts, $(w: w, j)$, is given by the number in the line j and column w. Thus $(12: 12, 7) = 65$. The value of the symbol $(w: i, j)$, *i. e.* the number of partitions of w things into j parts, none of which shall be greater than i, can also be found from the table in the majority of cases, viz: when the greater of i and j is $\geqq \frac{w-4}{2}$, by the following rule: suppose $i \geqq j$, for if not it only needs to be remembered that $(w: i, j) = (w: j, i)$. Then to find the value of $(w: i, j)$, subtract from the value of $(w: w, j)$ as given by the table the sum of the first $(w-i)$ elements in the $(j-1)^{th}$ line and add to it 0, 1 or 2 according as

$i \geqq \frac{w-2}{2}$, $= \frac{w-3}{2}$ or $= \frac{w-4}{2}$. Thus the value of $(14: 9, 7)$ is $105 - (1+1+2+3+5) + 0 = 93$; or again $(15: 6, 6) = 110 - (1+1+2+3+5+7+10+13+18) + 1 = 51$.

The proportion of the number of times this rule gives the value of $(w: i, j)$ is $\frac{3}{4}$ when w is indefinitely large. But for ordinary values of w it is much larger. Thus out of the 275 possible expressions when $w \leqq 12$ all are given by this rule: of the 530 expressions when $w \leqq 15$ all are given except the following six $(13: 4, 4)$, $(14: 4, 4)$, $(15: 3, 5)$, $(15: 4, 4)$, $(15: 4, 5)$ and $(15: 5, 5)$ which are readily seen to be equal respectively to 3, 2, 1, 1, 6 and 13. And generally those expressions not given by this rule are the easiest to calculate by the ordinary mechanical means.

These formulae were also obtained. If $N =$ the whole number of expressions for any particular value of w,

$$N = \frac{w^2 - 2w + 6}{2} - \sum_{n=2}^{n=\infty} E\left(\frac{w - (n^2 + n + 1)}{n+1}\right) \text{ if } w \text{ is even, and}$$

$$N = \frac{w^2 - 2w + 5}{2} - \sum_{n=2}^{n=\infty} E\left(\frac{w - (n^2 + n + 1)}{n+1}\right) \text{ if } w \text{ is odd.}$$

$E(x)$ signifying the greatest integer in x, positive values of $E(x)$ only being taken into account. Thus if $w = 100$ we readily find that $N = 4809$.

Finally this value for ΣN was obtained, for w even

$$\sum_{w=1}^{w=n} N = \frac{2n^3 - 3n^2 + 28n - 24}{12} - \frac{1}{2} \sum_{i=3}^{i=\infty} (ia_i(a_i - 1) + 2a_i\phi_i n)$$

and for w odd

$$\sum_{w=1}^{w=n} N = \frac{2n^3 - 3n^2 + 28n - 27}{12} - \frac{1}{2} \sum_{i=3}^{i=\infty} (ia_i(a_i - 1) + 2a_i\phi_i n)$$

where $a_i = E\left(\frac{n - i^2 + i - 1}{i}\right)$, only positive values being taken into account, and $\phi_i n = n - iE\left(\frac{n-1}{i}\right)$, that is $\phi_i n$ is the excess of n over the greatest multiple of i which is less than n. Thus as an example, if $n = 25$

$a_3 = E\left(\frac{25-7}{3}\right) = 6$, $a_4 = E\left(\frac{25-13}{4}\right) = 3$, $a_5 = E\left(\frac{25-21}{5}\right) = 0$, $\phi_3 25 = 1$, $\phi_4 25 = 1$, $\phi_5 25 = 5$. Substituting we have $\sum_{w=1}^{w=25} N = 2438$.

April meeting.

On Associative Imaginaries, by PROF. CAYLEY.

The imaginaries (x, y) defined by the equations

$$\begin{aligned} x^2 &= ax + by \\ xy &= cx + dy \\ yx &= ex + fy \\ y^2 &= gx + hy \end{aligned}$$

will not be in general associative: to make them so, we must have 8 double relations corresponding to the combinations $x^3, x^2y, xyx, xy^2, yx^2, yxy, y^2x, y^3$ respectively, viz: the first of these gives $x.x^2 - x^2.x = 0$, that is $0 = x(ax + by) - (ax + by)x, = b(yx - xy), = b[(e-c)x + (f-d)y]$; that is $0 = b(e-c)$ and $0 = b(f-d)$: and similarly for each of the other terms: we thus obtain apparently 16, but really only 12 relations between the 8 coefficients a, b, c, d, e, f, g, h, viz: the relations so obtained are $b(c-e) = 0$ (twice), $b(f-d) = 0$, $g(c-e) = 0$, $g(f-d) = 0$ (twice), $bg - ed = 0$ (twice), $bg - ef = 0$ (twice),
$c^2 + dg - ag - ch = 0$, $d^2 + bc - ad - bh = 0$, $e^2 + fg - ag - eh = 0$, $f^2 + be - af - bh = 0$,
$a(c-e) - cf + de = 0$, $h(f-d) - cf + de = 0$.

