## ESSAYS ON

# POLITICAL CONSTRAINTS AND FISCAL POLICY 

by

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#### Abstract

This dissertation contributes to the literature on the relationship between political constraints and fiscal policy outcomes. In the first two chapters, we focus on the level of detail of a government's budget, analyzing two budgetary institutions, the number of line-item appropriations and the number of appropriation bills. In the third chapter, we investigate commodity tax competition between two countries that differ in size and transportation cost. Finally, in the fourth chapter, we analyze investments in a state's fiscal capacity, letting public goods accumulate.

In Chapter 1, we analyze a budget's level of detail, focusing on the number of line-item appropriations. The executive authority receives the right for discretionary spending via appropriation bills approved by the legislative authority. An appropriation bill is composed of line-item appropriations that restrict the allocation of the authorized budget. We take Baron and Ferejohn (1989) as a starting point to develop a model of legislative bargaining in which two legislators decide on the number of line-item appropriations, choosing between one- and two-item budgets, and on the provision of two public goods. A trade-off between flexibility and commitment emerges in choosing the number of line-item appropriations. While a low probability of polarization between legislators leads to a one-item budget, a high probability of polarization leads to a two-item budget. Moreover, a high probability of polarization between legislators increases government spending. We extend our model to analyze the line-item veto right and flexible line-item appropriations.


In Chapter 2, we continue to investigate a budget's level of detail, concentrating
on the number of appropriation bills. In the first chapter, we assumed that the appropriation-bill count is fixed, but the number of line-items in a bill is flexible. In this chapter, we assume that an appropriation bill's line-item count is fixed but the number of bills is flexible. Moreover, in the previous chapter, a budget's level of detail and its size and composition are determined at the same time. In this chapter, a budget's level of detail is determined first, and its size and composition are fixed subsequently. Specifically, two legislators first decide on the number of appropriation bills, choosing between one- and two-bill budgeting, and subsequently on the provision of two public goods. While two-bill budgeting is more costly, it offers the executive authority commitment opportunity. We show that when polarization between legislators is high, public-good provision is higher under twobill budgeting than under one-bill budgeting. Moreover, a high polarization and more equal distribution of political power between legislators encourages two-bill budgeting. We extend our model to analyze group-specific transfers.

In Chapter 3, we analyze commodity tax competition between two countries. Kanbur and Keen (1993) and Nielsen (2001) offer two models to analyze commodity tax competition between two countries that differ in size. Nielsen's model provides a simpler setting as it gives continuous best-response functions. In both models there always exists a unique equilibrium in pure strategies in which the larger country sets a higher tax rate. However, in both models transportation costs are assumed to be equal in two countries. We relax this assumption. We show that this leads to discontinuity of the best-response correspondences in Nielsen's model. Moreover, existence of equilibrium in pure strategies is no longer guaranteed in either model. We give necessary and sufficient conditions for the existence of
equilibrium in pure strategies in both models. Additionally, we show that, when an equilibrium in pure strategies does not exist, there can exist an equilibrium in mixed strategies in which the larger country sets a lower tax rate with a positive probability in both models.

Finally, in Chapter 4, we turn our attention to a state's capacity to raise taxes. Besley and Persson (2010) propose a model in which two groups in a society decide on government policy - taxes and provision of a public good - and investment in state capacity to collect taxes, also known as fiscal capacity. One of the authors' main results is that an increase in the expected value of the public good increases investment in fiscal capacity. However, the authors assume that the public good depreciates completely in a period. As Battaglini and Coate (2007) state, most public goods accumulate over time. We add accumulation of the public good to the Besley-Persson model. We show that an increase in the expected value of the public good can decrease fiscal-capacity investment in this case. Moreover, an increase in the depreciation rate of the public good increases it.

Keywords: Discretionary spending, appropriation bill, line-item appropriation, line-item veto, executive authority, legislative authority, public good, polarization, commodity tax competition, cross-border shopping, transportation cost, fiscal capacity, durable public goods.

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## General Introduction

Historically, economists have developed fiscal policy recommendations assuming that governments raise taxes and spend money to benefit society. However, policy decisions are taken by self-interested politicians whose interests may contradict those of the rest of society. In these situations, the rules that govern political processes - that is, institutions - discipline politicians in their policy choices. Considering these facts, in his 1986 Nobel lecture, while delivering the central message of Knut Wicksell, who inspired his research, James M. Buchanan stated "Economists should cease proffering policy advice as if they were employed by a benevolent despot, and they should look to the structure within which political decisions are made." Indeed, there is now a large economics literature analyzing fiscal policy under political constraints. The main focus of this literature is the effects of institutions and politicians' motivations on fiscal policy outcomes. In this literature, political and economic behavior are acquired from the same individual preferences. Policies affect economic behavior that generates economic outcomes in the markets and economic outcomes affect political behavior that generates policies under institutional constraints. Moreover, an emerging literature extends the analysis of fiscal policy to cover institutions and explain their emergence. Research in this area is usually called as "political economy". This dissertation aims to contribute to the theoretical side of the political economy literature analyzing fiscal policy and the determinants of institutions.

Chapter 1 focuses on the preparation of a government's budget. An important tradition in the political economy literature assumes that politicians are motivated
by policy outcomes. ${ }^{1}$ A large set of papers in this tradition analyze bargaining in legislatures over economic policy among legislators representing the interests of different groups in society. ${ }^{2}$ Particularly, many papers, such as Leblanc et al. (2000), Volden and Wiseman (2007) and Battaglini and Coate (2007, 2008), study bargaining over the budget. These authors assume that, during the budget negotiations, legislators decide on the composition of the budget in complete detail. However, the Organisation for Economic Co-operation and Development (OECD) surveys show that the government budgets are prepared at different levels of detail, as measured by the number of items in the budget, and leave some of the spending decisions to the executive authority. In this chapter, we analyze the effects of the budget's level of detail on the size and composition of government spending and how the budget's level of detail is determined.

In our analysis, two legislators bargain over the budget of a government department. One of the legislators proposes a budget that can be specified at two different levels of detail. After the other legislator approves the budget, the proposer decides on the allocation of the tax revenue for the provision of the public goods. The public goods' payoffs for the legislators are random and determined after the budget is approved. If they take payoffs from different public goods, we say that there is polarization between the legislators. As a result, while a less detailed budget provides more flexibility to the proposer, the more detailed budget provides more commitment opportunity for the provision of public goods. We show that the proposer offers a less detailed budget if the probability of polarization is low and a more detailed budget if the probability of polarization is high. This is because

[^0]the proposer actually always prefers a less detailed budget over a more detailed one. However, for the responder to approve a less detailed budget, the preferences of the proposer and responder over the public goods should be similar. Hence, the probability of polarization should be low. If the probability of polarization is high, then to get the approval of the responder, the proposer offers a more detailed budget. Furthermore, we show that a high probability of polarization increases government spending, since it requires provision of more kinds of public goods.

Chapter 2 approaches the level of detail in a government's budget from a different perspective. In the first chapter, we assume that the number of government departments is fixed but the level of detail for a department is flexible. In the second chapter, we assume that the budget's level of detail for a department is fixed but the number of government departments is flexible. So, if there are more departments, the government's budget is more detailed. This perspective allows us to analyze the effects of new political factors on the budget's level of detail.

Our analysis in this chapter is conducted in two stages. In the first stage the legislators bargain over the number of government departments and over the size and composition of the government's budget in the second stage. In particular, in the first stage, a legislator proposes how many government departments to have, choosing between one and two. If the other legislator accepts the proposal, the political process continues obeying this rule; otherwise, a status-quo number of government departments prevails. In the second stage, depending on their political power, one of the legislators is chosen to propose a budget taking the number of government departments as given. After the other legislator approves the budget, the proposer decides on the allocation of the tax revenue for the provision of public
goods. We assume that building up a new department requires investment; in other words, it is a costly process. Thus, while having more departments increases the costs for the legislators, it provides a more detailed commitment opportunity to the budget-proposing legislator for the provision of public goods. We show that if polarization is low, public-good provision is the same under both department counts. This is because low polarization implies that the legislators' preferences for public goods are similar. Therefore, under any budgetary level of detail, the proposer makes an offer that results in his optimal policy. However, if polarization is high, public-good provision is lower under one government department. In this case, the preferences of the legislators on public goods are dissimilar. Under a less-detailed budget, the proposer cannot commit to the provision of the public goods that the responder prefers. Thus, he can get approval only for a small budget, which decreases the payoffs to both legislators. Given the effects of the number of government departments on the provision of public goods, the legislators choose to have one department when polarization is low: If polarization is low, the number of departments does not affect the provision of public goods and having two departments is costly. If polarization is high, the choice of the number of government departments depends on the status-quo number of departments and the distribution of the political power. More specifically, the status quo of two departments and more equal distribution of political power stimulates the choice of two departments.

Chapter 3 concentrates on fiscal policy interactions among countries. A branch of political economy literature assumes that the politicians are motivated by the rents that they can extract by holding office. ${ }^{3}$ Many papers that we can classify

[^1]in this branch analyze international tax competition when the tax base is mobile between countries. ${ }^{4}$ Particularly, an important portion of these papers, like Kanbur and Keen (1993), Trandel (1994), Ohsawa (1999) and Nielsen (2001), concentrates on commodity tax competition. In the common setup of these papers, revenuemaximizing politicians in different countries compete using the tax rate on a single commodity when consumers can engage in cross-border shopping by paying a travel cost. Politicians face the following trade-off in choosing the tax rate: on the one hand, a higher tax rate increases the tax revenue collected from the consumers shopping within the country; on the other hand, by causing more consumers to shop in other countries, it shrinks the tax base.

The seminal model in this literature is provided by Kanbur and Keen (1993) for a two-country setting. Later, Nielsen (2001) provided a simpler model permitting analysis of more complicated situations. The main results of the two papers are that there always exists a unique equilibrium in pure strategies and the larger country chooses a higher tax rate. However, these authors assume that the travel costs are the same in the two countries. In this chapter, we relax this assumption. In particular, we show that if the travel costs are different in the two countries, an equilibrium in pure strategies is no longer guaranteed to exist. We give necessary and sufficient conditions for the existence of equilibrium in pure strategies. Moreover, we show that there can exist an equilibrium in mixed strategies in which the larger country chooses a lower tax rate with a positive probability.

Finally, in Chapter 4 we turn our attention to the capacity of a state to raise (1986) and Polo (1998).
${ }^{4}$ See, for example, Edwards and Keen (1996) and Eggert and Sørensen (2008).
taxes. In general, the economics literature assumes that the state has enough capacity to raise taxes at any rate. However, effective tax collection requires investment in administrations such as the IRS in the United States, and in systems for monitoring tax compliance. Thus, historians have long been investigating state capacity to collect taxes as a fact to be explained. ${ }^{5}$ Now, an emerging literature in political economy also develops a similar approach. The bulk of the research in this emerging literature is composed of the papers of Timothy Besley and Torsten Persson (see, for example, Besley and Persson, 2009, 2010, 2011; Besley et al., 2013). A main result of this emerging literature is that risk of external conflict (war) increases investments in state capacity to collect taxes. However, these authors assume that investments in defense activities in the current period yield no benefit in the future. In this chapter, we relax this assumption and let defense activities accumulate over time, although they can depreciate at some rate.

Our analysis starts with two time periods and two groups in the society. In the first period, the incumbent group that holds political power decides on the current tax rate and the spending of tax revenue. The incumbent's choice of the tax rate is limited by the state's capacity to raise taxes. The tax revenue can be transferred back to citizens or invested in defense activities or in state capacity to collect more taxes in the next period. In the second period, the political power changes hands with some probability. The new incumbent decides on the second period's tax rate and the spending of the tax revenue. The incumbents spend tax revenue for transfers only if there is peace, and if they do, they transfer all the money to people in their own group. In the first period, investing the tax revenue in defense activities increases the incumbent's payoff in both periods by

[^2]accumulation. Investing it in state capacity increases only its second-period payoff. This payoff is possible if and only if the incumbent stays in power or there is a war. We show that a decrease in the depreciation rate of defense activities decreases investment in state capacity because it increases the benefit of investing in defense activities. In addition, we show that an increase in the risk of external conflict can decrease investment in state capacity, contrary to the general result in the literature. This is because an increase in the risk of external conflict increases the benefit of investment in both defense activities and state capacity. Thus, investment in state capacity increases if and only if the latter effect dominates.

In summary, in our dissertation, we take a first step to analyze the determinants of the level of detail of a government's budget and its effects on the size and composition of government spending. In our analysis, we highlight the effects of polarization and distribution of political power between the groups in society, and the status-quo organization of government. In addition, we investigate the role of asymmetric travel costs in commodity tax competition between two countries. We explore the effects of such costs on the existence of equilibria and the predictions of the workhorse models in the literature. Lastly, we focus on a state's capacity to collect taxes. We incorporate the accumulating nature of defense activities into the seminal models in the literature and analyze its implications for the main conclusions of these papers.

## Chapter 1

## Line-item Appropriations and

## Government Spending

### 1.1 Introduction

An executive authority chooses fiscal policy under the constraint of a government budget. A government budget is composed of mandatory and discretionary spending: mandatory spending is governed by formulas or criteria set by law and discretionary spending by annual appropriation bills negotiated by the legislative authority. Thus, institutions governing negotiations of appropriation bills are important for fiscal-policy outcomes. In this paper, we focus on one of these institutions, namely the number of line-item appropriations.

Line-item appropriations are the constituents of an appropriation bill. They restrict allocation of the budget authorized by the appropriation bill. As a simple example, assume that the Department of Education (DoE) provides only two public services, primary and higher education. The DoE receives authority to spend money
through appropriation bills approved by the legislative authority. An appropriation bill for the DoE can be written at two different levels of detail. The bill can have one item, which only specifies the total amount of money that can be spent on education services; alternatively, it can have two items, specifying the amounts that can be spent on the two education services separately. A one-item budget provides more flexibility to the executive authority in choosing education policies, as it allows the executive authority to allocate the DoE's total budget between the two services. Therefore, the number of line-item appropriations determines legislative versus executive control over government policies.

Empirical data presents some interesting facts about the number of line-item appropriations. As seen in Figure 1.1, the number of line-item appropriations differs extensively among countries and over time. ${ }^{1}$ In 2012, while countries like the Slovak Republic, Chile, and the United Kingdom had less than 250 line-item appropriations, countries like Turkey and Portugal had more than 40,000 line-item appropriations in their budgets. Between 2007 and 2012, while countries like Austria, Italy, and Slovenia decreased their number of line-item appropriations, countries like Mexico, Belgium, and Japan increased theirs. In the United States, in 2007, the number of line-item appropriations was 1514; in 2012, this number was 1700 . At this point, one may ask if the number of line-item appropriations is correlated with any economic variable, as the variations in the number of line-item appropriations may be purely an accounting issue. In Figure 1.2, we show that

[^3]this is not the case, by presenting the line-item counts and discretionary budget shares of six US government departments between 1990 and 2015. ${ }^{2}$ The correlation between number of line-item appropriations and the discretionary budget share is 0.73 for the Department of the Treasury and 0.22 for the Department of Defense. In contrast, it is -0.76 for the Department of Labor, -0.33 for the Department of Housing and Urban Development, -0.3 for the Department of Education and -0.13 for the Department of Health and Human Services. So, we observe both negative and positive correlations between the number of line-item appropriations and the discretionary budget share of US government departments. There is a growing literature on the effects of institutions on public finance (see, for example, Persson et al., 2000; Lizzeri and Persico, 2001; Milesi-Ferretti et al., 2002; Bowen et al., 2014). However, to the best of our knowledge, there is no analysis of the number of line-item appropriations in the literature.

We take a first step in analyzing the effects of the number of line-item appropriations on the size and composition of government spending, and how the number of line-item appropriations is determined. For this purpose, we take Baron and Ferejohn (1989) as a departure point to develop a model of legislative bargaining in which two legislators decide on the number of line-item appropriations and the government policy - the income tax rate and the provision of two public goods. We consider the two legislators as constituting the legislative authority. The decision

[^4]

Figure 1.1: Number of line-item appropriations contained in each country's budget.