From the first four equations it appears that either $b = 0$ and $g = 0$ or else $c = e$ and $d = f$: for brevity I attend only to the latter case, giving the commutative system

$$\begin{aligned} x^2 &= ax + by, \\ xy = yx &= cx + dy, \\ y^2 &= gx + hy; \end{aligned}$$

in order that this may be associative, we must still have the relations

$$\begin{aligned} bg &= cd, \\ c^2 + dg - ag - ch &= 0, \\ d^2 + bc - ad - bh &= 0, \end{aligned}$$

which are all three of them satisfied by $g = \frac{cd}{b}$, $h = \frac{d^2 + bc - ad}{b}$, viz: we thus have the associative and commutative system

$$\begin{aligned} x^2 &= ax + by, \\ xy = yx &= cx + dy, \\ y^2 &= \frac{cd}{b}x + \frac{d^2 + bc - ad}{b}y. \end{aligned}$$

I did not perceive how to identify this system with any of the double algebras of B. Peirce's Linear Associative Algebra, see pp. 120-122 of the Reprint, *American Journal of Mathematics*, t. IV (1881); but it has been pointed out to me by Mr. C. S. Peirce, that my system in the general case $ad - bc$ not $= 0$, is expressible as a mixture of two algebras of the form (a_1), see p. 120; whereas if $ad - bc = 0$, it is reducible to the form (c_2), see p. 122. The object of the present Note is to exhibit in the simple case of two imaginaries the whole system of relations which must subsist between the coefficients in order that the imaginaries may be associative; that is the system of equations which are solved implicitly by the establishment of the several multiplication tables of the memoir just referred to.

Note on Singular Solutions, by H. M. Perry.

The differential equation of the first order and degree for which $u=0$ is a singular solution, may always be written,

$$y' = \phi_1 + y'_u ,$$

where y'_u is the value of y' derived from $u=0$, and ϕ_1 is a function of x and u which vanishes when $u=0$.

The differential equation of the second order becomes after substituting $y' = \phi_1 + y'_u$, $u = 0$,

$$y'' = \frac{\partial\phi_1}{\partial u}\frac{\partial u}{\partial y}\phi_1 + y''_u = \phi_2 + y''_u ;$$

similarly, if $\phi_2 = 0$ when $u = 0$,

$$y''' = \frac{\partial\phi_2}{\partial u}\frac{\partial u}{\partial y}\phi_1 + y'''_u = \phi_3 + y'''_u ,$$

and in general if $\phi_1, \phi_2, \ldots \phi_{r-1} = 0$ when $u = 0$,

$$y^r = \frac{\partial\phi_{r-1}}{\partial u}\frac{\partial u}{\partial y}\phi_1 + y^r_u .$$

Hence the following results are easily deduced:

If n is the least power to which u occurs in ϕ_1, $u=0$ is a singular solution for differential equations of all orders less than $\frac{1}{1-n}$.

If $\frac{1}{1-n}$ lies between two integers r and $r+1$, $u=0$ satisfies the rth differential equation and makes the $(r+1)$th and all higher derivatives infinite.

If $\frac{1}{1-n}$ is negative, $u = 0$ satisfies all differential equations.

If $0 < n < \frac{1}{2}$, $u = 0$ is the equation of a curve tangent to the curves of the complete primitive at their cusps.

On Cubic Curves, by F. Franklin.

The investigation in this paper was based upon a consideration of the mixed polars of a pair of points with respect to the system of cubics represented by the equation $x^3 + y^3 + z^3 + 6lxyz = 0$; by this means a number of known properties of cubics were very simply and consecutively obtained, together with some properties believed to be new. The communication will be made the basis of a paper in a forthcoming number of the *American Journal of Mathematics*.

SYNOPSIS OF THE RECENT SCIENTIFIC JOURNALS

Published here.

American Journal of Philology. Edited by Professor Gildersleeve. *Vol. II. No. 8.* 1881.

Article I.—On the Fragments of Sophocles. By Robinson Ellis.

In this paper Professor Ellis has emended and illustrated some thirty odd fragments of Sophocles with especial reference to Professor Campbell's recent edition.

Article II.—Virgil's instructions for Ploughing, Fallowing, and the Rotation of Crops. By Clement L. Smith.

Professor Smith combats the view accepted by Heyne and by most recent editors of the Georgics, that the first of Virgil's four ploughings (Georg. I, 48) falls in the autumn, a whole year before seed-time. The autumn ploughing—for the sake of which Wagner assumes that vv. 47-49 could have been no part of the original poem—seems to Professor Smith purely imaginary, and from an examination of the Roman prose writers on husbandry the conclusion is reached that Virgil's four ploughings are: (1) in early spring, (2) in April, to kill the weeds, (3) in summer, (4) in the autumn just before sowing. In the matter of fallowing and rotation of crops Virgil's instructions are quite consistent with Varro's rule for fallowing in alternate years and with the two year system, which the modest dimensions of the ordinary Italian farm probably made very general. Yet he chooses to leave his language (Georg. I, 71-83) vague and it applies as well to a three year system as to any system which secures for the land the relaxation it needs, by relieving it 'every other time' from the burden of bearing the heavy grain crop. The difficulty which arises from the incongruity of Virgil's presentation of the advantages of rotation over fallowing, it is proposed to remove by transposing vv. 82, 83, and putting them immediately after 76. In this way we should have first a presentation of two ways of relieving the exhausted land, (vv. 71-76) with a statement of the advantage of the second over the first, (82 f.) then would come the reason why such relief is needed, (77 f.) and from that a skilful transition to the subject of manuring.

Article III.—The Semitic vowel A. By C. H. Toy.

In this article Professor Toy has gathered some materials for the history of the first Semitic vowel *a*, giving first the cases in which it has maintained itself, and then the changes it has undergone. So it is preserved in the primitive or underived nouns, in the monosyllabic triliterals of all the dialects, in dissyllabic triliterals of most—the Arabic being the richest. So in certain derived nouns, in the original form of the perfect of the verb, and in certain pronominal forms and most simple particles. It has been degraded into—1. *ă, ĕ.* 2. Into *i.* It has been shortened, it has been reduced to a sh'wa, it has disappeared. Then again it has been broadened into *ā*, diphthongized into *ē*, and finally subjected to sporadic modifications in the Hebrew *Hiphil.* There has thus been, from the earliest known times, a very considerable movement of this vowel, which closely resembles vowel modification, in other families of languages, particularly the European, and so far points to the oneness of the phonetic principles that control the various groups of human speech.