Figure 1.2: Number of line-item appropriations and discretionary budget share for six US government departments.
process follows several steps. First, nature chooses one of the legislators to propose a budget. We consider the selected legislator as constituting the executive authority. The proposer can choose to propose either a one- or two-item budget. A one-item budget specifies only the income tax rate; a two-item budget specifies the income tax rate and how the tax revenue should be allocated for the provision of the two public goods. If the other legislator accepts the budget proposal, then nature determines from which public good each of the legislators takes utility. If they take utility from different public goods, we say that there is polarization between the legislators. After nature's choice of the preferences, the proposer decides on the provision of the two public goods under the constraint of the approved budget. If the responder rejects the budget proposal, a status-quo policy is implemented.

We show that, while the proposer offers a one-item budget under a low probability of polarization, he offers a two-item budget under a high probability of polarization. To understand why, notice that given an income tax rate, the proposer prefers a one-item budget over a two-item budget. This is because a one-item budget provides the flexibility to choose the allocation of the total tax revenue for the provision of the public goods: He can allocate the whole tax revenue for the provision of the public good that gives him positive utility, which depends on nature's choice of his preference parameter. Yet, for the responder to accept a budget proposal, his expected utility from the proposer's policy choice should be higher than his utility from the status-quo policy. Therefore, the probability of polarization between proposer and responder should be low for a one-item budget proposal to be accepted. If the probability of polarization between legislators is high, then the legislators will take utilities from the different public goods with
a high probability. Therefore, the responder's expected utility from a one-item budget will be less than his utility from the status-quo policy. As a result, he will reject a one-item budget proposal. Thus, if the probability of polarization between legislators is high, the proposer offers a two-item budget, which forces him to commit to the provision of both public goods. Additionally, under a high probability of polarization, government spending is higher than under a low probability of polarization. This is because while the public good that is less valuable to the proposer is not provided under a low probability of polarization, it is provided at a positive level under a high probability of polarization.

In an extension of our model, we analyze a situation in which each line-item appropriation can be accepted or rejected separately. Such line-item veto powers are given to many state governors in the US. However, there are few theoretical models that analyze the effects of the line-item veto (see, for example, Carter and Schap, 1987, 1900; Dearden and Husted, 1993). Furthermore, we do not observe the line-item veto in the legislative decision-making process. So, it becomes interesting to ask why legislators do not have the line-item veto powers. However, to the best of our knowledge, there is no paper in the literature that addresses this question. We offer a legislative-bargaining model to answer these questions. We show that a line-item veto power given to the responder leads to no public-good provision and decreases the payoff of both legislators.

In another extension of our model, we analyze a situation in which the proposer can make transfers between the line-item appropriations after the responder accepts the budget. Such transfers are allowed in the US and in some other OECD countries to a limited extent. So, it is important to understand their welfare effects. We
address this question by allowing flexible line-item appropriations in our model. We show that this always increases the proposer's payoff, but can effect the responder's both positively and negatively. In particular, it increases the responder's payoff if and only if nature's choice probability of the responder's preference parameter is high enough.

Our model predicts that a higher probability of polarization between legislators leads to a larger number of line-item appropriations. We check this prediction of our model, first, with the US government budget data that we collected for its six departments. To measure polarization in the US, we use the partisan conflict measure of Azzimonti (2016). We show that there is a positive correlation between the level of partisan conflict and the total number of line-item appropriations for the six US government departments. This confirms the prediction of our model. We check the same prediction of our model also with the OECD budget survey data on the number of line-item appropriations across countries. Alesina et al. (1999) uses ethnic fractionalization to measure polarization in the US. Following them, we use the "lack of ethnic tension" measure of the PRS Group's International Country Risk Guide (ICRG) to measure lack of polarization in the countries surveyed by the OECD. We show that there is a negative correlation between lack of ethnic tension and the number of line-item appropriations. This provides another confirmation for the prediction of our model.

The remainder of the paper is organized as follows. In Section 1.2, we summarize the related literature. In Section 1.3, we present our model. In Section 1.4, we define equilibrium for our model and analyze the determinants and the effects of the number of line-item appropriations. In Section 1.5, we extend our model to
analyze the line-item veto and flexible line-item appropriations. In Section 1.6, we discuss some empirical evidence about the main prediction of our model. Finally, Section 1.7 concludes.

### 1.2 Related Literature

Our model is based on the legislative bargaining model of Baron and Ferejohn (1989), which is applied to public finance by Persson (1998), followed such papers as Leblanc et al. (2000), Volden and Wiseman (2007), and Battaglini and Coate (2008). These papers assume that the legislative authority decides on all dimensions of government policy. We contribute to this literature by analyzing a model that lets the proposer choose to decide on some dimensions of government policy after the legislative process. We show that this may decrease the control of the responder over government policy.

There is a vast literature on the provision of public goods under politicaleconomy frictions as surveyed in Persson and Tabellini (2000). A subset of this literature analyzes the effects of different political institutions on the provision of public goods. For example, Persson et al. (2000), Lizzeri and Persico (2001), and Milesi-Ferretti et al. (2002) analyze the provision of public goods under different electoral systems. Bowen et al. (2014) investigate the provision of a public good under two budgetary institutions: mandatory and discretionary spending programs. Our paper adds to this literature by focusing on the number of line-item appropriations.

A growing literature analyzes the determinants and effects of the limits on executive power. A recent paper in this literature that is relevant to ours is Besley
et al. (2014). They analyze an infinite horizon model in which the executive authority determines provision of a public good, group-specific transfers and future limits of executive power in each period. They show that a higher probability of losing office leads to stronger executive constraints and this increases the provision of the public good. We add to this literature by analyzing the constraints on the executive power induced by line-item appropriations. The Besley et al. (2014) model does not allow consideration of the effects of polarization on the provision of public goods. We show that constraints on executive power increase the provision of public goods only when the probability of polarization is high. Other papers in this literature are Lagunoff (2001), Aghion et al. (2004), Maskin and Tirole (2004), Ticchi and Vindigni (2010), Acemoglu et al. (2013), Robinson and Torvik (2013) and Karakas (2016).

A growing theoretical and empirical literature analyzes the effects of polarization on economic outcomes. For example, on the theoretical side, Alesina et al. (1999) investigate the effects of polarization on provision of public goods, Azzimonti (2011) on investment and government spending and Azzimonti and Talbert (2014) on macroeconomics fluctuations. Esteban and Ray $(1994,2008,2011)$ provide a theory of measurement of polarization and analyze the effects of polarization on conflict. On the empirical side, Lindqvist and Östling (2010) investigate the effects of polarization on government spending, and Alt and Lassen (2006) its effects on electoral cycles. Our paper connects to this literature by theoretically analyzing the effects of the probability of polarization on the choice of the line-item count.

### 1.3 Model

### 1.3.1 Economic Environment

The society is separated into two groups of citizens, each with unit mass and indexed by $i \in\{1,2\}$. Each citizen has an endowment of one unit of labor denoted by $l$. There is a consumption good denoted by $z$, and two public goods denoted by $g$ and indexed by $j \in\{1,2\}$. The consumption good is produced with the technology $f(l)=l$ and the public goods with the technology $f_{j}(l)=\alpha_{j} l$ where $\alpha_{j}>0$ for each $j$.

A citizen's utility function in group $i$ is given by

$$
u_{i}\left(z, g_{1}, g_{2} ; \beta_{i}\right)=z+\beta_{i} h\left(g_{1}\right)+\left(1-\beta_{i}\right) h\left(g_{2}\right),
$$

where $h: \mathbb{R}_{+} \rightarrow \mathbb{R}_{+}$is an increasing and strictly concave function with $h(0)=0$. We assume that $\beta_{i} \in\{0,1\}$ for each $i$. If $\beta_{1} \neq \beta_{2}$, then we say that there is polarization among citizens.

There is a competitive labor market. Thus, the assumption on the production technology of the consumption good implies that the wage rate is equal to 1 .

### 1.3.2 Government Policies

A government policy is described by the triplet $\left(\tau, l_{1}, l_{2}\right)$, where $\tau$ is the income tax rate and $l_{j}$ is the amount of labor allocated for the production of public good
$j$. The set of feasible government policies is given by

$$
P=\left\{\left(\tau, l_{1}, l_{2}\right) \in[0,1] \times\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}^{2}: l_{1}+l_{2}=2 \tau \text { and } l_{1}+l_{2} \leq L\right\}
$$

where $L$ is the maximum amount of labor that can be hired for the production of public goods. Feasibility requires $L \leq 2$, which is the total endowment of labor in the economy. We assume that, for production of a public good, labor can only be hired at discrete levels; that is, $l_{j} \in\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}$ for each $j$ where $n \in \mathbb{N}$ and $n \geq 2 .{ }^{3}$ We also assume that tax revenue can only be spent for production of public goods and not wasted. Let

$$
Q=\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}^{2}
$$

denote the set of labor-hiring policies.
An economic environment with government policies can be summarized by

$$
\Gamma=\left(\left(\alpha_{j}\right)_{j \in\{1,2\}}, h, n, L\right)
$$

### 1.3.3 Political Process

Government policy decisions are made by a legislature consisting of representatives of both groups. As described in Figure 1.3, nature first selects one of the legislators to propose a budget $\mathbf{b}$. The proposer can choose to propose either a one- or a two-item budget.

[^5]One-Item Budget.-A one-item budget is composed of only an income tax rate, $\tau$. Let

$$
B_{s}=\left\{0, \frac{1}{2 n} L, \frac{2}{2 n} L, \ldots, \frac{1}{2} L\right\}
$$

denote the set of feasible one-item budget proposals. If the proposer chooses to propose a one-item budget, the other legislator responds by either accepting or rejecting the proposal. If he accepts the proposal, nature determines $\beta_{i}$ for each $i$ in the next stage. The probability that nature chooses $\beta_{i}=1$ is $\lambda_{i} \in(0,1)$. Subsequently, the proposer chooses a labor-hiring policy, $\mathbf{q}=\left(l_{1}, l_{2}\right)$. Let

$$
Q_{s}(\mathbf{b})=\left\{\left(l_{1}, l_{2}\right) \in\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}^{2}: l_{1}+l_{2}=2 \tau\right\}
$$

denote the set of feasible labor-hiring policies given a one-item budget, $\mathbf{b}=\tau \in B_{s}$. If the responder rejects the budget proposal, then the status-quo policy $\mathbf{q}^{s}=$ $\left(\tau^{s}, l_{1}^{s}, l_{2}^{s}\right)=(0,0,0)$ is implemented.

Two-Item Budget.-A two-item budget is composed of an income tax rate, $\tau$, and the provision of both public goods $\left(g_{1}, g_{2}\right)$. Since the only input is labor, a two-item budget can be specified with an income tax rate $\tau$ and amounts of labor $\left(l_{1}, l_{2}\right)$ that will be hired for the provision of public goods. Let

$$
\begin{aligned}
B_{d}= & \left\{\mathbf{b}_{d}=\left(\tau, l_{1}, l_{2}\right) \in\left\{0, \frac{1}{2 n} L, \frac{2}{2 n} L, \ldots, \frac{1}{2} L\right\} \times\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}^{2}\right. \\
& \left.: l_{1}+l_{2}=2 \tau\right\}
\end{aligned}
$$

denote the set of feasible two-item budget proposals. If the proposer chooses to propose a two-item budget, then the other legislator responds by either accepting


Figure 1.3: Political process.
or rejecting the proposal. If he accepts the proposal, nature determines $\beta_{i}$ for each $i$ in the next stage. The probability that nature chooses $\beta_{i}=1$ is $\lambda_{i}$. Subsequently, the proposer implements the labor-hiring policy specified in the budget proposal. For convenience of notation, let

$$
Q_{d}(\mathbf{b})=\left\{\left(l_{1}, l_{2}\right)\right\}
$$

denote the set of feasible labor-hiring policies given a two-item budget $\mathbf{b}=$ $\left(\tau, l_{1}, l_{2}\right) \in B_{d}$. If the responder rejects the budget proposal, then the status-quo policy $\mathbf{q}^{s}$ is implemented.

### 1.4 Political Equilibrium

A strategy for legislator $i,\left(b_{i}, a_{i}, c_{i}\right)$, is composed of a budget-proposal strategy, $b_{i} \in B_{s} \cup B_{d}$, a budget-acceptance strategy, $a_{i}: B_{s} \cup B_{d} \rightarrow\{0,1\}$, and a labor-hiring strategy, $c_{i}:\left(B_{s} \cup B_{d}\right) \times\{0,1\} \rightarrow Q$, such that $c_{i}\left(\mathbf{b}, \beta_{i}\right) \in Q_{\sigma}(\mathbf{b})$ for each $\mathbf{b} \in B_{\sigma}$, $\sigma \in\{s, d\}$ and $\beta_{i} \in\{0,1\}$. Legislator $i$ 's budget-acceptance strategy, $a_{i}(\mathbf{b})$, takes the value 1 if legislator $i$ accepts the budget proposal, $\mathbf{b}$, offered by legislator $i^{\prime} \neq i$, and 0 otherwise.

We consider subgame-perfect equilibria. We focus on equilibria such that $a_{i}(\mathbf{b})=1$ when legislator $i$ is indifferent between $c_{i^{\prime}}(\mathbf{b}, \cdot)$ and $\mathbf{q}^{s}$, and $a_{i}\left(b^{i^{\prime}}\right)=1$ for all $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$. The first condition requires that a responding legislator accepts any proposal that he is indifferent between accepting and rejecting, and the second condition requires that the equilibrium proposals are always accepted. A formal definition of an equilibrium is given in Appendix A.1.

We first present our assumptions and then our results. Formal proofs of all results are given in Appendix A.2.

Assumption 1.1. Production technologies of the two public goods are the same; specifically, $\alpha_{1}=\alpha_{2}=\alpha>0$.

Assumption 1.1 is not necessary for getting results from our model. However, it makes it easier to state our results.

Assumption 1.2. Any two different government policies give different utilities.
Specifically, $u_{i}\left(1-\tau, \alpha l_{1}, \alpha l_{2} ; \beta_{i}\right) \neq u_{i}\left(1-\tau^{\prime}, \alpha l_{1}^{\prime}, \alpha l_{2}^{\prime} ; \beta_{i}\right)$ for any $\left(\tau, l_{1}, l_{2}\right),\left(\tau^{\prime}, l_{1}^{\prime}, l_{2}^{\prime}\right)$ $\in P$ with $\left(\tau, l_{1}, l_{2}\right) \neq\left(\tau^{\prime}, l_{1}^{\prime}, l_{2}^{\prime}\right), \beta_{i} \in\{0,1\}$, and $i \in\{1,2\}$.

Assumption 1.2 enables the model to attain a unique equilibrium. We make this assumption to avoid nongeneric cases that complicate the statement and proof of our results.

Before stating our third assumption, we define a collection of sets that $h$ can belong to. Let

$$
\begin{aligned}
H_{0}= & \left\{y \in \mathcal{H}: \frac{1}{2 n} L>y\left(\alpha \frac{2}{n} L\right)-y\left(\alpha \frac{1}{n} L\right)\right\}, \\
H_{m}= & \left\{y \in \mathcal{H}: y\left(\alpha \frac{m+1}{n} L\right)-y\left(\alpha \frac{m}{n} L\right)>\frac{1}{2 n} L\right. \\
& \left.>y\left(\alpha \frac{m+2}{n} L\right)-y\left(\alpha \frac{m+1}{n} L\right)\right\}
\end{aligned}
$$

for each $m=1, \ldots, n-2$ and

$$
H_{n-1}=\left\{y \in \mathcal{H}: y(\alpha L)-y\left(\alpha \frac{n-1}{n} L\right)>\frac{1}{2 n} L\right\},
$$

where $\mathcal{H}$ is the set of increasing and strictly concave functions from $\mathbb{R}_{+}$to $\mathbb{R}_{+}$. For any $m=1, \ldots, n-1$, if $h \in H_{m}$, we have $h\left(\alpha \frac{k+1}{n} L\right)-h\left(\alpha \frac{k}{n} L\right)>\frac{1}{2 n} L$ for all $k=1, \ldots m$ and $h\left(\alpha \frac{k^{\prime}}{n} L\right)-h\left(\alpha \frac{k^{\prime}-1}{n} L\right)<\frac{1}{2 n} L$ for all $k^{\prime}=m+2, \ldots n$ because of the strict concavity of $h$. Thus, as the index of the set that $h$ belongs to increases, the value of public goods increases. The expression on the left of the inequality in $H_{m}$ denotes the benefit of increasing the labor allocated for production of a public good from $\frac{m}{n} L$ to $\frac{m+1}{n} L$. The expression on the right denotes the benefit of increasing the labor allocated for production of a public good from $\frac{m+1}{n} L$ to $\frac{m+2}{n} L$. The expression in the middle is the cost, that is, the citizen's tax burden of any such increase. Therefore, if $h \in H_{m}$ and $\beta_{i}=1$, then it is optimal for legislator $i$ to
allocate $\frac{m+1}{n} L$ amount of labor for production of $g_{1}$ for any $m=1, \ldots, n-1$. If he allocates less than $\frac{m+1}{n} L$, he can increase his utility by allocating $\frac{1}{n} L$ more labor, since the tax burden of such an increase is less than its benefit. On the other hand, if he allocates more than $\frac{m+1}{n} L$, he can increase his utility by allocating $\frac{1}{n} L$ less labor, since, in this case, the decrease in the tax burden caused by such a change is greater than the loss of benefit it causes.