Article IV.—On the position of Rhematic to. By H. E. Shepherd.

The insertion of an adverb between *to* and the infinitive has been noted in a number of reputable writers. Dr. Fitzedward Hall has discovered this peculiarity in at least a score of well-known authors, extending from Chaucer and Wickliffe to George Eliot and Mr. Ruskin. The author who most abounds in this use is Bishop Pecock, the author of the "Repressor" (A. D. 1456), and Professor Shepherd has culled from this work a number of phrases which illustrate almost every feature of his usage, and has noted a number of examples from reputable authors, English and American, among them some of our most accomplished essayists and philologists.

Article V.—Assyriological Notes. By J. F. McCurdy.

These notes refer to כשר, קשר, and ברה

Article VI.—On πρίν in the Attic Orators. By the Editor.

πρίν is a comparative formation. From this follows its negative sense (*before = not yet.*) The negative sense carries with it the prevalent use of the aorist tense. This principle has not been clearly formulated in the text books of Greek grammar, although the fact has sometimes been emphasized. There is no exception worth considering to the rule that in classic Greek πρὶν ἄν with subj. and πρίν with opt. require a negative in the lead. Exceptions sometimes cited are no exceptions; the negative sense has escaped those who have adduced the examples. One real exception must be emended out of existence. On the other hand, πρίν with the indicative may follow a positive clause, but many of the positive clauses cited are really negative, and even when a positive clause is used, the context must show the equivalency of πρίν *before* to ἕως *until.* After this criticism on the ordinary presentation, the author of the paper proceeds to show briefly the history of the usage. The dominant construction in the entire period is πρίν with the aorist infinitive, an anomalous combination. The various theories as to the origin of this construction are briefly discussed, and this discussion is followed by the statistics of the oratorical usage of πρίν, with a presentation of the normal Greek usage under each head. Exceptions are treated at greater or less length. In the orators, as might have been expected, a certain crystallization is noticeable. Of individual peculiarities there is not much to say. Demosthenes uses the particle simply; Isokrates in his more formal orations employs πρότερον-πρίν freely for the sake of balance; Lysias is sharp and masculine.

There are three elaborate *Reviews*, one of Storm's *Englische Philologie* by Professor GARNETT, one of the works of *Spyridon Lambros* by Mr. THOMAS DAVIDSON, and one of Gustav Meyer's *Griechische Grammatik* by Dr. BLOOMFIELD, in which the reviewer has introduced a *Note on the Theory of Sonant Coefficients.* Professor LANMAN has noticed with some detail Peile's *Tale of Nala.* The other reviews concern Mezger's *Pindar's Siegeslieder*, Arnoldt's *Chor im Agamemnon des Aeschylus*, Seuffert's *Deutsche Literaturdenkmale*, and Otis's *Elementary German.*

Reports are made of the *Rheinisches Museum*, *Fleckeisen's Jahrbücher*, *Philologus*, Kölbing's *Englische Studien.*

In the *Correspondence* there is a letter from Mr. A. S. COOK on *The Philological Society's English Dictionary.*

The management of the journal at the close of this, the second volume, offers to new subscribers the two back volumes for $2.00 a volume, payable in advance.

No. 9, which is the opening number of Vol. III of the *Journal of Philology,* is in press and contains:

Articles on *Nonius Marcellinus,* by Professor NETTLESHIP; on the *Insertion of an adverb between* TO *and the Infinitive,* by Dr. FITZEDWARD HALL; on *Final* AS *in Sanskrit,* by Dr. MAURICE BLOOMFIELD; on the *Phonetics of the Ormulum,* by Mr. BLACKBURN; on the *MSS. of Terence,* by Dr. WARREN; on *"For-Sake,"* by Professor GARNETT.

Reviews and Notes of Deffner's *Zakonische Grammatik, Anecdota Oxoniensa,* MacMahon's *Translation of Brêdif's Demosthenes,* Campbell's *Sophocles,* Ellis's *Ibis,* Hayman's *Odyssey, Transactions of Cambridge Philological Society, Recent Publications in Norse Philology.*

Reports of *Hermes, Mnemosyne, Anglia, Romania, Journal de Philologie.*

Correspondence (letters from L. CAMPBELL, L. R. PACKARD, A. D. SAVAGE).

Studies from the Johns Hopkins Biological Laboratory. Edited by PROFESSOR MARTIN. *Vol. II. No.* 2.

Article I.—List of Medusæ found at Beaufort, N. C., during the Summers of 1880 *and* 1881. By W. K. BROOKS.

During the two summers forty-three species were found; twenty-seven of these had previously been described by A. Agassiz and McCrady as occurring at Charleston, S. C. Eight of the species found were entirely new to science.

Article II.—On the Origin of the so-called "Test-Cells" in the Ascidian Ovum. By J. PLAYFAIR MCMURRICH. With plate.

The author worked last summer at the marine zoölogical station of the University, using the eggs of two species. He comes to the conclusion that the "test-cells" are simply masses of albuminous matter containing two or three granules of food-yolk, and originate by separation of bits of egg protoplasm forced out during its contraction. The meaning of the process cannot be elucidated until we know the life-histories of more of the lower Ascidia.

Article III.—A Contribution to the study of the Bacterial Organisms commonly found upon exposed Mucous Surfaces and in the Alimentary Canal of healthy persons. By GEORGE M. STERNBERG, Surgeon U. S. A. With three heliotype plates from micro-photographs.