Assumption 1.3. As the value of public goods increases, the utility of the minimum provision of a public good also increases; specifically, if $h \in H_{m}$, then $h\left(\alpha \frac{1}{n} L\right)>$ $\frac{m+2}{2 n} L$ for each $m \in\{0, \ldots, n-1\} .{ }^{4}$

Assumption 1.3 states that as the index of the set that $h$ belongs to increases, the utility from the minimum provision of a public good also increases. We make this assumption to be able to obtain closed form solutions for our equilibrium.

The probability of polarization between legislators is given by the function $p:(0,1)^{2} \rightarrow(0,1)$ defined by $p\left(\lambda_{1}, \lambda_{2}\right)=\lambda_{1}\left(1-\lambda_{2}\right)+\left(1-\lambda_{1}\right) \lambda_{2}$ for all $\lambda_{1}, \lambda_{2} \in(0,1)$. As we see in Figure 1.4, if both $\lambda_{1}$ and $\lambda_{2}$ are close to 0 or 1 , then the probability of polarization is lower than when $\lambda_{1}$ is close to 0 and $\lambda_{2}$ to 1 or vice versa. Moreover, we have $p\left(\lambda_{1}, \lambda_{2}\right)=\frac{1}{2}$ if $\lambda_{i}=\frac{1}{2}$ for some $i$. We use these facts to get Lemma 1.1.

Lemma 1.1. If both $\lambda_{1}$ and $\lambda_{2}$ are close to 0 or 1, the probability of polarization is lower than when $\lambda_{1}$ is close to 0 and $\lambda_{2}$ to 1 or vice versa. Specifically, if $\lambda \in\left(0, \frac{1}{2}\right)$, $p\left(\lambda_{1}, \lambda_{2}\right)<p\left(\lambda_{1}^{\prime}, \lambda_{2}^{\prime}\right)$ for any $\lambda_{1}, \lambda_{2} \in(0, \lambda)$ or $\lambda_{1}, \lambda_{2} \in(1-\lambda, 1)$ and $\lambda_{1}^{\prime} \in(0, \lambda)$ and $\lambda_{2}^{\prime} \in(1-\lambda, 1)$, or vice versa.

It is easier to understand our lemma using the contours of the probability

[^6]

Figure 1.4: Probability of polarization.
of polarization presented in Figure 1.5. For any $\lambda \in\left(0, \frac{1}{2}\right)$, we can divide the $\lambda_{1} \lambda_{2}$-plane into six zones. If $\lambda_{1}$ and $\lambda_{2}$ are in the zones that are next to the points $(0,0)$ or $(1,1)$, as in panels (a) and (b), respectively, the probability of polarization is lower than when they are in the zones close to the points $(0,1)$ or $(1,0)$, as in panels (c) and (d), respectively. So, given a $\lambda \in\left(0, \frac{1}{2}\right)$, we say that there is a low probability of polarization if $\lambda_{1}, \lambda_{2} \in(0, \lambda)$ or $\lambda_{1}, \lambda_{2} \in(1-\lambda, 1)$, and a high probability of polarization if $\lambda_{1} \in(0, \lambda)$ and $\lambda_{2} \in(1-\lambda, 1)$ or vice versa. We use this terminology to present our first proposition.

Proposition 1.1. Let $\Gamma$ be an economic environment that satisfies Assumptions 1.1-1.3. If $\lambda_{i}$ is close to 0 or 1 for each $i \in\{1,2\}$, then $\Gamma$ has a unique equilibrium. Specifically, there exists $\lambda_{\Gamma} \in\left(0, \frac{1}{2}\right)$ such that, if $\lambda_{i} \in\left(0, \lambda_{\Gamma}\right) \cup\left(1-\lambda_{\Gamma}, 1\right)$ for each $i \in\{1,2\}, \Gamma$ has a unique equilibrium $\left(b_{i}, a_{i}, c_{i}\right)_{i \in\{1,2\}}$. Moreover, the following provisions hold.
(i) If the probability of polarization is low, the legislators propose a one-item budget. Specifically, if $\lambda_{i}, \lambda_{i^{\prime}} \in\left(0, \lambda_{\Gamma}\right)$ or $\lambda_{i}, \lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$ for each $i, i^{\prime} \in\{1,2\}$


Figure 1.5: Contours of the probability of polarization.
with $i \neq i^{\prime}, b_{i}, b_{i^{\prime}} \in B_{s}$.
(ii) If the probability of polarization is high, the legislators propose a two-item budget. Specifically, if $\lambda_{i} \in\left(0, \lambda_{\Gamma}\right)$ and $\lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$ for each $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}, b_{i}, b_{i^{\prime}} \in B_{d}$.

We characterize the equilibrium of our model only when $\lambda_{i}$ is close to 0 or 1 for both $i$. In Proposition 1.1, we formally state this condition by requiring $\lambda_{i}$ to be smaller than a lower bound $\lambda_{\Gamma}$ or greater than the upper bound $1-\lambda_{\Gamma}$ for both $i$. Under this condition, a unique equilibrium exists for any economic environment that satisfies Assumptions 1.1-1.3. This is easy to prove by backward induction.

If the probability of polarization is low, then the legislators propose a one-item budget. On the other hand, if the probability of polarization is high, then they
propose a two-item budget. To understand why, notice that under a one-item budget, the proposer can choose how to allocate the tax revenue for the provision of public goods contingent on nature's choice of his preference parameter. This provides him the flexibility to allocate the whole tax revenue for the provision of the public good that gives him positive utility. Thus, given an income tax rate, a proposer always prefers a one-item budget over a two-item budget. Yet, at the equilibrium, a budget proposal must be accepted. For a responder to accept a one-item budget proposal, he should get higher expected utility from the proposal than the status-quo policy. For this, the probability that the proposer and the responder will have the same preference parameters should be high. This is achieved when the probability of polarization between legislators is low. If the probability of polarization between legislators is high, then the probability that the proposer and the responder will have the same preference parameters will be low. Thus, under a one-item budget, with a high probability, the proposer will allocate the whole tax revenue for the provision of a public good that will not give any utility to the responder. Therefore, the expected utility of the responder from a one-item budget will be less than his utility from the status-quo policy. As a result, the responder will reject a one-item budget proposal. In this case, at the equilibrium, the proposer offers a two-item budget, which provides him the opportunity to commit to the provision of both public goods.

Proposition 1.2. Let $\Gamma$ be an economic environment that satisfies Assumptions 1.1-1.3. Assume further that the probability of polarization is low. Specifically, $\lambda_{i}, \lambda_{i^{\prime}} \in\left(0, \lambda_{\Gamma}\right)$ or $\lambda_{i}, \lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$ for each $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$. Let $\left(b_{i}, a_{i}, c_{i}\right)_{i \in\{1,2\}}$ be the unique equilibrium of $\Gamma$. Then, only one of the public goods
is provided and its provision increases as the value of public goods increases. Specifically, if $h \in H_{m}, b_{i}=\frac{m+1}{2 n} L, c_{i}\left(b_{i}, 1\right)=\left(\frac{m+1}{n} L, 0\right)$ and $c_{i}\left(b_{i}, 0\right)=\left(0, \frac{m+1}{n} L\right)$ for all $m \in\{0, \ldots, n-1\}$ and $i \in\{1,2\}$.

In Proposition 1.2, we examine the equilibrium budget proposals and government policies more closely when the probability of polarization is low. In this case, since at the equilibrium a one-item budget is accepted, the proposer chooses how to allocate the tax revenue for the provision of public goods contingent on nature's choice of his preference parameter. Therefore, he allocates the whole tax revenue for provision of $g_{1}$ when his preference parameter is 1 and for provision of $g_{2}$ when his preference parameter is 0 . Moreover, the income tax rate and the provision of public goods increase as the value of public goods increases. Since the probability of polarization is low, a proposer chooses the income tax rate without any constraint. Thus, if $h \in H_{m}$, he proposes the income tax rate $\frac{m+1}{2 n} L$ for each $m \in\{0, \ldots, n-1\}$. Proposition 1.3. Let $\Gamma$ be an economic environment that satisfies Assumptions 1.1-1.3. Assume further that the probability of polarization is high. Specifically, $\lambda_{i} \in\left(0, \lambda_{\Gamma}\right)$ and $\lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$ for each $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$. Let $\left(b_{i}, a_{i}, c_{i}\right)_{i \in\{1,2\}}$ be the unique equilibrium of $\Gamma$. Then, both public goods are provided and public-good provision increases as the value of public goods increases. Specifically, the following hold.
(i) If $h \in H_{m}, b_{i}=\left(\frac{m+2}{2 n} L, \frac{1}{n} L, \frac{m+1}{n} L\right)$ and $b_{i^{\prime}}=\left(\frac{m+2}{2 n} L, \frac{m+1}{n} L, \frac{1}{n} L\right)$ for all $m \in\{0,1, \ldots, n-2\}$.
(ii) If $h \in H_{n-1}, b_{i}=\left(\frac{1}{2} L, \frac{1}{n} L, \frac{n-1}{n} L\right)$ and $b_{i^{\prime}}=\left(\frac{1}{2} L, \frac{n-1}{n} L, \frac{1}{n} L\right)$.

In Proposition 1.3, we examine the equilibrium budget proposals more closely when the probability of polarization is high. In this case, the proposer offers a
two-item budget ensuring the provision of both public goods. Moreover, the income tax rate and public-good provision increase as the value of public goods increases. Assume that legislator $i$ is the proposer with $\lambda_{i} \in\left(0, \lambda_{\Gamma}\right)$ and legislator $i^{\prime}$ is the responder with $\lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$. If $h \in H_{m}$, legislator $i$ proposes a budget with income tax rate equal to $\frac{m+2}{2 n} L$ for all $m \in\{0, \ldots, n-2\}$. By taking advantage of being the proposer, he allocates most of the tax revenue for the provision of the public good that is more valuable to him - while he allocates $\frac{m+1}{n} L$ labor for provision of $g_{2}$, he allocates $\frac{1}{n} L$ labor for $g_{1}$. Thus, as the value of public goods increases, only the provision of $g_{2}$ increases. If $h \in H_{n-1}$, the income tax rate and the amount of labor allocated for provision of $g_{2}$ remain at $\frac{1}{2} L$ and $\frac{n-1}{n} L$, respectively. This is because the maximum amount of labor that can be allocated for the provision of public goods is $L$.

Corollary 1.1. Let $\Gamma$ be an economic environment that satisfies Assumptions 1.1-1.3. Government spending is higher under a high probability of polarization than under a low probability of polarization. Specifically, $\tau_{i}=\frac{m+1}{2 n}$ if $\lambda_{i}, \lambda_{i^{\prime}} \in\left(0, \lambda_{\Gamma}\right)$ or $\lambda_{i}, \lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$, and $\tau_{i}=\frac{m+2}{2 n}$ if $\lambda_{i} \in\left(0, \lambda_{\Gamma}\right)$ and $\lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$ for each $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$ and $m \in\{0, \ldots, n-2\}$.

As we state in Corollary 1.1, a high probability of polarization leads to more government spending than a low probability of polarization. In particular, if $h \in H_{m}$, then under a low probability of polarization the government spending is equal to $\frac{m+1}{n}$ and under a high probability of polarization it is equal to $\frac{m+2}{n}$ for any $m \in\{0, \ldots, n-2\}$. To understand why, note that the provision of the public good that is more valuable to the proposer is the same under both a low and a high probability of polarization. However, while under a low probability of polarization
the public good that is less valuable to the proposer is not provided, under a high probability of polarization it is provided at a positive level.

### 1.5 Extensions

### 1.5.1 Line-Item Veto

In this part, we allow the responder a line-item veto right, so that he can accept or reject each item separately in a two-item budget. The line-item veto right exists in the United States for many state governors. There is a large empirical literature on the effects of the line-item veto right on government spending as surveyed in Carter and Schap (1990). However, there are few theoretical models that analyze the line-item veto right (see, for example, Carter and Schap, 1987, 1990; Dearden and Husted, 1993). Our model does not capture the properties of a system with a governor completely. Nevertheless, for theoretical curiosity, we believe it is still worth considering the line-item veto right in a legislative-bargaining model. Moreover, one can ask why the line-item veto right does not exist in the legislative decision-making process.

In our analysis, to attain a unique equilibrium and to simplify to present our results, we assume that if the proposer is indifferent between one- and two-item budgets, he proposes the one-item budget. This assumption is effective only when the proposer prefers to offer a budget with income tax rate equal to zero. Moreover, we only focus on a high probability of polarization, since under a low probability of polarization a one-item budget is offered, so allowing the line-item veto right does not effect the equilibrium.

Proposition 1.4. Let $\Gamma$ be an economic environment that allows the line-item veto right and satisfies Assumptions 1.1-1.3. Assume further that the probability of polarization is high. Specifically, $\lambda_{i} \in\left(0, \lambda_{\Gamma}\right)$ and $\lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$ for each $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$. Г has a unique equilibrium in which the public-good provision is zero.

Comparing Propositions 1.3 and 1.4, we see that allowing the line-item veto right changes the equilibrium substantively. In particular, when the line-item veto right is not allowed, both public goods are provided at a positive level, whereas when it is allowed, public-good provision is zero. To understand why, note that, because the probability of polarization is high, the responder rejects the provision of the public good that will give a positive utility to the proposer with a high probability when the line-item veto right is allowed. Given this response and a high probability of polarization, the proposer makes an offer such that no public good is provided.

### 1.5.2 Flexible Line-Item Appropriations

In this part, we allow for flexible line-item appropriations, so that the proposer can choose to make transfers between the line-item appropriations after the budget is approved by the responder. Transfers between the line-item appropriations are allowed in the United States and in some other OECD countries to a limited extent. For example, the US government Department of Labor 2015 appropriation bill Section 102 states, "Not to exceed 1 percent of any discretionary funds (...) which are appropriated for the current fiscal year for the Department of Labor in this Act may be transferred between a program, project, or activity ...."

Our aim is to investigate the welfare effects of allowing for flexible line-item appropriations. To this end, we modify a two-item budget in a way that allows for transfers between the line-item appropriations. So, we obtain a flexible two-item budget $\left(\tau, l_{1}, l_{2}, k\right)$ where $\tau$ is the income tax rate, $\left(l_{1}, l_{2}\right)$ are the amounts of labor that will be hired for the provision of the public goods, and $k$ is the maximum amount of labor that can be transferred from $l_{1}$ to $l_{2}$ or vice versa after the budget is approved.

In our analysis, we focus only on a high probability of polarization because, under a low probability of polarization, a one-item budget is offered, so allowing for a flexible two-item budget does not effect the equilibrium.

Proposition 1.5. Let $\Gamma$ be an economic environment that allows for a flexible twoitem budget and satisfies Assumptions 1.1-1.3. Assume further that the probability of polarization is high. Specifically, $\lambda_{i} \in\left(0, \lambda_{\Gamma}\right)$ and $\lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$ for each $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime} . \Gamma$ has a unique equilibrium in which legislators propose a flexible two-item budget.