The author describes the bacteria found in the mouth, and near the openings of other cavities of the body; and gives an account of the results of their culture outside the body in various nutritive media. The object of the paper is to contribute to a knowledge of what harmless varieties exist in the healthy body, with reference to the now generally accepted doctrine that some forms of bacteria when living in the body cause serious diseases.

Article IV.—A Fatal Form of Septicæmia in the Rabbit produced by the subcutaneous injection of Human Saliva. By GEORGE M. STERNBERG, Surgeon U. S. A. With one heliotype plate.

The author's saliva contains a micrococcus which causes it to kill rabbits with all the symptoms of septicæmia within twenty-four hours. The subcutaneous areolar tissue of rabbits dying from this is found infiltrated with bloody serum and swarming with micrococci. These can be artificially cultivated in *bouillon* made from the flesh of a healthy rabbit; and a small drop of the sixth generation of the artificial culture possesses all the virulence of the original specimen.

Article V.—Experiments with Disinfectants. By GEORGE M. STERNBERG, Surgeon U. S. A.

Taking the infected *bouillon* referred to above, the author added to it various percentages of different reputed disinfecting agents, and then injected it into rabbits, with the object of seeing if the virulence of the micrococcus had been destroyed.

As a result he puts the disinfectant in three groups. (1) Those efficient in proportion of 0.5 per cent. or less. Among them are iodine, chromic acid, thymol, ferric chloride, sodium hyposulphite, &c., &c. (2) Those inefficient when present in not larger proportion than 0.5 per cent. but efficient in proportion under 2 per cent. In this group, with others, are included carbolic acid, salicylate of soda, zinc chloride, potassium permanganate, and borax. (3) Substances which fail to disinfect when present in the proportion of 2 per cent. In this class we find glycerine, alcohol, pyrogallic acid, alum, &c., &c.

Article VI.—Observations on the Direct Influence of Variations of Arterial Pressure upon the Rate of Beat of the Mammalian Heart. By H. NEWELL MARTIN.

The experiments were made upon dogs, whose hearts were completely isolated physiologically from all the rest of the body, except the lungs. The heart was supplied with defibrinated blood of a constant temperature, supplied to the right auricle from a Marriotte's flask, and therefore under constant pressure. The result arrived at was that variations of aortic pressure, between 25 and 140 millimetres of mercury, had no influence whatever upon the pulse rate.

Article VII.—The Influence of Variations of Arterial Pressure upon the Pulse Rate in the Frog and the Terrapin. By WM. H. HOWELL and MACTIER WARFIELD. With one plate.

The method employed and the results obtained were essentially the same as those described in connection with Article VI. (See p. 203.)

Article VIII.—Some Notes on the Development of Arbacia punctulata. By H. GARMAN and B. P. COLTON. With two plates.

The eggs were artificially fertilised, and their development followed until the young sea-urchins emerged from the plutei. The authors describe in detail the structure and habits of the pluteus in its various stages; and also the structure of the young sea-urchin.

Article IX.—On the Structure and Significance of some Aberrant forms of Lamellibranchiate Gills. By K. MITSUKURI.

The species studied were Nucula proxima and Yoldia limatula. The gills in these species are extremely simple, and the results of their study, combined with those of other workers on more complex forms, make it tolerably certain that the primitive Lamellibranchiate gill was filamentary.

Article X.—Observations on the Early Stages of some Polychætous Annelides. By EDMUND B. WILSON. With four plates.

This article contains descriptions and figures of the early stages of five genera of Annelides, viz: *Clymenella, Arenicola, Chætopterus, Spiochætopterus,* and *Diopatra.* It is shown that the segmentation of the egg is, in its earlier stages, of the same type with some *Oligochæta, Hirudinea, Gephyrea, Mollusca, Turbellaria,* etc., the egg dividing into two, four, and then into eight spherules, of which four are ectodermic and four mainly entodermic. In later stages there are indications of a process which shows some similarity to delamination and suggests the way in which the latter mode of development may have been derived from epibolic invagination.

The larvæ of *Clymenella* and *Arenicola* are shown to be *Telotrochæ*, which have probably been modified by the larvæ having abandoned a free-swimming life. The young *Diopatra* is an "Atrocha," but shows indications of having been derived from a Telatrochöus form. *Chœtopterus* and *Spiochœtopterus* are typical Mesotrochæ; the latter is permanently monotrochal, the latter only in its earlier stages. In *Chœtopterus* a provisional belt of cilia appears, which is replaced afterwards by a permanent belt behind it.

Article XI.—The Origin of the Eggs of Salpa. By W. K. BROOKS. With one plate.

In this paper the author brings forward new facts which support a view formerly promulgated by him, in accordance with which each salpa of a chain is a male, which before its separation from the body of the solitary salpa, receives one egg from it. (See p. 202).

No. 3 of Volume II. of *Studies from Biological Laboratory* is in press, and contains:

Article I.—Observations on the Blood-flow in the Coronary Arteries of the Heart. By H. N. MARTIN and WM. T. SEDGWICK.

Article II.—The Influence of Digitaline upon the Work of the Heart. By H. H. DONALDSON and M. WARFIELD.

Article III.—On a New Species of Pilidium. By E. B. WILSON.

Article IV.—Observations on Blood Pressure and Pulse in a Case of Open Ductus Arteriosus in a Dog. By WM. H. HOWELL and FRANK DONALDSON.

Article V.—On the Polar Effects of Weak Induction Currents. By H. SEWALL.

Article VI.—On the Development of Starch Granules. By A. F. W. SCHIMPER.

American Journal of Mathematics. Edited by PROFESSOR SYLVESTER. *Vol. IV. Nos.* 2 and 3.