As we state in Proposition 1.5, when a flexible two-item budget is allowed, there exists a unique equilibrium. Calling our original two-item budget $\left(\tau, l_{1}, l_{2}\right)$ a strict two-item budget, let $u_{p}^{*}\left(u_{p}^{* *}\right)$ denote the equilibrium payoff of the proposer when we have a strict (flexible) two-item budget and a high probability of polarization. Similarly, let $u_{r}^{*}$ and $u_{r}^{* *}$ denote the equilibrium payoffs of the responder. Let $\lambda_{r}$ denote the probability that nature chooses the preference parameter of the responder to be 1 .

Proposition 1.6. Let $\Gamma$ be an economic environment that satisfies Assumptions 1.1-1.3. Assume further that the probability of polarization is high. Specifically,
$\lambda_{i} \in\left(0, \lambda_{\Gamma}\right)$ and $\lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$ for each $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$. The following hold.
(i) A flexible two-item budget increases the proposer's payoff compared to a strict two-item budget. Specifically, $u_{p}^{* *}>u_{p}^{*}$.
(ii) A flexible two-item budget increases the responder's payoff compared to a strict two-item budget if and only if nature's choice probability of his preference parameter is high enough. Specifically, there exists $\tilde{\lambda}_{\Gamma} \in\left(0, \lambda_{\Gamma}\right]$ such that $u_{r}^{* *}>u_{r}^{*}$ if and only if $\lambda_{r} \in\left(0, \tilde{\lambda}_{\Gamma}\right) \cup\left(1-\tilde{\lambda}_{\Gamma}, 1\right)$.

When a flexible two-item budget is allowed, the proposer prefers to choose the amount of labor that can be transferred between the line-item appropriations as high as possible. However, for his budget proposal to be accepted, he must promise the responder that the provision of the public good will give him a positive utility with a high probability. So, he promises the minimum provision of that public good and keeps the rest of the tax revenue transferable between the two public goods. As we state in Proposition 1.6, this increases the proposer's payoff since he can use some part of the tax revenue contingent on nature's choice of his preference parameter. In contrast, the responder's payoff is affected in two opposite directions. On the one hand, his payoff increases when both legislators take positive utility from the public good that will give positive utility to the responder with a high ex ante probability. On the other hand, his payoff decreases when he takes a positive utility from the other public good. While the former effect dominates when nature's choice probability of the responder's preference parameter is high enough, the latter effect dominates otherwise.

### 1.6 Empirical Discussion

Our model predicts that a higher probability of polarization between legislators leads to a larger number of line-item appropriations. We check this prediction of our model first using the US government budget data presented in Figure 1.2. For the US, Azzimonti (2016) provides a measure of lawmakers' disagreement about policy using newspaper articles. The author calls this measure as "partisan conflict". The measure takes the value 100 in 1990 and increases as the conflict among lawmakers increases. We use the partisan conflict measure of Azzimonti (2016) to measure polarization in the US. In Figure 1.6, we present a graph of partisan conflict and the total number of line-item appropriations of the six US government departments included in Figure 1.2. The correlation between the two variables is 0.26 . So, as partisan conflict increases, the total number of line-item appropriations increases in the US. This confirms the prediction of our model.

We check the same prediction of our model also using the OECD budget survey data on the number of line-item appropriations across countries presented in Figure 1.1. To measure polarization in the countries surveyed by the OECD, we follow the approach of Alesina et al. (1999), who use ethnic fractionalization to measure polarization in the US. The authors justify their choice by the arguments of a large literature which states that conflicts over the provision of public goods are mainly determined by racial divisions. ${ }^{5}$ Similar to Alesina et al. (1999), we use ethnic tension to measure polarization. For this purpose, we employ the "lack of ethnic tension" measure of the ICRG of PRS Group. For a country, this variable takes

[^7]

Figure 1.6: Partisan conflict in the US and the total number of line-item appropriations for the six US government departments.
the value 1 if ethnic tension is maximum and 6 if ethnic tension is minimum, even though such differences may still exist in the country. ${ }^{6}$ In Figure 1.7, we present a scatter of lack of ethnic tension and the number of line-item appropriations excluding the outlier countries with a high number of line-item appropriations. ${ }^{7}$

The correlation between the two variables is -0.24 . So, the countries that have a higher level of ethnic tension have a larger number of line-item appropriations in
their budgets. This provides another confirmation for our prediction. ${ }^{8}$

[^8]

Figure 1.7: Lack of ethnic tension and the number of line-item appropriations for countries included in the OECD budget surveys without outliers.

### 1.7 Conclusion

In this paper, we analyze a bargaining model in which two legislators decide on the number of line-item appropriations and the provision of two public goods. Legislators choose between one- and two-item budgets. We show that while a low probability of polarization between legislators results in a one-item budget, a high probability of polarization results in a two-item budget. This is because given an income tax rate, the proposer prefers a one-item budget to a two-item budget, since a one-item budget provides the flexibility to choose the provision of the public goods. However, for the responder to accept a one-item budget proposal, the probability of polarization between legislators should be low. If the probability of polarization between legislators is high, then to get the approval of the responder the proposer needs to commit to the provision of the public good that will give positive utility to the responder with a high probability. Thus, he offers a two-item
budget promising the provision of both public goods. Additionally, we show that a high probability of polarization increases government spending, since under a high probability of polarization the public good that is less valuable to the proposer needs to be provided at a positive level.

Several extensions of our model seem interesting for future research. First, we assume that citizens are either polarized or not, and labor can only be hired in discrete levels. It would be interesting to remove these assumptions. Moreover, we characterize our equilibrium only when nature's choice probabilities of the preference parameters are close to either zero or one. How does the equilibrium look for other values of the nature's choice probabilities of the preference parameters?

It would also be interesting to extend our model to more than two legislators and public goods. Although there are papers in the literature that analyze legislative bargaining models of public finance with more than two legislators (e.g., Battaglini and Coate, 2007; Persson et al., 2000), they either assume there is only one public good or all dimensions of the government policy are determined by the legislative authority.

Finally, line-item appropriations can be investigated for presidential and parliamentary systems separately. For this purpose, our model can be combined with the model of Persson et al. (2000).

## Chapter 2

## Appropriation Bills and Public

## Goods

### 2.1 Introduction

The effects of institutions on the size and composition of government spending have been the subject of many papers in the political economy literature. Most of these papers analyzed the effects of different electoral institutions (e.g., Persson et al., 2000; Lizzeri and Persico, 2001; Miles-Ferretti et al., 2002). Recently, Bowen et al. (2014) focused on a budgetary institution and compared the effects of mandatory versus discretionary spending programs on government spending. In Chapter 1 , we added to this literature by investigating the determination and effects of another budgetary institution, namely the number of line-item appropriations. In this chapter, we continue our investigation of the determination of budgetary institutions and their effects on government spending by analyzing the number of appropriation bills.

Appropriation bills are written for each government department separately. Thus, as new departments are set up or existing departments merge, the number of appropriation bills changes. Setting up a new department is costly, yet it provides the executive authority an opportunity to commit to the provision of public goods separately. As a simple example, assume that there is a Department of Education and Health (DoEH). For budget negotiations, an executive authority proposes a single bill to the legislature to grant spending authority to the DoEH. However, for a department providing a large number of public services, it may not be possible to write in complete detail how the authorized money will be allocated for the provision of public services in a single bill. This may be caused by the organizational structure of the department - many public services may need to be provided by a single office or a program - or the committee that writes the bill can have limited time and human resources. ${ }^{1}$ Thus, after the bill is accepted, the executive authority can allocate the authorized money according to its own preferences. However, if there are two separate departments, say a Department of Education and a Department of Health, the executive authority must propose two separate bills to the legislature and must allocate the authorized money as specified in the bills. In two separate bills, the executive can propose a more detailed bill for both education and health services. Therefore, a high number of appropriation bills increases the legislative control over government policies.

At this point, one might ask if, in the empirical data, changes in the number of appropriation bills lead to any different economic outcome. To answer this question

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Figure 2.1: United States Department of Education and Department of Health and Human Services discretionary budget shares.
we can look at two cases in the United States. The first case is the Department Education and Department of Health and Human Services. Until 1979, the two departments were merged under the name of Department of Education, Health and Human Services. As seen in Figure 2.1, before the separation, the discretionary budget shares of the two departments were almost equal. However, after the separation, the budget share of the Department of Education has almost always been below that of the Department of Health and Human Services. The second case is the Department of Homeland Security. Until it was established in 2002, the functions of the department were allocated among different government agencies. As seen in Figure 2.2, the average discretionary budget share of the department during these years was $1.7 \%$. However, after 2002 this number increased to $4.1 \%$. Doubtless, the September 11 attacks in 2001 had an impact on the department's budget share, yet we note that this increase was accompanied by the inception of a new department with its own appropriation bill.

There is a growing literature on the effects of institutions on public finance


Figure 2.2: United States Department of Homeland Security discretionary budget share.
(Persson et al., 2000; Lizzeri and Persico, 2001; Milesi-Ferretti et al., 2002; Bowen et al., 2014). However, to the best of our knowledge, there is no analysis of the number of appropriation bills in the literature. We take a first step to understand the effects of the number of appropriation bills on the size and composition of government spending and how the number of appropriation bills is determined. For this purpose, using the Baron and Ferejohn (1989) approach, we develop a model of legislative bargaining in which two legislators first decide on the budgeting rule (the number of appropriation bills) and subsequently on the government policy income tax rate and provision of two public goods. We consider the two legislators as constituting the legislative authority. Legislators attach different values to different public goods and we refer to the size of this difference as the level of polarization between legislators. In the first stage of our model, a legislator is randomly selected to propose a budgeting rule. If the other legislator accepts the offer, it is implemented in the second stage; otherwise, a status-quo budgeting rule prevails. We consider two budgeting rules: one-bill budgeting, in which only the
income tax rate is determined, and two-bill budgeting, in which both the income tax rate and provision of two public goods are determined. In the second stage, a new legislator is randomly selected to make a budget proposal. He has to obey the budgeting rule determined in the first stage. If the other legislator accepts the proposal, it is implemented, otherwise a status-quo government policy prevails. We consider the legislator who is selected to propose a budget as constituting the executive authority. The probability of being selected as the executive authority captures the legislators' political power.

The effects of the two budgeting rules on government policy are determined according to the degree of polarization between legislators. If polarization is low, then the two budgeting rules lead to the same income tax rate and provision of both public goods. This is because when polarization is low, the best government policy for the budget proposer gives the other legislator a greater payoff than status-quo policy gives. Hence, under both budgeting rules, the budget proposer makes an offer that leads to his best government policy. If polarization is high, then under one-bill budgeting, the income tax rate and provision of both public goods are zero; under two-bill budgeting, they are positive. This is because of different commitment opportunities that the two budgeting rules provide. Under one-bill budgeting, the budget proposer can only commit to an income tax rate. Thus, if his budget proposal is accepted, then he allocates the whole tax revenue for provision of the public good that is more valuable to him. However, such a government policy gives the other legislator a payoff that is less than what he gets from the status-quo policy. Therefore, the responding legislator rejects any budget proposal with a positive income tax rate. On the other hand, under two-bill budgeting,
the budget proposer can commit to an income tax rate and the provision of both public goods. This allows him to promise the provision of the public good that is more valuable to the other legislator. Hence, a budget proposal with a positive income tax rate and provision of both public goods can be accepted.

Determination of an implemented budgeting rule requires consideration of other model parameters besides the level of polarization. If polarization is low, then one-bill budgeting is implemented. This is because both budgeting rules lead to the same government policy and two-bill budgeting is costly. If polarization is high, then, first, the result depends on the cost of implementing two-bill budgeting. However, an interesting case emerges when this cost is moderate. In this case, if the status-quo budgeting rule is one-bill budgeting, then two-bill budgeting is implemented if and only if both legislators have high enough political power. To understand why, note that because the status-quo budgeting rule is one-bill budgeting, for a legislator to propose two-bill budgeting and for the other legislator to accept such a proposal, they should both get a higher expected payoff under twobill budgeting than under one-bill budgeting. Since the legislator who proposes the budget gets a higher payoff than the other legislator, this requires both legislators to have a high enough probability of proposing the budget. On the other hand, if the status-quo budgeting rule is two-bill budgeting, then two-bill budgeting is implemented if and only if one legislator has high enough political power. In this case, since the status-quo budgeting rule is two-bill budgeting, for a legislator to propose two-bill budgeting and for the other legislator to accept such a proposal, it is enough that one legislator gets a higher expected payoff under two-bill budgeting than under one-bill budgeting.

In the previous chapter, we analyzed the number of line-item appropriations, which are the constituents of an appropriation bill. Here, we would like make the differences between the two chapters clear. First, in the previous chapter, we took the number of appropriation bills as fixed and analyzed the choice of the number of line-item appropriations. In this chapter, we take the number of line-item appropriations as fixed and analyze the choice of the number of appropriation bills. Second, in the previous chapter, the legislators' preferences are determined after they bargain over the budget. In this chapter, the legislators' preferences are given at the beginning of the game. Third, in the previous chapter, we study only complete polarization and no polarization between legislators. In this chapter, we allow moderate levels of polarization between these two extremes. Fourth, in the previous chapter, the budgeting rule and the budget are determined simultaneously. In this chapter, the budgeting rule is determined first, and the budget is determined subsequently. This generates two new parameters which affect the equilibrium in this chapter - the status-quo budgeting rule and the distribution of political power.

Besides their modeling assumptions, the two chapters also give different results. In the previous chapter, if the probability of polarization is low, then a one-item budget is chosen. If the probability of polarization is high, then a two-item budget is chosen. In this chapter, if polarization is low, then, similarly, a one-bill budget is chosen. However, if polarization is high, then the choice of budgeting rule depends on the cost of two-bill budgeting, the status-quo budgeting rule, and the legislators' political power. Depending on the values of these parameters either a one- or a two-bill budget can be chosen.

In addition, in this chapter, in a simple extension of our model, we investigate
the effects of group-specific transfers - monetary transfers of tax revenues to citizens of a specific group - on provision of public goods and efficiency. According to the general result in the literature, group-specific transfers decrease provision of public goods and lead to inefficient outcomes. We consider two situations: In the first one, group-specific transfers are allowed, and in the second one, they are not allowed. In both cases, we take the budgeting rule as one-bill budgeting. We show that if polarization is high, then allowing for group-specific transfers increases provision of public goods and provides Pareto improvement in efficiency. More specifically, when group-specific transfers are not allowed, the income tax rate and public-good provision are zero. Yet, when group-specific transfers are allowed, the income tax rate and provision of a public good are positive. While this does not decrease the payoff of any citizen, it increases the payoff of the citizens that are in the group of the legislator proposing the budget.

The remainder of the paper is organized as follows. In Section 2.2, we summarize the related literature. In Section 2.3, we present our model. In Section 2.4, we define the equilibrium for our model and analyze the determinants and the effects of the two budgeting rules. In Section 2.5, we extend our model to analyze group-specific transfers. Section 2.6 concludes.

### 2.2 Related Literature

A large literature investigates the provision of public goods under political-economy frictions (see Persson and Tabellini, 2000, for a survey). A subset of this literature focuses on the effects of different political institutions on public-good provision. For example, Persson et al. (2000), Lizzeri and Persico (2001), and Milesi-Ferretti et
al. (2002) investigate public-good provision under different electoral systems, and Bowen et al. (2014) analyze public-good provision under two budgetary institutions: mandatory versus discretionary spending programs. We contribute to this literature by analyzing the effects of the number of appropriation bills on the provision of public goods.

An expanding literature analyzes the determinants and effects of limits on executive power. Among them the most relevant paper is Besley et al. (2014). They analyze an infinite-horizon model in which, in each period, the executive power determines the provision of a public good, group-specific transfers, and limits of executive power in the next period. They show that a higher probability of losing office leads to stronger executive constraints, and this increases provision of the public good. We add to this literature by analyzing the constraints on executive power induced by appropriation bills. The Besley et al. (2014) model does not capture the effect of polarization on public-good provision. We show that constraints on executive power increase provision of public goods only when polarization is high. In the Besley et al. (2014) equilibrium, there is no link between the executive constraints in two consecutive periods. In our equilibrium, the status-quo budgeting rule is an important determinant of the implemented budgeting rule. Other papers in this literature are Lagunoff (2001), Aghion et al. (2004), Maskin and Tirole (2004), Ticchi and Vindigni (2010), Acemoglu et al. (2013), Robinson and Torvik (2013), and Karakas (2016).