Article I.—Linear Associative Algebra. By the late BENJAMIN PEIRCE. With Notes and Addenda, by C. S. PEIRCE, son of the author.

The consent of the family of the late Professor Benjamin Peirce has been kindly given to the publication in the *American Journal of Mathematics*, of his valuable and unique memoir on Linear Associative Algebra, of which only a small number of copies in lithograph were taken in the author's lifetime, for distribution among his friends. This publication will, it is believed, supply a want which has been long and widely felt, and bring within the reach of the general mathematical public a work which may almost be entitled to take rank as the *Principia* of the philosophical study of the laws of algebraical operation. The text has been revised and annotated by Mr. C. S. Peirce; in his foot-notes will be found, besides other matter, expressions for the units of the various algebras in terms of the algebra of relatives. The *Addenda* are as follows:

I. On the Uses and Transformations of Linear Algebra. By BENJAMIN PEIRCE. (Presented to the American Academy of Arts and Sciences, May 11, 1875.)

II. On the Relative Forms of the Algebras. By C. S. PEIRCE.

III. On the Algebras in which Division is Unambiguous. By C. S. PEIRCE.

This paper occupies the whole of No. 2, and more than one-third of No. 3, of the Journal.

Article II.—On Tchebycheff's Theory of the Totality of the Prime Numbers comprised within given Limits. By J. J. SYLVESTER.

If it be admitted that Legendre's approximate formula for the number of prime numbers inferior to a given number, which has been confirmed by direct enumeration of the number of prime numbers contained in the first few millions, can be extended to those remote regions of number which transcend the limits and even the possibilities of human experience, it will follow as a consequence that the average density of the distribution of prime numbers in the neighborhood of a large quantity x approximates to $\frac{1}{\log x}$, and consequently that the number of primes included between x and $(1+\varepsilon)\,x$, or if we like to say so, between $x+A$ and $(1+\varepsilon)x+B$, will be approximately equal to $\frac{\varepsilon x}{\log x}$, and therefore will become indefinitely great, however small ε may be taken. Although there can hardly be a doubt that such is the fact, no step had been taken previous to Tchebycheff's researches towards establishing this proposition demonstratively. Tchebycheff has succeeded in proving it, not, it is true, in an absolute sense, but for all values of ε exceeding the fraction $\frac{1}{5}$. He has done more, inasmuch as he has given formulae for actually ascertaining a number x for all values superior to which there will be at least any specified number K of primes included between $x+A$ and $(1+\varepsilon)\,x+B$ when ε has any positive value superior to $\frac{1}{5}$, and A and B are any quantities positive or negative. The object of what follows is to make a further advance in the same direction, and to show upon Tchebycheff's own principles that the proposition remains true when ε is conditioned no longer to be inferior to the fraction $\frac{1}{5}$, but to the fraction $\frac{1}{6}+\frac{1}{4642\frac{10}{11}}$, so that the excess above unity (the region, so to say, of darkness) is reduced to scarcely more than five-sixths of its value for the first named fraction. This conclusion is arrived at by aid exclusively of Tchebycheff's own formulae.

Tchebycheff's method may be regarded as the *first* approximation to the inferior and superior limits of a quantity ψx subject to the conditions

$$Vx > Ax + F\log x,$$
$$Vx < Ax + F_1 \log x,$$

where $Vx = \psi x - \psi\frac{x}{6} + \psi\frac{x}{7} - \psi\frac{x}{10}$ etc., (see Serret's *Cours d'Algèbre supérieure*, 4th Ed., Vol. 2, pp. 230-233), and to the further conditions that ψx is not less than $\psi x'$ if $x > x'$, and that $\psi x = 0$ when $x < 1$.

The limits obtained for ψx depend exclusively on these definitions, and would be applicable to any function ψx whatever that satisfied them.

The advance made in this article consists in pursuing the approximation through an indefinite number of steps, in each of which use is made of the limits attained in the preceding one, so as to bring the superior and inferior limits to ψx continually nearer and nearer to each other as regards the *principal* term (a multiple of x) which enters into each of them: the remaining terms over and above this multiple of x in the expressions for the limits always continue to be positive-integer powers of log x, and consequently the ratio of the limits becomes as nearly as we please identical with the ratio of the principal terms (*i. e.* of their coefficients) when x is taken sufficiently great: this ratio as given in the first approximation is $\frac{6}{5}$, but as the approximation is continued continually converges to but never reaches the fraction above referred to, viz: $\frac{7}{6}+\frac{1}{4642\frac{10}{11}}$.

Such, and such only, is the small but not unimportant contribution here supplied to Tchebycheff's remarkable theory. As no allusion is made to the possibility of this contraction of the limits in a work published so recently as 1879, by an author so competent as M. Serret, the writer presumes that it has hitherto remained unnoticed.

As a matter of fact, resting on evidence (although empirical in its character) leaving on the mind scarcely a shadow of doubt of the correctness of the conclusion to which it points, the true asymptotic value of $\frac{\psi x}{x}$ when ψx signifies the quantity so designated in M. Serret's article is unity. The mean value of the superior and inferior limits to this value given by the ordinary Tchebycheff formulæ is 1.01342 . . . ; the mean value given by the limits found in this article is .999697. . . . The deviation of the mean from the presumable true value of the ratio is accordingly less than .00303 in the one case and more than .01342 in the other, so that the effect of the seemingly slight change in the limits effected in this article is to give a result almost 45 times nearer to the truth than that given by the mean value of the limits previously determined. The article concludes with a demonstration that if by any method it should at any time be found possible to cause the superior and inferior limits of the asymptotic value of $\frac{\psi x}{x}$ to differ from each other by a quantity capable of being made indefinitely small, the quantity towards which these limits converge in opposite directions can be no other than *unity*.

Article III.—Specimen of a Literal Table for Binary Quantics, otherwise a Partition Table. By PROFESSOR CAYLEY.

"The table, commencing 1; b; c, b^2; d, bc, b^3; . . . is in fact a Partition Table, viz. considering the letters b, c, d, . . . as denoting 1, 2, 3, . . . respectively, it is 1^0; 1; 2, 11; 3, 12, 111; . . . a table of the partitions of the numbers 0, 1, 2, 3, . . . , expressed however in the literal form, in order to its giving the literal terms which enter into the coefficients of any covariant of a binary quantic. The table ought to have been made and published many years ago, before the calculation of the covariants of the quintic; and the present publication of it is, in some measure, an anachronism: but I in fact felt the need of it in some calculations in regard to the sextic; and I think the table may be found useful on other occasions. I have contented myself with calculating the table up to $s = 18$, that is, so as to include in it all the partitions of 18: it would, I think, be desirable to extend it further, say to $z = 26$, or even beyond this point, but perhaps without introducing any new letters, (that is, so as for the the higher numbers to give only the partitions with a largest part not exceeding 26)."

The above is followed by remarks upon the employment of the table, and an explanation of the method used in its construction; the table itself occupies about four and one-half pages.

Article IV.—Note on Hansen's General Formulae for Perturbations. By G. W. HILL.

Hansen's expression for the perturbations of the mean anomaly is modified in such a way that it does not involve the element he denominates as h. In the second place, a formula is given for determining the residual perturbations of the radius vector, which does not require the performance of any integration additional to those already made in getting the perturbations of the mean anomaly. This formula is given in its rigorous form and also as simplified when either second or third order terms may be neglected. It is absolutely necessary to make use of this formula in getting the constant term of the second coördinate; and advantageous even for getting the portion which involves simply the powers of the time. Hansen has nowhere given explicitly this equation, although his analysis implicitly involves it.

Article V.—On the Solution of a certain class of Difference and Differential Equations. By J. J. SYLVESTER.

Very many years ago the writer of this paper fell upon a class of equations in finite differences, of which the *general solution* can be found. See the posthumous edition of Boole's Finite Differences, edited by Mr. Moulton, pp. 229-231.

The equation in question has the persymmetrical determinant

$$\begin{vmatrix} U_x, & U_{x+1} \cdots\cdots & U_{x+i}, \\ U_{x+1}, & U_{x+2} \cdots\cdots & U_{x+i+1}, \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ U_{x+i}, & U_{x+i+1} \cdots\cdots & U_{x+2i} \end{vmatrix}$$

(i being any number), for its left hand and cm^x for its right-hand member. Instead of $U_x, U_{x+1} \ldots\ldots U_{x+2i}$, may be written $y, \frac{d}{dx}y, \ldots, \left(\frac{d}{dx}\right)^{2i} y$, and the corresponding differential equation will readily be seen to admit of an analogous solution. In the present article it is shown that leaving unaltered the left-hand member of the equation so formed (whether expressed in terms of U_x and its augmentatives, or of y and its derivatives) but writing, in the right-hand member, instead of the single exponential a linear function of $i+1$ exponentials, a *particular* solution of the equation so modified can be found for all values of i provided only that the exponentials in question m_1^x, $m_2^x, \ldots, m_i^x$ remain distinct from each other. In the case of any of these becoming equal (unless they all become so at once, in which case the original equation admitting of a known general solution comes back again) or what is the same thing in case of any of the coefficients of the linear functions vanishing, (unless they all but one vanish) the solution becomes illusory; but if instead of taking any group of the exponentials and making them absolutely equal, they be regarded as differing from each other by infinitesimal quantities, such group or each of several of such groups will be replaced by a rational integral function in x of an order inferior by unity to the number of coalescing exponentials in the group, multiplied by a single exponential; and the solution of the equation so modified may be elicited from the solution applicable to the general form of the equation. The cases of $i = 2$, $i = 3$, are worked out in full, and there would not be the slightest difficulty in obtaining analogous solutions for the differential equations $yy'' - y'^2 = Aa^x + Bb^x + Cc^x$, $yy''y'''' - yy'''^2 - y'^2y'''' + 2y'y''y''' - y''^3 = Aa^x + Bb^x + Cc^x + Dd^x$

Article VI.—On the Analytical Forms called Trees. By PROFESSOR CAYLEY.

This article contains a new formula, by generating functions, for the number of distinct trees of N knots; the formula being derived from the consideration of the centre or bicentre of number, instead of the centre or bicentre of length, which was the basis of the investigation previously given by the author, (*Brit. Assoc. Report*, 1875).

Articles VII, VIII, IX, X.—Notes.

1°. *On Symbols of Operation.* By PROFESSOR CROFTON.

2°. *On Segments made on Lines by Curves.* By MISS CHRISTINE LADD.

3°. *On the Multiplication of the* $(n-1)^{th}$ *power of a Symmetric Determinant of the* n^{th} *Order by the Second Power of any Determinant of the same Order.* By THOMAS MUIR.

4°. *On Newton's Method of Approximation.* By F. FRANKLIN.

Article XI.—Simple and Uniform Method of Obtaining Taylor's, Cayley's and Lagrange's Series. By J. C. GLASHAN.

Article XII.—Forms of Rolle's Theorem. By J. C. GLASHAN.

CONTENTS:—

I. *Functions of a Simple Variable.* 1. Fundamental Form (Rolle's). 2. Extension of Rolle's Form. 3. Lagrange's Form. 4. Cauchy's Form. 5. Special case of Cauchy's Form. 6. Extension of Cauchy's Form. 7. Forms expressed by definite integrals. 8. Remainder in Taylor's Theorem. 9. Remainder in Cayley's Theorem. 10. Remainder in Lagrange's Series.