A growing literature of constitutional economics analyzes the constraints under which the members of a society operate. Voigt (1997) presents a review of the positive branch of this literature. Our paper connects to this literature by letting
legislators choose first the budgeting rule and subsequently the budget. Some recent papers in this literature are Koray (2000), Persson (2002), Messner and Polborn (2004), Barbera and Jackson (2004), and Maggi and Morelli (2006).

We base our model on the legislative-bargaining approach of Baron and Ferejohn (1989), which is applied to public finance by Persson (1998). Persson (1998) is followed by such papers as Leblanc et al. (2000), Volden and Wiseman (2007) and Battaglini and Coate (2008). However, these papers assume that legislative authority decides on all dimensions of government policy. We contribute to this literature by letting the proposer decide on some dimensions of government policy after the legislative process.

Lockwood (2004) analyzes a model similar to ours, but his purpose of analysis is different. Specifically, he analyzes decentralization via federal and unitary referenda. Moreover, he assumes that all dimensions of government policy are decided by the legislative authority. Hence, he does not analyze the determinants and the effects of the number of appropriation bills, which are central to our paper.

Many papers cited above and Battaglini and Coate (2007) analyze the effects of group-specific transfers on public-good provision. The general result is that group-specific transfers decrease it and lead to inefficient outcomes. We add to this literature by showing that allowing for group-specific transfers increases public-good provision and provides Pareto improvement in efficiency if polarization is high and one-bill budgeting is implemented.

### 2.3 Model

### 2.3.1 The Economic Environment

The society is separated into two groups of citizens each with unit mass population and indexed by $i \in\{1,2\}$. Each citizen has an endowment of one unit of labor denoted by $l$. There is a single consumption good denoted by $z$ and two public goods denoted by $g$ and indexed by $j \in\{1,2\}$. The consumption good is produced with the technology $f(l)=l$ and the public goods with the technology $f_{j}(l)=\alpha_{j} l$ where $\alpha_{j}>0$ for each $j$.

A citizen's utility function in group 1 is

$$
u_{1}\left(z, g_{1}, g_{2}\right)=z+\beta h\left(g_{1}\right)+(1-\beta) h\left(g_{2}\right)
$$

and in group 2 is

$$
u_{2}\left(z, g_{1}, g_{2}\right)=z+(1-\beta) h\left(g_{1}\right)+\beta h\left(g_{2}\right),
$$

where $h: \mathbb{R}_{+} \rightarrow \mathbb{R}_{+}$is an increasing and strictly concave function with $h(0)=0$. Citizens differ only in their preferences for public goods. Their disagreement is parameterized by $\beta$, which we interpret as the level of polarization between the two groups. We assume that $\frac{1}{2} \leq \beta \leq 1$. Notice that legislator 1 always values public good 1 more than or as much as public good 2 and vice versa for legislator 2 .

There is a competitive labor market. Thus, the assumption on the production technology of the consumption good implies a wage rate equal to 1 .

### 2.3.2 Government Policies

A government policy is described by the triplet $\left(\tau, l_{1}, l_{2}\right)$, where $\tau$ is the income tax rate and $l_{j}$ is the amount of labor allocated for production of public good $j$. The set of feasible government policies is given by

$$
P=\left\{\left(\tau, l_{1}, l_{2}\right) \in[0,1] \times\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}^{2}: l_{1}+l_{2}=2 \tau \text { and } l_{1}+l_{2} \leq L\right\}
$$

where $L$ is the maximum amount of labor that can be allocated for the production of public goods. Feasibility requires that $L \leq 2$, which is the total endowment of labor in the economy. We assume that, for the production of public good $j$, labor can only be hired at discrete levels; that is, $l_{j} \in\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}$ where $n \in \mathbb{Z}_{+} .{ }^{2}$ We also assume that tax revenue can only be spent for production of public goods and not wasted. Let

$$
Q=\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}^{2}
$$

denote the set of labor-hiring policies.
The economic environment with government policies can be summarized by

$$
\Gamma=\left(\left(\alpha_{j}\right)_{j \in\{1,2\}}, \beta, h, n, L\right)
$$

[^10]
### 2.3.3 The Political Process

Government policy decisions are made by a legislature consisting of representatives of both groups. The decision process includes two stages. In the first stage, as described in Figure 2.3, nature selects one of the legislators to propose a budgeting rule, $\sigma \in\{s, d\}$, where $s$ and $d$ represents one- and two-bill budgeting, respectively. The other legislator responds by either accepting or rejecting the proposal. If he accepts, in the second stage, the proposed budgeting rule is implemented. Otherwise, a status-quo budgeting rule, $\sigma^{s} \in\{s, d\}$, prevails. Implementing onebill budgeting has a cost, $c_{s}=0$, and two-bill budgeting has a different cost, $c_{d}>0$. In the second stage, as described in Figure 2.4, nature first selects one of the legislators to propose a budget. The probability that nature selects legislator $i$ is $\lambda_{i}$. We interpret $\lambda_{i}$ as the political power of legislator $i$. The proposer has to follow the budgeting rule determined in the first stage. Composition of a budget and the remainder of the process under each budgeting rule is described in the following subsections.

One-Bill Budgeting. - Under one-bill budgeting, a budget is only composed of an income tax rate $\tau$. Let

$$
B_{s}=\left\{0, \frac{1}{2 n} L, \frac{2}{2 n} L, \ldots, \frac{1}{2} L\right\}
$$

denote the set of feasible budget proposals under one-bill budgeting.
Under this rule, the proposer offers a budget $\mathbf{b}_{s} \in B_{s}$. The other legislator responds by either accepting or rejecting it. In the case of acceptance, the proposer
chooses a labor-hiring policy, $\mathbf{q}=\left(l_{1}, l_{2}\right)$. Let

$$
Q_{s}\left(\mathbf{b}_{s}\right)=\left\{\left(l_{1}, l_{2}\right) \in\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}^{2}: l_{1}+l_{2}=2 \tau\right\}
$$

denote the set of feasible labor-hiring policies given a budget $\mathbf{b}_{s}=\tau \in B_{s}$. If the responder rejects the proposal, then the status-quo policy, $\mathbf{q}^{s}=\left(\tau^{s}, g_{1}^{s}, g_{2}^{s}\right)=$ $(0,0,0)$, is implemented.

Two-Bill Budgeting. - Under two-bill budgeting, a budget is composed of an income tax rate $\tau$ and provision of both public goods $\left(g_{1}, g_{2}\right)$. Since labor is the only input, a budget can be specified with an income tax rate $\tau$ and amounts of labor $\left(l_{1}, l_{2}\right)$ that will be hired for public-good provision. Let

$$
\begin{aligned}
B_{d}= & \left\{\mathbf{b}_{d}=\left(\tau, l_{1}, l_{2}\right) \in\left\{0, \frac{1}{2 n} L, \frac{2}{2 n} L, \ldots, \frac{1}{2} L\right\} \times\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}^{2}:\right. \\
& \left.l_{1}+l_{2}=2 \tau\right\}
\end{aligned}
$$

denote the set of feasible budget proposals under two-bill budgeting.
Under this rule, the proposer offers a budget, $\mathbf{b}_{d}=\left(\tau, l_{1}, l_{2}\right) \in B_{d}$. The other legislator responds by either accepting or rejecting it. In case of acceptance, the proposer implements the labor-hiring policy $\mathbf{q}=\left(l_{1}, l_{2}\right)$. For convenience of notation, let

$$
Q_{d}\left(\mathbf{b}_{d}\right)=\left\{\left(l_{1}, l_{2}\right)\right\}
$$

denote the set of feasible labor-hiring policies given a budget $\mathbf{b}_{d}=\left(\tau, l_{1}, l_{2}\right) \in B_{d}$. If the responding legislator rejects the proposal, then the status-quo policy, $\mathbf{q}^{s}$, is implemented.


Figure 2.3: First stage of the political process.


Figure 2.4: Second stage of the political process.

### 2.4 Political Equilibrium

A strategy for legislator $i,\left(v^{i}, \rho^{i},\left(b_{\sigma}^{i}, a_{\sigma}^{i}, c_{\sigma}^{i}\right)_{\sigma \in\{s, d\}}\right)$, is composed of a budgeting-rule proposal strategy, $v^{i} \in\{s, d\}$, a budgeting-rule acceptance strategy, $\rho^{i}:\{s, d\} \rightarrow$ $\{0,1\}$, and, for each budgeting rule $\sigma \in\{s, d\}$, a budget-proposal strategy, $b_{\sigma}^{i} \in B_{\sigma}$, a budget-acceptance strategy, $a_{\sigma}^{i}: B_{\sigma} \rightarrow\{0,1\}$, and a labor-hiring strategy, $c_{\sigma}^{i}: B_{\sigma} \rightarrow Q$, such that $c_{\sigma}^{i}\left(\mathbf{b}_{\sigma}\right) \in Q_{\sigma}\left(\mathbf{b}_{\sigma}\right)$ for each $\mathbf{b}_{\sigma} \in B_{\sigma}$. Legislator $i$ 's budgeting-rule acceptance strategy, $\rho^{i}(\sigma)$, takes the value 1 if legislator $i$ accepts the budgeting-rule proposal, $\sigma$, offered by legislator $i^{\prime} \neq i$, and 0 otherwise. Legislator $i$ 's budget-acceptance strategy, $a_{\sigma}^{i}\left(\mathbf{b}_{\sigma}\right)$, takes the value 1 if legislator $i$ accepts the budget proposal, $\mathbf{b}_{\sigma}$, offered by legislator $i^{\prime} \neq i$, and 0 otherwise.

We consider subgame-perfect equilibria. We restrict attention to equilibria in which (i) $\rho^{i}(\sigma)=1$ when legislator $i$ is indifferent between $\sigma$ and $\sigma^{s}$, (ii) $a_{\sigma}^{i}\left(\mathbf{b}_{\sigma}\right)=1$ when legislator $i$ is indifferent between $c_{\sigma}^{i^{\prime}}\left(\mathbf{b}_{\sigma}\right)$ and $\mathbf{q}^{s}$, (iii) $\rho^{i}\left(v^{i^{\prime}}\right)=1$ and (iv) $a_{\sigma}^{i}\left(b_{\sigma}^{i^{\prime}}\right)=1$ for all $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$. That is, a responding legislator accepts any proposal that he is indifferent between accepting and rejecting, and equilibrium proposals are always accepted. A formal definition of an equilibrium is given in Appendix B.1.

We first present our assumptions and then our results. Formal proofs of all propositions are given in Appendix B.2.

Assumption 2.1. Production technologies of the two public goods are the same; specifically, $\alpha_{1}=\alpha_{2}=\alpha>0$.

To get results from our model, we do not need Assumption 2.1. However, it makes it easier to state our results.

Assumption 2.2. Public goods are valuable enough for citizens; specifically, $h\left(\alpha \frac{1}{n} L\right)>\frac{1}{n} L$.

Assumption 2.2 is a standard one in the literature stating that the value of public goods is high enough for citizens.

Assumption 2.3. Any two different government policies give different utilities; specifically, $u_{i}\left(1-\tau, \alpha l_{1}, \alpha l_{2}\right)-c_{\sigma} \neq u_{i}\left(1-\tau^{\prime}, \alpha l_{1}^{\prime}, \alpha l_{2}^{\prime}\right)-c_{\sigma^{\prime}}$ for any $\left(\tau, l_{1}, l_{2}\right),\left(\tau^{\prime}, l_{1}^{\prime}, l_{2}^{\prime}\right)$ $\in P$ such that $\left(\tau, l_{1}, l_{2}\right) \neq\left(\tau^{\prime}, l_{1}^{\prime}, l_{2}^{\prime}\right), \sigma, \sigma^{\prime} \in\{s, d\}$ and $i \in\{1,2\}$.

Assumption 2.3 ensures unique equilibria. We make this assumption to avoid nongeneric cases that complicate the statement and proof of our results. For any $\mathbf{y}=\left(y_{1}, y_{2}\right) \in \mathbb{R}^{2}$, let $\mathbf{y}_{t}=y_{t}$ for any $t \in\{1,2\}$.

Proposition 2.1. Let $\Gamma$ be an economic environment that satisfies Assumptions 2.1-2.3. The environment has a unique equilibrium $\left(v^{i}, \rho^{i},\left(b_{\sigma}^{i}, a_{\sigma}^{i}, c_{\sigma}^{i}\right)_{\sigma \in\{s, d\}}\right)_{i \in\{1,2\}}$. Assume further that $n=2$. Then the following hold.
(i) If polarization is low, then the two budgeting rules lead to the same provision of both public goods; specifically, if $\beta<\underline{\beta}$, then $c_{s}^{i}\left(b_{s}^{i}\right)=c_{d}^{i}\left(b_{d}^{i}\right)$ for any $i \in\{1,2\}$,
(ii) If polarization is moderate, then under one-bill budgeting one public good is provided at a lower level, and the other public good is provided at the same level compared to two-bill budgeting; specifically, if $\underline{\beta} \leq \beta \leq \bar{\beta}$, then $\left(c_{s i^{\prime}}^{i}\left(b_{s}^{i}\right), c_{s i}^{i}\left(b_{s}^{i}\right)\right)=$ $\left(0, \frac{1}{2} L\right)$ and $\left(c_{d i^{\prime}}^{i}\left(b_{d}^{i}\right), c_{d i}^{i}\left(b_{d}^{i}\right)\right)=\left(\frac{1}{2} L, \frac{1}{2} L\right)$ for any $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$,
(iii) If polarization is high, then both public goods are provided at a lower level under one-bill budgeting compared to two-bill budgeting; specifically, if $\max \{\underline{\beta}, \bar{\beta}\}<$ $\beta$, then $c_{s}^{i}\left(b_{s}^{i}\right)=(0,0)$ and $c_{d}^{i}\left(b_{d}^{i}\right)=\left(\frac{1}{2} L, \frac{1}{2} L\right)$ for any $i \in\{1,2\}$, where $\underline{\beta}=$ $\max \left\{\frac{h\left(\alpha \frac{1}{2} L\right)}{h(\alpha L)}, 1-\frac{L}{2 h(\alpha L)}\right\}$ and $\bar{\beta}=1-\frac{L}{4 h\left(\alpha \frac{1}{2} L\right)}$.

In Proposition 2.1, we compare public-good provision under two budgeting rules assuming that $n=2$. This assumption enables us to characterize the equilibrium in a closed form. If polarization is low - that is, $\beta<\underline{\beta}$ - then the two budgeting rules lead to the same provision of both public goods because, when polarization is low, the best government policy for the legislator who proposes the budget gives the other legislator a payoff greater than what he gets from the status-quo policy. Thus, under both budgeting rules, the proposer offers a budget that results in his favorite government policy.

If polarization is moderate - that is, $\beta \leq \beta \leq \bar{\beta}$ - then under one-bill budgeting one public good is provided at a lower level and the other public good is provided at the same level compared to two-bill budgeting. To understand why, assume that nature selects legislator 2 to propose a budget. Under a moderate polarization, we have

$$
\begin{equation*}
u_{2}\left(1-\frac{1}{4} L, 0, \alpha \frac{1}{2} L\right)<u_{2}\left(1-\frac{1}{2} L, \alpha \frac{1}{2} L, \alpha \frac{1}{2} L\right)<u_{2}\left(1-\frac{1}{2} L, 0, \alpha L\right) \tag{2.1}
\end{equation*}
$$

and

$$
\begin{equation*}
u_{1}\left(1-\frac{1}{2} L, 0, \alpha L\right)<1<u_{1}\left(1-\frac{1}{4} L, 0, \alpha \frac{1}{2} L\right) . \tag{2.2}
\end{equation*}
$$

In this case, polarization is high enough so that the best government policy for legislator 2 is composed of choosing the highest income tax rate and allocating the whole tax revenue for provision of public good 2. However, this policy gives legislator 1 a payoff less than what he gets from the status-quo policy. Therefore, under two-bill budgeting, legislator 2 proposes a budget that results in his secondbest government policy, which is providing $\alpha \frac{1}{2} L$ of both public goods. On the
other hand, under one-bill budgeting, legislator 2 cannot commit to provision of two public goods separately. Hence, he proposes a budget that results in his thirdbest government policy which is only providing $\alpha \frac{1}{2} L$ of public good 2. Because polarization is also low enough legislator 1 accepts this proposal.