II. *Functions of a Complex Variable.* 11. General Form. 12. Special Form. 13. Remainder in Taylor's Theorem.

This article covers about eleven pages, of which the last four will appear in No. 4 of the *Journal.*

American Chemical Journal. Edited by PROFESSOR REMSEN. Contents of *Vol. IV, No.* 2. May, 1882.

Article I.—On Certain Substances obtained from Turmeric. — Curcumin. By C. LORING JACKSON and A. E. MENKE.

Article II.—Dibromiodacrylic and Chlorbromiodacrylic Acids. By C. F. MABERY and RACHEL LLOYD.

Article III.—Preliminary Notice on Orthoiodbenzylbromide and its Derivatives. By C. F. MABERY and F. C. ROBINSON.

Article IV.—Chlortribrompropionic Acid. By C. F. MABERY and H. C. WEBER.

Article V.—Influence of Peptones and certain Inorganic Salts on the Diastatic Action of Saliva. By R. H. CHITTENDEN and J. S. ELY.

Article VI.—On the Determination of Reverted Phosphates. By THOMAS S. GLADDING.

Article VII.—Columbite and Orthite from Amelia County, Virginia. By F. P. DUNNINGTON.

Article VIII.—A New Mineral from Colorado. By F. W. CLARKE and N. W. PERRY.

Article IX.—Oxidation of Metatoluene-sulphamide. By IRA REMSEN and CHASE PALMER.

A Report on Progress in Agricultural Science. By H. P. ARMSBY.

Reviews of the Treatises on Organic Chemistry of Roscoe and Schorlemmer, and of Wislicenus. By THE EDITOR.

Notes: On a New Gas Regulator, Processes for Direct Coppering of Castings of Iron and Steel; Artificial Quinine.

Recent Publications.

MARYLAND HISTORICAL SOCIETY AND PEABODY INSTITUTE.

The following *Notes* are reprinted from *The Nation*, New York, March 16 and March 30, 1882.

Deposit of the Colonial Archives of the State with the Maryland Historical Society.

The colonial and revolutionary archives of Maryland are about to be transferred from the Land Office at Annapolis to the keeping of the Maryland Historical Society in Baltimore, in accordance with a bill lately passed by the Maryland Senate (March 16), and House of Delegates (March 12)—a bill which was the outgrowth of a memorial presented to the Legislature by the above Society, and already mentioned in these columns. These archives embrace the acts and resolves of the Province of Maryland; the council books of the Proprietary; the journals and correspondence of the Council of Safety; letters from Maryland delegates in the Continental Congress; sixty-two manuscript letters from George Washington; letters from Knox, Marion, Gates, Lincoln, Greene, Lee, Lafayette, Pulaski, Steuben, De Kalb, De Grasse, Rochambeau; and papers relating to the Maryland militia, which did such efficient service at the North under Smallwood, and at the South under John Eager Howard. While many valuable records of colonial and Revolutionary Maryland have been lost or damaged from lack of proper care, those which yet remain will be faithfully preserved by the Maryland Historical Society, whose library is already a treasure-house of Maryland laws, documents, manuscripts, family papers, pamphlets, broadsides, and newspaper files (covering the entire Revolutionary period, and extending as far back as 1728). For the safe keeping and easy consultation of the State papers and most valuable Society documents, a fire-proof room, adjoining the main part of the library, will probably be constructed, for members of the Society are now endeavoring to obtain funds for this purpose. The State has made an appropriation sufficient to begin the work of "arranging, editing, and publishing" the archives, together with "other documents pertaining to the history of Maryland." The State will be the owner of all volumes published by its aid, but the Society is empowered to sell these publications at cost price, and to add the proceeds to the State Publication Fund, simply reporting progress, receipts, and expenditures, from year to year, to the Legislature. Thus an undertaking which in the hands of politicians might perhaps have become a piece of printer's jobbery, as some States have found to their cost, is removed from all temptation and made a scientific enterprise, enlisting not only legislative aid, but also the resources of a learned Society and the pride of all good citizens. The enterprise is kept upon a State footing, honorable to Maryland, while entirely in the hands of a trustworthy record commission.

The Maryland Historical Society, when it shall have made the archives of the State generally accessible to students, will be in a position to encourage original research. It has a publication fund of $20,000, left by George Peabody for the printing of papers elucidating the history of Maryland or of the country at large. At a recent meeting, the Society voted a considerable appropriation from its treasury toward securing copies of certain Maryland documents, missing in the Annapolis collection, and yet preserved in England, for it was the custom of the proprietary Government to send duplicates of all important Acts of provincial legislation to the mother country. The "Stevens Index of Maryland documents in the State Paper Office, London," will doubtless be of great service to Maryland in recovering some of her long-lost records. This index, or calendar, contains descriptions and abstracts of 1,729 Maryland documents, covering the period from 1626 to 1780, and thus admirably supplementing the Annapolis collection. The index was prepared many years ago at the suggestion and expense of Henry Stevens, and was by him sold for £100 to George Peabody, who presented it to the Maryland Historical Society. This gift, of trifling cost when compared with other benefactions to Baltimore by the same hand, will yet, like the Peabody Institute, increase and multiply in power in proportion to the growing appreciation of its significance.

Catalogue of the Library of the Peabody Institute.