If polarization is high - that is, $\max \{\underline{\beta}, \bar{\beta}\}<\beta$ - then under one-bill budgeting both public goods are provided at a lower level compared to two-bill budgeting. To explain why, let us continue to assume that nature selects legislator 2 to propose a budget. Under a high polarization, we have

$$
\begin{equation*}
u_{2}\left(1-\frac{1}{2} L, \alpha \frac{1}{2} L, \alpha \frac{1}{2} L\right)<\min \left\{u_{2}\left(1-\frac{1}{4} L, 0, \alpha \frac{1}{2} L\right), u_{2}\left(1-\frac{1}{2} L, 0, \alpha L\right)\right\} \tag{2.3}
\end{equation*}
$$

and

$$
\begin{equation*}
u_{1}\left(1-\frac{1}{2} L, 0, \alpha L\right)<u_{1}\left(1-\frac{1}{4} L, 0, \alpha \frac{1}{2} L\right)<1 . \tag{2.4}
\end{equation*}
$$

In this case, polarization is high enough so that the first two best government policies for legislator 2 require only the provision of public good 2. However, any such policy gives legislator 1 a payoff less than what he gets from the status-quo policy. Thus, under two-bill budgeting, legislator 2 proposes a budget that results in his third-best government policy, which is providing $\alpha \frac{1}{2} L$ of both public goods. However, under one-bill budgeting, legislator 2 cannot commit to provision of two public goods separately and allocates the whole tax revenue for provision of public good 2. Therefore, he offers a budget with income tax rate equal to zero which is the only budget that legislator 1 accepts under one-bill budgeting.

Without assuming that $n=2$, we cannot characterize the equilibrium in a closed form. However, we can still generalize our results when polarization is low
and high.

Proposition 2.2. Let $\Gamma$ be an economic environment that satisfies Assumptions 2.1-2.3 and let $\left(v^{i}, \rho^{i},\left(b_{\sigma}^{i}, a_{\sigma}^{i}, c_{\sigma}^{i}\right)_{\sigma \in\{s, d\}}\right)_{i \in\{1,2\}}$ be its unique equilibrium. There exists $\underline{\boldsymbol{\beta}}, \overline{\boldsymbol{\beta}} \in\left(\frac{1}{2}, 1\right)$ such that $\underline{\boldsymbol{\beta}} \leq \overline{\boldsymbol{\beta}}$ and the following hold.
(i) If polarization is low, then the two budgeting rules lead to the same provision of both public goods; specifically, if $\beta<\boldsymbol{\beta}$, then $c_{s}^{i}\left(b_{s}^{i}\right)=c_{d}^{i}\left(b_{d}^{i}\right)$ for any $i \in\{1,2\}$.
(ii) If polarization is high then both public goods are provided at a lower level under one-bill budgeting compared to two-bill budgeting; specifically, if $\beta>\overline{\boldsymbol{\beta}}$, then $c_{s}^{i}\left(b_{s}^{i}\right)=(0,0)$ and $c_{d}^{i}\left(b_{d}^{i}\right) \in \mathbb{R}_{++}^{2}$ for any $i \in\{1,2\}$.

If polarization is low - that is, $\beta<\underline{\boldsymbol{\beta}}$ - then the two budgeting rules lead to the same provision of both public goods. The intuition behind this result is the same as with Proposition 2.1 part ( $i$ ). In particular, when polarization is low the best government policy for the legislator who proposes the budget gives the other legislator a payoff greater than that of the status-quo policy. Thus, under both budgeting rules, the proposer offers a budget that leads to his best government policy.

If polarization is high - that is, $\beta>\overline{\boldsymbol{\beta}}-$ then both public goods are provided at a lower level under one-bill budgeting compared to two-bill budgeting. In this case, polarization is high enough so that the best government policy for the legislator who proposes the budget is to allocate the whole tax revenue for provision of the public good that is more valuable to him. On the other hand, such a policy gives the other legislator a payoff that is less than that of the status-quo policy. In this situation, under two-bill budgeting, the proposer can offer a budget that promises provision of both public goods at $\alpha \frac{1}{n} L$ and the responder accepts this proposal.

Thus, we know that under two-bill budgeting both public goods are provided at a positive level. However, under one-bill budgeting, the proposer cannot commit to provision of two public goods separately. Hence, he offers a budget with income tax rate equal to zero.

Up to this point, we have taken the budgeting rules as given and analyzed their effects on public-good provision. We now focus on the determination of the budgeting rule in the game's first stage. Let $\Gamma$ be an economic environment and $\left(v^{i}, \rho^{i},\left(b_{\sigma}^{i}, a_{\sigma}^{i}, c_{\sigma}^{i}\right)_{\sigma \in\{s, d\}}\right)_{i \in\{1,2\}}$ be its unique equilibrium. Define

$$
u_{i \sigma}^{i} \equiv u_{i}\left(1-\tau_{\sigma}^{i}, \alpha c_{\sigma 1}^{i}\left(b_{\sigma}^{i}\right), \alpha c_{\sigma 2}^{i}\left(b_{\sigma}^{i}\right)\right)
$$

and

$$
u_{i^{\prime} \sigma}^{i} \equiv u_{i^{\prime}}\left(1-\tau_{\sigma}^{i}, \alpha c_{\sigma 1}^{i}\left(b_{\sigma}^{i}\right), \alpha c_{\sigma 2}^{i}\left(b_{\sigma}^{i}\right)\right)
$$

for any $\sigma \in\{s, d\}$ where $\tau_{\sigma}^{i}$ is the first component of $b_{\sigma}^{i}$; that is, $u_{i \sigma}^{i}$ and $u_{i^{\prime} \sigma}^{i}$ are the equilibrium payoffs of legislators $i$ and $i^{\prime}$, respectively, when legislator $i$ is the budget proposer under budgeting rule $\sigma$. By Assumption 2.1, we have $u_{1 \sigma}^{1}=u_{2 \sigma}^{2}=\bar{u}_{\sigma}$ and $u_{1 \sigma}^{2}=u_{2 \sigma}^{1}=\underline{u}_{\sigma}$ for any $\sigma \in\{s, d\}$. It is clear that $\bar{u}_{\sigma} \geq \underline{u}_{\sigma}$.

Proposition 2.3. Let $\Gamma$ be an economic environment that satisfies Assumptions 2.1-2.3 and $\left(v^{i}, \rho^{i},\left(b_{\sigma}^{i}, a_{\sigma}^{i}, c_{\sigma}^{i}\right)_{\sigma \in\{s, d\}}\right)_{i \in\{1,2\}}$ be its unique equilibrium. Then the following hold.
(i) If polarization is low, or both polarization and the cost of two-bill budgeting are high, then legislators propose one-bill budgeting; specifically, if $\beta<\underline{\boldsymbol{\beta}}$, or $\beta>\overline{\boldsymbol{\beta}}$ and $\bar{u}_{d}<1+c_{d}$, then $v^{i}=s$ for any $i \in\{1,2\}$.
(ii) If polarization is high, the cost of two-bill budgeting is moderate, and the
status-quo budgeting rule is one-bill budgeting, then legislators propose two-bill budgeting if and only if the political power of each legislators is high enough; specifically, if $\beta>\overline{\boldsymbol{\beta}}, \underline{u}_{d}<1+c_{d}<\bar{u}_{d}$, and $\sigma^{s}=s$, then $v^{i}=d$ if and only if $\bar{u}_{d}-\left(1+c_{d}\right) \geq \lambda_{i}\left(\bar{u}_{d}-\underline{u}_{d}\right) \geq 1+c_{d}-\underline{u}_{d}$ for any $i \in\{1,2\}$.
(iii) If polarization is high, the cost of two-bill budgeting is moderate, and the status-quo budgeting rule is two-bill budgeting, then legislators propose two-bill budgeting if and only if the political power of at least one legislator is high enough; specifically, if $\beta>\overline{\boldsymbol{\beta}}, \underline{u}_{d}<1+c_{d}<\bar{u}_{d}$, and $\sigma^{s}=d$, then $v^{i}=d$ if and only if $\bar{u}_{d}-\left(1+c_{d}\right) \geq \lambda_{i}\left(\bar{u}_{d}-\underline{u}_{d}\right)$ or $\lambda_{i}\left(\bar{u}_{d}-\underline{u}_{d}\right) \geq 1+c_{d}-\underline{u}_{d}$ for any $i \in\{1,2\}$.
(iv) If polarization is high and the cost of two-bill budgeting is low, then legislators propose two-bill budgeting; specifically, if $\beta>\overline{\boldsymbol{\beta}}$ and $1+c_{d}<\underline{u}_{d}$, then $v^{i}=d$ for any $i \in\{1,2\}$.

As we state in Proposition 2.3, if polarization is low - that is, $\beta<\underline{\boldsymbol{\beta}}$ - then legislators propose one-bill budgeting. This is because when polarization is low, both budgeting rules lead to the same provision of public goods and two-bill budgeting is costly to implement. If polarization is high - that is, $\beta>\overline{\boldsymbol{\beta}}-$ then expected payoff of legislator $i$ is 1 under one-bill budgeting and $\lambda_{i} \bar{u}_{d}+\left(1-\lambda_{i}\right) \underline{u}_{d}-c_{d}$ under two-bill budgeting. Thus, if cost of two-bill budgeting is high - that is, $1+c_{d}>\bar{u}_{d}$ - then legislators propose one-bill budgeting; if it is low - that is, $1+c_{d}<\underline{u}_{d}$ - then they propose two-bill budgeting. If cost of two-bill budgeting is moderate; that is, $\bar{u}_{d} \geq 1+c_{d} \geq \underline{u}_{d}$, then implemented budgeting rule depends on the status-quo budgeting rule and the political power of the legislators. In particular, if the status-quo budgeting rule is one-bill budgeting, then legislators propose two-bill budgeting if and only if both legislators have high enough political
power. To understand why, note that since the status-quo budgeting rule is onebill budgeting, for a legislator to propose two-bill budgeting and for the other legislator to accept this proposal, both legislators should get a higher expected payoff under two-bill budgeting than under one-bill budgeting. Since the legislator who proposes the budget gets a higher payoff than the other legislator, this requires both legislators to have a high enough probability of proposing the budget. On the other hand, if the status-quo budgeting rule is two-bill budgeting, then the legislators propose two-bill budgeting if and only if at least one of the legislators has enough political power. In this case, because the status-quo budgeting rule is two-bill budgeting, for a legislator to propose two-bill budgeting and for the other legislator to accept such a proposal it is enough that one of the legislators gets a higher expected payoff under two-bill budgeting than under one-bill budgeting.

In our model, we assume that there are two public goods and each appropriation bill has one item. When our DoEH example is considered, these assumptions may seem restrictive for the environment that we want to analyze. At this point, we introduce a more general model that may be a better description of the environment that we want to analyze and for which our main results continue to hold. In our extended model, we assume two groups of public goods, in each group there two goods and an appropriation bill has two items.

We denote the each group of public goods by $t \in\{A, B\}$ and the goods in group $t$ by $\left(g_{t 1}, g_{t 2}\right)$. A citizen's utility function in group 1 is
$u_{1}\left(z, g_{A 1}, g_{A 2}, g_{B 1}, g_{B 2}\right)=z+\beta h\left(g_{A 1}\right)+(1-\beta) h\left(g_{A 2}\right)+\beta h\left(g_{B 1}\right)+(1-\beta) h\left(g_{B 2}\right)$
and in group 2 is
$u_{2}\left(z, g_{A 1}, g_{A 2}, g_{B 1}, g_{B 2}\right)=z+(1-\beta) h\left(g_{A 1}\right)+\beta h\left(g_{A 2}\right)+(1-\beta) h\left(g_{B 1}\right)+\beta h\left(g_{B 2}\right)$.

A government policy is described by a quintuple ( $\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}$ ), where $\tau$ is the income tax rate and $l_{t j}$ is the amount of labor allocated for the production of public good $j$ in group $t$. For the provision of each group of public goods, the maximum amount of labor that can be hired is $L \leq 1$.

Under one-bill budgeting, a budget, $\left(\tau, l_{A}, l_{B}\right)$, is composed of an income tax rate $\tau$ and the amount of labor $l_{t}$ that will be hired for the provision of public goods in each group $t$. If a one-bill budget is accepted, the proposer decides how to allocate the labor $l_{t}$ for the provision of the public goods $\left(g_{t 1}, g_{t 2}\right)$ for each group $t$.

Under two-bill budgeting, a budget ( $\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}$ ) is composed of an income tax rate $\tau$ and the amounts of labor $\left(l_{t 1}, l_{t 2}\right)$ that will be hired for the production of the public goods $\left(g_{t 1}, g_{t 2}\right)$ in each group $t$. If a two-bill budget is accepted, the proposer implements the labor-hiring policy specified in the budget.

We give a complete description of our extended model and results in Appendix B.3. Here, we would like to state that under assumptions analogous to Assumptions 2.1-2.3, results similar to Propositions 2.2 and 2.3 continue to hold.

### 2.5 Group-Specific Transfers

In this section, we allow the government policy to include group-specific transfers that is, monetary transfers of tax revenue to the citizens of a specific group. As Battaglini and Coate (2007) stated, the conventional wisdom is that allowing for
group-specific transfers decreases provision of public goods and leads to inefficient outcomes. This is also the general result in the literature - one exception is the aforementioned paper in which the authors show that legislative decision making can be efficient in the long run despite allowing group-specific transfers. We show that, in our model, if polarization is high, then allowing for group-specific transfers increases public-good provision and improves efficiency under one-bill budgeting.

When group-specific transfers are allowed, a government policy is described by a quintuple $\left(\tau, l_{1}, l_{2}, s_{1}, s_{2}\right)$ where $\tau$ is the income tax rate, $l_{j}$ is the amount of labor allocated for provision of public good $j$ and $s_{i}$ is the transfer to citizens of group $i$. The set of feasible government policies is given by
$\tilde{P}=\left\{\left(\tau, l_{1}, l_{2}, s_{1}, s_{2}\right) \in[0,1] \times\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}^{2} \times[0,2]^{2}: l_{1}+l_{2}+s_{1}+s_{2}=2 \tau\right\}$
and the status-quo policy is $\tilde{\mathbf{q}}^{s}=\left(\tau^{s}, l_{1}^{s}, l_{2}^{s}, s_{1}^{s}, s_{2}^{s}\right)=\mathbf{0}$.
We assume that the budgeting rule is given as one-bill budgeting. Thus, a budget $\tilde{\mathbf{b}}_{s}=\left(\tau, s_{1}, s_{2}\right)$ is composed of income tax rate $\tau$ and transfers to citizens of each group $\left(s_{1}, s_{2}\right)$. The set of feasible budget proposals is given by

$$
\begin{equation*}
\tilde{B}_{s}=\left\{\left(\tau, s_{1}, s_{2}\right) \in[0,1] \times[0,2]^{2}: 2 \tau-s_{1}-s_{2} \in\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}\right\} \tag{2.6}
\end{equation*}
$$

and, for a given budget $\tilde{\mathbf{b}}_{s}=\left(\tau, s_{1}, s_{2}\right) \in \tilde{B}_{s}$, the set of feasible labor-hiring policies is given by

$$
\begin{equation*}
\tilde{Q}_{s}\left(\tilde{\mathbf{b}}_{s}\right)=\left\{\mathbf{q}=\left(l_{1}, l_{2}\right) \in\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}^{2}: l_{1}+l_{2}=2 \tau-s_{1}-s_{2}\right\} . \tag{2.7}
\end{equation*}
$$

A strategy for legislator $i,\left(\tilde{b}_{s}^{i}, \tilde{a}_{s}^{i}, \tilde{c}_{s}^{i}\right)$, is composed of a budget-proposal strategy $\tilde{b}_{s}^{i} \in \tilde{B}_{s}$, a budget-acceptance strategy $\tilde{a}_{s}^{i}: \tilde{B}_{s} \rightarrow\{0,1\}$, and a labor-hiring strategy $\tilde{c}_{s}^{i}: \tilde{B}_{s} \rightarrow Q$ such that $\tilde{c}_{s}^{i}\left(\tilde{\mathbf{b}}_{s}\right) \in \tilde{Q}_{s}\left(\tilde{\mathbf{b}}_{s}\right)$ for each $\tilde{\mathbf{b}}_{s} \in \tilde{B}_{s}$. Legislator $i$ 's budgetacceptance strategy $\tilde{a}_{s}^{i}\left(\tilde{\mathbf{b}}_{s}\right)$ takes the value 1 if legislator $i$ accepts the budget proposal $\tilde{\mathbf{b}}_{s}$ offered by legislator $i^{\prime} \neq i$, and 0 otherwise. We consider subgameperfect equilibria. We restrict attention to equilibria in which (i) $\tilde{a}_{s}^{i}\left(\tilde{\mathbf{b}}_{s}\right)=1$ when legislator $i$ is indifferent between $\tilde{c}_{s}^{i^{\prime}}\left(\tilde{\mathbf{b}}_{s}\right)$ and $\tilde{\mathbf{q}}^{s}$ and (ii) $\tilde{a}_{s}^{i}\left(\tilde{b}_{s}^{i^{\prime}}\right)=1$ for all $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$. That is, a responding legislator accepts any proposal that he is indifferent between accepting and rejecting, and equilibrium proposals are always accepted.

Let $\left(\tilde{b}_{s}^{i}, \tilde{a}_{s}^{i}, \tilde{c}_{s}^{i}\right)_{i \in\{1,2\}}$ be an equilibrium when group-specific transfers are allowed. Define $\tilde{u}_{i s}^{i} \equiv u_{i}\left(1-\tilde{\tau}_{s}^{i}, \alpha \tilde{c}_{s 1}^{i}\left(\tilde{b}_{s}^{i}\right), \alpha \tilde{c}_{s 2}^{i}\left(\tilde{b}_{s}^{i}\right)\right)$ and $\tilde{u}_{i^{\prime} s}^{i} \equiv u_{i^{\prime}}\left(1-\tilde{\tau}_{s}^{i}, \alpha \tilde{c}_{s 1}^{i}\left(\tilde{b}_{s}^{i}\right), \alpha \tilde{c}_{s 2}^{i}\left(\tilde{b}_{s}^{i}\right)\right)$, where $\tilde{\tau}_{s}^{i}$ is the first component of $\tilde{b}_{s}^{i}$. By Assumption 2.1, we have $\tilde{u}_{1 s}^{1}=\tilde{u}_{2 s}^{2}=\tilde{u}_{s}$ and $\tilde{u}_{1 s}^{2}=\tilde{u}_{2 s}^{1}={\underset{\sim}{u}}_{s}$. It is clear that $\tilde{u}_{s} \geq{\underset{\sim}{u}}_{s} \geq 1$.

Proposition 2.4. Let $\Gamma$ be an economic environment that satisfies Assumptions 2.12.3. Let the budgeting rule be one-bill budgeting. Then there exists an equilibrium when group-specific transfers are allowed. Let $\left(\tilde{b}_{s}^{i}, \tilde{a}_{s}^{i}, \tilde{c}_{s}^{i}\right)_{i \in\{1,2\}}$ be an equilibrium when group-specific transfers are allowed and $\left(b_{s}^{i}, a_{s}^{i}, c_{s}^{i}\right)_{i \in\{1,2\}}$ be the unique equilibrium when group-specific transfers are not allowed. If $n \geq 2$, there exists $\tilde{\boldsymbol{\beta}} \in\left(\frac{1}{2}, 1\right)$ such that if $\beta>\tilde{\boldsymbol{\beta}}$, then the following hold.
(i) Allowing for group-specific transfers increases the income tax rate and publicgood provision; specifically, $\left(\tau_{s}^{i}, c_{s i^{\prime}}^{i}\left(b_{s}^{i}\right), c_{s i}^{i}\left(b_{s}^{i}\right)\right)=(0,0,0)$ and $\left(\tilde{\tau}_{s}^{i}, \tilde{c}_{s i^{\prime}}^{i}\left(\tilde{b}_{s}^{i}\right), \tilde{c}_{s i}^{i}\left(\tilde{b}_{s}^{i}\right)\right) \in$ $\mathbb{R}_{++} \times\{0\} \times \mathbb{R}_{++}$for any $i, i^{\prime} \in\{1,2\}$ with $i^{\prime} \neq i$.
(ii) Allowing for group-specific transfers provides Pareto improvement in effi-
ciency; specifically, $\bar{u}_{s}<\tilde{u}_{s}$ and $\underline{u}_{s} \leq \underline{u}_{s}$.

When group-specific transfers are not allowed and polarization is high, under one-bill budgeting public-good provision is zero, since one-bill budgeting does not provide the opportunity to commit to provision of public goods separately. Allowing for group-specific transfers provides a new policy dimension to commit to in the budget proposal, which results in positive provision of a public good. In particular, when group-specific transfers are allowed, the proposer offers a budget with a positive income tax rate and transfers to citizens of the responding legislator's group. He chooses the amount of transfers high enough so that the responding legislator accepts the proposal and low enough so that the whole tax revenue is not spent on transfers. Thus, a positive amount of the public good that is more valuable to the proposer can be provided. Compared to the situation when group-specific transfers are not allowed, this does not decrease the payoff of any citizen and increases the payoff of the citizens in the proposer's group.

### 2.6 Conclusion

In this paper, we analyze a bargaining model in which two legislators first decide on the number of appropriation bills in the budget, and subsequently on provision of two public goods. While deciding on the number of appropriation bills, legislators choose between one- and two-bill budgeting. We show that if polarization between legislators is low, then the two budgeting rules lead to the same provision of both public goods. On the other hand, if polarization between legislators is high, then under one-bill budgeting both public goods are provided at a lower level than under two-bill budgeting. Additionally, we show that if polarization between
legislators is low, then the implemented budgeting rule is one-bill budgeting. If polarization between the legislators is high, then the interesting case emerges when the cost of two-bill budgeting is moderate. In this case, the implemented budgeting rule depends on the status-quo budgeting rule and the distribution of political power between the legislators. In particular, compared to a status quo of one-bill budgeting, a two-bill budgeting status quo allows for a more biased distribution of political power between legislators to result in implementation of two-bill budgeting.

Several extensions of our model seem interesting for future research. First, we can extend our model to an infinite horizon and let the status-quo budgeting rule be determined endogenously by the previous period's implemented budgeting rule. This would contribute to the growing literature of infinite-horizon political-economy models. This literature includes, for example, Krusell and Rios-Rull (1999), Hassler et al. (2003), and Azzimonti (2011) that analyze infinite-horizon political economy models of policymaking, and Acemoglu and Robinson (2001) that analyze an infinite-horizon model of political regime change.

We characterize our equilibrium only for high and low levels of polarization. How does the equilibrium look for moderate levels of polarization? Moreover, in our model with four public goods, we assume that the degree of polarization between legislators for each group of public goods is the same. It would be interesting to allow for different degrees of polarization between legislators for each group of public goods.

It would also be interesting to extend our model to more than two legislators. Although there are papers in the literature that analyze legislative bargaining
models of public finance with more than two legislators - such as Battaglini and Coate (2007) and Persson et al. (2000) - they either assume that there is only one public good or all dimensions of the government policy are determined by the legislative authority.

## Chapter 3

## Commodity Tax Competition

## When Countries Differ in Size

## and Transportation Cost

### 3.1 Introduction

Free movement of goods across countries has been putting pressure on the commodity tax rates of national governments for a long time by causing the movement of tax bases. As a result, various discussions have arisen among the exposed countries on the coordination of tax policies.

Motivated by these facts, one of the first papers that analyzed commodity taxation in an international setting is Kanbur and Keen (1993). In this seminal paper, the authors proposed a model of commodity tax competition with crossborder shopping between two countries. The countries have the same geographical size but different population densities. Consumers demand at most a unit of a
consumption good which they can buy either from their own country or from the neighbor country by paying a travel cost. The governments of the two countries compete in tax rates on the consumption good to maximize their revenues. Although the smaller country's best-response correspondence is discontinuous, there always exists an equilibrium in pure strategies in which the smaller country chooses a lower tax rate. Later, Nielsen (2001), proposed a similar model with the difference that the countries have the same population density but different geographical sizes. Still in Nielsen's model, there always exists an equilibrium in pure strategies in which the smaller country chooses a lower tax rate. However, Nielsen's model provided a simpler setting with a continuous best-response function for both countries. This let Nielsen analyze more-complicated environments and policies.

Nonetheless, both models assume that the transportation costs in the two countries are the same. In this paper, we relax this assumption in both models. We show that this leads to discontinuous best-response correspondences in Nielsen's model. Moreover, the existence of equilibrium in pure strategies is no longer guaranteed in either model. We give necessary and sufficient conditions for the existence of equilibrium in pure strategies in both models. Finally, we show that if there is no equilibrium in pure strategies, there can exist an equilibrium in mixed strategies in which the smaller country chooses a lower tax rate with a positive probability in both models.

The paper proceeds as follows. In Section 3.2, we summarize related literature. In Section 3.3, we present our model and analyze the equilibrium for the Nielsen and Kanbur-Keen (KK) environments separately. In Section 3.4, we conclude.

### 3.2 Related Literature

There is now a large theoretical literature on international tax competition as surveyed in Keen and Konrad (2013). A subset of this literature focuses on commodity tax competition. One of the first papers on the subject is Mintz and Tulkens (1986) in which the authors analyze a general equilibrium model of two countries trading in two goods. Their model provides a general setting with few restrictions on consumer preferences. However, this comes with some costs and equilibrium in pure strategies is not shown to exist generally. Even if it exits, Mintz and Tulkens (1986) report few general characteristics or comparative statistics results. Kanbur and Keen (1993) provide a simpler model in which there always exists an equilibrium in pure strategies and that allows consideration of coordination policies such as tax harmonization or minimum tax rates. Nielsen (2001) provides an even simpler model with continuous best-response functions. Moreover, he analyzes the effects of transportation cost for goods and border inspection on taxes. Both KK and Nielsen assume that the populations are distributed uniformly in a country. Trandel (1994) analyzes the effects of nonuniform population distributions in a two-country setting. Ohsawa (1999) analyzes a model with a finite number of countries located on a line with the same geographical sizes and population densities. He shows that there always exists an equilibrium in pure strategies and the equilibrium tax rates exhibit a $U$ shape. Moreover, he analyzes a two-country model that includes the Nielsen (2001) model as a special case. ${ }^{1}$

All the papers mentioned above assume that the transportation costs in differ-

[^11]ent countries are the same. Haufler (1996) analyze a model that allows different transportation costs in two countries, but the transportation costs depend on the amount of the good transported. In our model, they depend on the distance the consumers travel. Devereux et al. (2007) analyze a model that generalizes the Kanbur and Keen (1993) and Nielsen (2001) models and allows different transportation costs in the two countries. They note that different ratios of transportation cost to population density in the two countries may cause nonexistence of equilibrium. However, they do not address this question and conduct their analysis assuming an equilibrium always exists.

Another common feature of the aforementioned papers is the assumption that the governments are revenue maximizers. Nielsen (2002) assumes that governments also care about the welfare of consumers and shows that, in this case, the smaller country can choose a higher tax rate.

As Kanbur and Keen (1993) state, issues similar to the ones that arise in international tax competition also arise in tax competition among federal states. However, in tax competition among federal states, the federal government also plays a role with its taxation power. Some papers in this literature are Arnott and Grieson (1981), Gordon (1983), and Wilson (1986).

### 3.3 Model

As in the KK and Nielsen models, we have a partial equilibrium model with two countries and a single good taxed by the governments in each country. The countries are located on the real line. The "foreign" country extends from -1 to $b$ and the "home" country from $b$ to 1 where $b \geq 0$. The population is distributed
uniformly in each country with a density $H$ in the foreign country and $h$ in the home country where $H \geq h>0$.

For the consumers in the foreign country the value of the good is $V$ and for the people in the home country it is $v$ where $V, v>0$. Each consumer buys only one unit of the good if its value is greater than or equal to its cost. We assume that the marginal cost of production is zero and there is free entry into the market in both countries. The foreign country imposes a unit tax $T$ on the goods sold within its borders and the home country a unit $\operatorname{tax} t$ where $T, t \geq 0$. If the border between the countries is closed, the consumers have to buy the good from the shops in their country that are located at each point. If the border is open, they can buy the good either from the shops in their country or from the shops in the other country at the border. If they buy the good from their own country, they pay no transportation cost. However, if they buy it from the other country, they pay a transportation cost of $D$ in the foreign country and a transportation cost of $d$ in the home country for each unit of distance that they travel, including the returning cost. So, a consumer in the foreign country living at the distance $s$ from the border buys the good from the home country if and only if

$$
\begin{equation*}
V-t-D s \geq V-T \tag{3.1}
\end{equation*}
$$

and

$$
V-t-D s \geq 0
$$

Similarly, a consumer in the home country living at the distance $s$ from the border
buys the good from the foreign country if and only if

$$
\begin{equation*}
v-T-d s \geq v-t \tag{3.2}
\end{equation*}
$$

and

$$
v-T-d s \geq 0
$$

Equation 3.1 implies that $s \leq \frac{T-t}{D}$, and Equation 3.2 that $s \leq \frac{t-T}{d}$.
If $b=0$ and $D=d$, we have the KK model. If $H=h=1$ and $D=d$, we have the Nielsen model.

Following KK and Nielsen, we assume that the objective of both governments is maximizing the tax revenue. As KK states, this can be motivated either by assuming that each country is governed by a rent-seeking politician or tax revenues are spent on a public good with a very high value for the people. When the border is open, both countries choose their tax rates simultaneously. We focus on the Nash equilibria of this game. In the following two subsections, we consider the effects of asymmetric transportation costs for the Nielsen and KK models separately. Throughout the paper, we assume that the value of the good is high enough for the people in both countries so that at the resulting tax rates all people want to buy the good. ${ }^{2}$

### 3.3.1 Nielsen Environment

In this subsection, we extend the Nielsen (2001) model by allowing for different transportation costs in the two countries. Thus, we assume that the population

[^12]densities in the two countries are the same.

Assumption 3.1. $H=h=1$.

The revenue function of the home country is given as

$$
r(T, t)= \begin{cases}t\left(1-b+\frac{T-t}{D}\right) & \text { if } t \leq T  \tag{3.3}\\ t\left(1-b-\frac{t-T}{d}\right) & \text { if } t \geq T\end{cases}
$$

Since Nielsen (2001) assumes that $D=d$, the revenue functions in his model do not need to be defined partially as in our model. As we state in Lemma 3.1, the best-response correspondence of the home country takes two different shapes depending on the value of the transportation costs in the two countries. The proof of all lemmas and propositions are given in Appendix C.1.

Lemma 3.1. Under Assumption 3.1, the best-response correspondence of the home country, $t(T)$, is given as follows. If $D \leq d$, we have

$$
t(T)= \begin{cases}\frac{d(1-b)+T}{2} & \text { if } T \leq(1-b) \sqrt{D d}  \tag{3.4}\\ \frac{D(1-b)+T}{2} & \text { if }(1-b) \sqrt{D d} \leq T\end{cases}
$$

and if $D>d$, we have

$$
t(T)= \begin{cases}\frac{d(1-b)+T}{2} & \text { if } T \leq d(1-b)  \tag{3.5}\\ T & \text { if } d(1-b)<T \leq D(1-b) \\ \frac{D(1-b)+T}{2} & \text { if } D(1-b)<T\end{cases}
$$

The home country's best-response correspondence when $D \leq d$ is given in


Figure 3.1: The home country's best-response correspondence when $D \leq d$ in the Nielsen environment.

Figure 3.1. As we see in the figure, it is discontinuous and multivalued when $T=(1-b) \sqrt{D d}$. This is because, when $T$ is low enough, it is better for the home country to choose a tax rate higher than the foreign country and have outward shopping. In this case, since it only collects taxes from its own citizens, its tax choice depends on the transportation cost in its borders, $d$. However, when $T$ is high enough, it is better for the home country to choose a tax rate lower than the foreign country and have inward shopping. In this case, since it tries to attract consumers from the foreign country, its tax choice will depend on the transportation cost in the foreign country, $D$, which is less than $d$. This creates the discontinuity in the best-response correspondence. The best-response correspondence of the home country when $D>d$ is given in Figure 3.2. As we see in the figure, it is piecewise linear. ${ }^{3}$ In Nielsen (2001), since the transportation costs are equal, the best-response functions are linear (without piecewise) and continuous.