The Peabody Institute in Baltimore has now in type something over 150 royal octavo pages of its catalogue, upon the preparation of which, by means of an analytical card catalogue, the working force of the library has been employed for the past thirteen years. It will require four or five years longer to complete the arrangement and printing of the catalogue, and it will then probably embrace over 4,000 pages, published in four or five volumes. The proof-sheets, covering thus far about two-thirds of the subjects and authors to be catalogued under the letter A, promise certain valuable improvements in the art of cataloguing public libraries in this country. Under a given subject, or author, will be found not merely an alphabetical arrangement of the main authorities and titles, but also an alphabetical grouping of the chief collateral material, monographs, essays, magazine articles, and the like, that may be found in the Peabody Library touching the subject or author in hand. For example, under the head of "Æsthetics" we find, first, an alphabetical list of authors who have written more or less systematic works upon this subject; and, second, in finer type, an alphabetical list of minor authorities, dissertations, and miscellaneous articles. The name of the author, if known, is the guiding principle of this arrangement, otherwise the catchword of the essay or monograph is given, like the names of authors, in bold heavy type, so as to attract the eye at once upon the closely-printed page. Under the head of an author, for example, "Arago, Dominique François Jean, 1786–1853," we find an alphabetical analysis of his "Œuvres complètes," seventeen volumes. Instead of reprinting the table of contents for each volume, the contents of the whole series are arranged alphabetically, the *catchword* (not necessarily the initial word) of the titles serving as a guide in the classification; for instance, "Sur l'*action calorifique* et l'action chimique de la lumière, v. 7; Sur les *chronomètres* et les pendules, v. 12." This system of registering articles by alphabetical catchwords becomes of immense value when applied to the publications of learned societies, like the literary collections of the French Institute, Cambridge Philosophical Transactions, the publications of the Royal Irish Academy, and of the academies of St. Petersburg, Berlin, Vienna, Brussels, and the archives of Munich, and even to English and American reviews. Most catalogues, if they take any notice at all of the contents of a long series of volumes, simply give the contents of each volume by itself, the result being, in the case of a very long series, that a student is sometimes obliged to look through several pages of chaotic titles in order to ascertain whether there is any material relating to the subject he may have in hand. This annoyance and grievous waste of time will be entirely spared if the Peabody idea is systematically carried out and subject-titles are arranged alphabetically with appended references to volumes, but without regard to the succession of volumes. It should be as easy to find one's way through a vast collection of monographs and special treatises as through a complete dictionary of the English or French language. The body of existing science should be an encyclopædia of knowledge, properly indexed for the use of students, so that they may add to its volume without duplicating the work of predecessors.

The practical difficulties and labor involved in such a classification are beyond all estimate, for the present state of the world's scientific papers is but little removed from chaos. The Peabody idea has not yet been applied to the classification of the special articles on natural science to be found in the journals of European academies, but very much has been done in the fields of literature, art, and history. The idea is capable of indefinite expansion, and is only a suggestion of what the art of cataloguing a great public library of research may one day become. The responsibility of the Peabody undertaking falls upon the Provost of the Institute, Dr. N. H. Morison, who, in his personal direction of this great work, is ably seconded by Mr. Philip R. Uhler, the official librarian of the Peabody. The pains these gentlemen have taken in simply laying the foundations for this catalogue is not, and cannot yet be, appreciated by the general public. The special, analytical card catalogue, registering not merely all books entered upon the public card catalogue, but all magazine articles, analyses of journals and literary collections, was an indispensable preliminary to a published catalogue. This work, when finished, will be a vast collection of bibliographies—literally thousands of classified lists—which will prove of the greatest value to Baltimore specialists, in showing what resources are already available, and will also be of the greatest convenience to the public at large.

A NOTE ON TRIPLE OBJECTIVES.

The last number of the *Vierteljahrsschrift der Astronomischen Gesellschaft* contains a review of a paper On Triple Objectives, with Complete Color Correction, which was published in the *American Journal of Science*, (1879). The reviewer has apparently mistaken a table of differences, which are derived from calculations entirely independent of all theory, as an indication of the inexactness of the method there pursued; at least such must be the conclusion from his having followed the table for comparison with a wholly irrelevant quotation of the maximum residual found by Professor Schmidt in applying a formula for expressing indices of refraction as a function of wave length. These differences depend almost exclusively upon errors in the measures of indices, which enter here multiplied by large coefficients. Had he referred to the table of residuals he would have found that, in the group of refractive indices discussed in both papers, the simpler empirical formula chosen gives no residual as large as that quoted from Professor Schmidt's work, and in the group of more accurately determined indices from an earlier paper by the same writer, the formula is much more satisfactory.

Again, the reviewer objects to the suggestion of an error in n_E of Fraunhofer's Flint 13, because Professor Schmidt did not recognize it, not observing that this is a matter of course since he used the fictitious values of the indices derived from his empirical formula, whereas the author used the values as given directly by the observations. If a similar course had been followed in this paper all the differences would have been zero.

CHARLES S. HASTINGS.

Johns Hopkins University, May 13, 1882.

The lectures of Professor SIMON NEWCOMB on the Principles of Taxation, announced for the current academic year, were postponed by reason of sickness and other causes. It is expected that they will be delivered in November next.

The CHESAPEAKE ZOÖLOGICAL LABORATORY, for the study of forms of marine life, began its session for the season, under the direction of Dr. Brooks, at Beaufort, N. C., on May 1. The usual number of students have been enrolled.

The SEMI-ANNUAL EXAMINATIONS will begin on May 22 and continue daily till June 5. The entrance examinations will be in progress from June 5 to 8.

The Johns Hopkins University Circulars are printed by Messrs. JOHN MURPHY & CO., 182 West Baltimore Street, Baltimore, from whom copies may be obtained. They may also be procured, as soon as published, from Messrs. CUSHINGS & BAILEY, No. 262 West Baltimore Street, Baltimore. Price 5 cents each.