Proposition 3.1. Under Assumption 3.1, there exists an equilibrium in pure

[^13]

Figure 3.2: The home country's best-response correspondence when $D>d$ in the Nielsen environment.
strategies if and only if $\frac{3+b}{3(1-b)} \geq \sqrt{\frac{d}{D}} \geq \frac{3-b}{3(1+b)}$. If it exists, it is unique and the equilibrium tax rates are given as

$$
t_{N}=D\left(1-\frac{b}{3}\right) \text { and } T_{N}=D\left(1+\frac{b}{3}\right) .
$$

As we state in Proposition 3.1, under Assumption 3.1, there exists an equilibrium in pure strategies if and only if the transportation costs are close enough to each other. If an equilibrium in pure strategies exists, it is unique and, at the equilibrium, the smaller country (home) chooses a lower tax rate. The interesting point is that while the value of equilibrium tax rates depend on the transportation cost only in the larger (foreign) country, the existence of equilibrium depends on the transportation costs in both countries. In Nielsen (2001), the existence of equilibrium in pure strategies is guaranteed because the transportation costs are equal in his model.

Proposition 3.2. Under Assumption 3.1, if an equilibrium in pure strategies does


Figure 3.3: An illustration of Part (i) of Proposition 3.2.
not exist, an equilibrium in mixed strategies can exist such that the larger country sets a lower tax rate with a positive probability. In particular,
(i) If $\frac{3-9 b}{3(1+b)}>\sqrt{\frac{d}{D}}$, then $t_{N}=(1+b) \sqrt{D d}, T_{N}=\frac{(1+b)(d+\sqrt{D d})}{2}$ with probability $\alpha_{N}$ and $T_{N}=\frac{(1+b)(D+\sqrt{D d})}{2}$ with probability $1-\alpha_{N}$, where $\alpha_{N}=\frac{\frac{3-b}{2}-\frac{3}{2}(1+b) \sqrt{\frac{d}{D}}}{(1+b) \frac{3}{2} \frac{D-d}{\sqrt{D d}}}$.
(ii) If $\sqrt{\frac{d}{D}}>\frac{3+9 b}{3(1-b)}$, then $t_{N}=\frac{(1-b)(D+\sqrt{D d})}{2}$ with probability $\beta_{N}, t_{N}=$ $\frac{(1-b)(d+\sqrt{D d})}{2}$ with probability $1-\beta_{N}$ and $T_{N}=(1-b) \sqrt{D d}$, where $\beta_{N}=\frac{\frac{3+b}{2}-\frac{3}{2}(1-b) \sqrt{\frac{D}{d}}}{(1-b) \frac{3}{2} \frac{d-D}{\sqrt{D d}}}$.

As we state in Proposition 3.2, under Assumption 3.1, if an equilibrium in pure strategies does not exist, we have an equilibrium in mixed strategies when the ratio of transportation costs are large or small enough. In contrast to the equilibrium in pure strategies, at the mixed-strategy equilibrium, the larger country can choose a lower tax rate with a positive probability. An illustration of the case in Part (i) of Proposition 3.2 is given in Figure 3.3. Since $\frac{d}{D}<1$, the bestresponse correspondence of the foreign country is discontinuous and multivalued at $t=t_{0}=(1+b) \sqrt{D d}$. This also allows us to have a mixed-strategy equilibrium. If $t=t_{0}$, the foreign country obtains the same revenue by choosing $T_{1}$ or $T_{2}$. So,
choosing $T_{1}$ with a probability $\alpha$ and $T_{2}$ with the probability $1-\alpha$ is a best response. If we can show that for some value of $\alpha$, choosing $t_{0}$ is also a best response for the home country, we have a mixed-strategy equilibrium. We show the existence of such an $\alpha$ with the following arguments. If the foreign country chooses $T_{1}$ with probability one, that is, $\alpha=1$, choosing $t_{1}$ is the best response for the home country. If the foreign country chooses $T_{2}$ with probability one, that is, $\alpha=0$, it is best response for the home country to choose $t_{2}$. Since, $(1+b) \sqrt{D d}$ is between $t_{1}$ and $t_{2}$, and the best response of the home country to $\alpha$ changes continuously with respect to it, there exists $\alpha_{N} \in(0,1)$ for which $t_{0}$ is a best response of the home country when $\alpha=\alpha_{N}$. Indeed, thanks to the quadratic form of the revenue functions, we can compute the value of $\alpha_{N}$. Moreover, since $d<D, t_{N}>T_{N}$ with probability $\alpha_{N}$. A similar situation arises in Part (ii) of Proposition 3.2 but the home country randomizes over two tax rates in this case since $\frac{d}{D}>1 .{ }^{4}$

### 3.3.2 KK Environment

In this subsection, we extend the model of Kanbur and Keen (1993) allowing for different transportation costs in the two countries. Thus, we assume that the geographical sizes of the two countries are the same.

Assumption 3.2. $b=0$.

[^14]The home country's revenue function is given as

$$
r(T, t)= \begin{cases}t h+t H \frac{T-t}{D} & \text { if } t \leq T  \tag{3.6}\\ t h\left(1-\frac{t-T}{d}\right) & \text { if } t \geq T\end{cases}
$$

The home country's best-response correspondence takes two different shapes depending on the ratio of population densities to transportation costs in the two countries. Let $\theta=h / H$.

Lemma 3.2. Under Assumption 3.2, the best-response correspondence of the home country, $t(T)$, is given as follows. If $D \theta \leq d$, we have

$$
t(T)= \begin{cases}\frac{d+T}{2} & \text { if } T \leq \sqrt{D \theta d}  \tag{3.7}\\ \frac{D \theta+T}{2} & \text { if } \sqrt{D \theta d} \leq T\end{cases}
$$

If $D \theta>d$, we have

$$
t(T)= \begin{cases}\frac{d+T}{2} & \text { if } T \leq d  \tag{3.8}\\ T & \text { if } d<T \leq D \theta \\ \frac{D \theta+T}{2} & \text { if } D \theta<T\end{cases}
$$

As we see in Figure 3.4, the home country's best-response correspondence is discontinuous at $T=\sqrt{D \theta d}$ when $D \theta \leq d$. It is clear from Equation 3.7 that the discontinuity will persist even if we assume that the transportation costs are the same in the two countries provided that the densities of the populations are different, as in the model of Kanbur and Keen (1993). However, as we show below, while there always exists an equilibrium in pure strategies when the transportation costs are the same in the two countries, this does not hold when the transportation


Figure 3.4: The home country's best-response correspondence when $D \theta \leq d$ in the KK environment.
costs are different. ${ }^{5}$ In Figure 3.5, we also present the home country's best-response correspondence when $D \theta>d$ for completeness.

Proposition 3.3. Under Assumption 3.2, there exists an equilibrium in pure strategies if and only if $\frac{2 / \theta+1}{3} \geq \sqrt{\frac{d}{D \theta}} \geq \frac{2 \theta+1}{3}$. If it exists, it is unique and the equilibrium tax rates are given as

$$
t_{K K}=\frac{D}{3}(2 \theta+1) \text { and } T_{K K}=\frac{D}{3}(2+\theta) .
$$

As we state in Proposition 3.3, under Assumption 3.2, there exists an equilibrium in pure strategies if and only if the ratio of transportation cost to population density, $\frac{d / h}{D / H}$, is close enough in the two countries. If an equilibrium in pure strategies exists, it is unique and at the equilibrium the smaller country (home) chooses a lower tax rate. As in the Nielsen environment, while the equilibrium tax rates depend on the transportation cost only in the larger (foreign) country, the existence of

[^15]

Figure 3.5: The home country's best-response correspondence when $D \theta>d$ in the KK environment.
equilibrium depends on the transportation costs in both countries. Existence of equilibrium in pure strategies in the Kanbur and Keen (1993) model is guaranteed since the transportation costs in the two countries are assumed to be equal.

Proposition 3.4. Under Assumption 3.2, if an equilibrium in pure strategies does not exist, an equilibrium in mixed strategies can exist such that the larger country sets a lower tax rate with a positive probability. In particular,
(i) If $2 \theta-1>\sqrt{\frac{d}{D \theta}}$, then $t_{K K}=\sqrt{d D / \theta}, T_{K K}=\frac{d / \theta+\sqrt{d D / \theta}}{2}$ with probability $\alpha_{K K}$ and $T_{K K}=\frac{D+\sqrt{d D / \theta}}{2}$ with probability $1-\alpha_{K K}$, where $\alpha_{K K}=\frac{h+\frac{H}{2}\left(3 \sqrt{\frac{d}{D \theta}}-1\right)}{\frac{3}{2}\left(\frac{h}{d}-\frac{H}{D}\right) \sqrt{\frac{D D}{\theta}}}$.
(ii) If $\sqrt{\frac{d}{D \theta}}>\frac{2}{\theta}-1$, then $t_{K K}=\frac{D \theta+\sqrt{D \theta d}}{2}$ with probability $\beta_{K K}, t_{K K}=\frac{d+\sqrt{D \theta d}}{2}$ with probability $1-\beta_{K K}$ and $T_{K K}=\sqrt{D \theta d}$, where $\beta_{K K}=\frac{H+\frac{h}{2}\left(3 \sqrt{\frac{D \theta}{d}}-1\right)}{\frac{3}{2}\left(\frac{H}{D}-\frac{h}{d}\right) \sqrt{D d \theta}}$.

As we state in Proposition 3.4, under Assumption 3.2, if an equilibrium in pure strategies does not exist, an equilibrium in mixed strategies can exist in which the larger (foreign) country chooses a lower tax rate with a positive probability. Figure 3.6 illustrates the case in Part (i) of Proposition 3.4. Since, $\frac{d}{D \theta}<1$, the best-


Figure 3.6: An illustration of Part (i) of Proposition 3.4.
response correspondence of the foreign country is discontinuous and multivalued at $t=\sqrt{\frac{d}{D \theta}}$. Following arguments similar to those we presented for Part (i) of Proposition 3.2, we can show that an equilibrium in mixed strategies exists in which the foreign country randomizes over two tax rates and the home country chooses the tax rate $\sqrt{\frac{d}{D \theta}}$. In Part (ii) of the proposition, since $\frac{d}{D \theta}>1$, the home country's best-response correspondence is discontinuous and multivalued at $T=\sqrt{D \theta d}$, and, at the equilibrium, it randomizes over two tax rates while the foreign country chooses the tax rate $\sqrt{D \theta d}$.

### 3.4 Conclusion

We analyzed two commodity tax-competition models with cross-border shopping between two countries, focusing on the effects of asymmetric transportation costs. Extending the model of Nielsen (2001), in our first model, we showed that asymmetric transportation costs lead to discontinuous best-response correspondences
and nonexistence of an equilibrium in pure strategies. Nevertheless, we showed that if an equilibrium in pure strategies does not exist, an equilibrium in mixed strategies can exist in which the smaller country chooses a lower tax rate with a positive probability, contrary to the conventional result in the literature.

In our second model, we extended the model of Kanbur and Keen (1993), which already had discontinuous best-response correspondences due to asymmetric population densities but still had a unique equilibrium in pure strategies. As in our first model, we showed that asymmetric transportation costs leads to nonexistence of an equilibrium in pure strategies, but if an equilibrium in pure strategies does not exist, there can exist an equilibrium in mixed strategies in which the smaller country chooses a higher tax rate with a positive probability.

Our future research plan is to extend these models to a general equilibrium setting with two goods in which the terms of trade are determined endogenously.

## Chapter 4

## Fiscal Capacity with Durable

## Public Goods

### 4.1 Introduction

A growing literature in economics focuses on the determinants of state capacity to raise revenue, known as fiscal capacity. Besley and Persson (2010) propose a model that represents the bare bones of the settings analyzed in this literature. In particular, the authors of the paper offer a two-period model in which two groups in the society decide on government policy - taxes and public-good provision - in each period and investment in fiscal and legal capacity of the state in the first period. From a historical perspective, they interpret the public good as the defense activities against external threats. Thus, if there is an external conflict, the public good takes a high value; otherwise, it takes a low value. One of the authors' main results is that an increase in the risk of external conflict increases investment in fiscal capacity - a common result in the literature. As Battaglini and Coate
(2007) state, most public goods, particularly national defense activities, accumulate over time. However, Besley and Persson assume that the public good depreciates completely in the first period, as do the other papers in the literature.

In this paper, we add accumulation of the public good to the Besley and Persson (2010) fiscal capacity model. This gives us two results. First, an increase in the public good's depreciation rate increases investment in fiscal capacity because the value of investing in fiscal capacity increases relative to the public good. Second, under certain conditions, an increase in the risk of external conflict decreases investment in fiscal capacity, contrary to the general result in the literature. This is because an increase in the risk of external conflict increases the value of investment in both fiscal capacity and in the public good. Thus, depending on the parameters of the model, either effect can dominate. We give the conditions for the dominance of each effect.

Our paper is organized as follows. In Section 4.2, we give a brief summary of related literature. In Section 4.3, we describe our model and present our results. Finally, in Section 4.4, we conclude.

### 4.2 Related Literature

The evolution of states' fiscal capacity and the effects of war on this capacity have long been investigated by political and economic historians (see, for example, Tilly, 1985, 1990; Levi, 1988; Brewer, 1989). In recent years, a growing literature in economics analyzes the determinants of a state's fiscal capacity. The bulk of the research in the economics literature is composed of the joint papers of Timothy Besley and Torsten Persson. In Besley and Persson (2010), the authors state
the positive correlation between the measures of fiscal and legal capacity using cross-country data. Additionally, they show that both measures are positively correlated with per capita income. They propose a two-period model that explains these correlations, highlighting the effects of external-conflict risk and political stability. Moreover, they analyze the effects of natural-resource income on internal conflict, through which it affects investments in legal and fiscal capacity.

Besley and Persson (2009) propose another two-period model to analyze the determinants of investment in legal and fiscal capacity. In this model, the authors provide a deeper microfoundation for the effects of legal capacity on income by modeling capital markets. Moreover, this model facilitates an analysis of the effects of inclusiveness of political institutions on the investments in legal and fiscal capacity. In this paper, they also provide conditional correlations between the outcome variables and the determinants proposed in their model.

In their book, Besley and Persson (2011) provide a more elaborate discussion of the empirical facts on fiscal and legal capacity, deeper micro foundations for their two-period model, various extensions to analyze the effects of different phenomena on investments in legal and fiscal capacity, and more empirical evidence to support the implications of their model.

Besley et al. (2013) propose an infinite horizon model to analyze investment in fiscal capacity. Departing from their previous papers, in this paper, the authors assume that the value of the public good is nonrandom and the cost of investment in fiscal capacity is linear. Nevertheless, they work with a more general quasilinear utility function. They show three steady states with different levels of fiscal capacity. Lastly, Cárdenas (2010) analyses a two-period model to consider the effects of
income inequality on investment in fiscal capacity.

### 4.3 Model

The society has a unit mass of population divided into two groups with equal sizes. There are two time periods denoted by $s=1,2$. In each period, $s$, an incumbent group, $I_{s}$, holds the political power and the other group, $O_{s}$, represents the opposition. The opposition in the first period takes the political power in the second period with a probability $\gamma$. In each period, $s$, the incumbent decides on a government policy composed of a tax rate, $t^{J_{s}}$, for each group, $J_{s} \in\left\{I_{s}, O_{s}\right\}$, and investment in a public good, $g_{s}$. At any period, $s$, the level of the public good is given as

$$
G_{s}=g_{s}+(1-d) G_{s-1},
$$

where $d \in[0,1]$ is a depreciation rate and $G_{0}$ is given exogenously. We assume no disinvestment in the public good; that is, $g_{s} \geq 0$. The government is constrained by its fiscal capacity, $\tau_{s}$, in choosing tax rates, so we have $t^{J_{s}} \leq \tau_{s}$ for each group, $J_{s}$. We will explain how the fiscal capacity is determined below. Notice that, to allow for transfers between the groups, we allow for negative tax rates. The government also has a natural-resource income $R_{s} \in\left\{R_{L}, R_{H}\right\}$, where $R_{H}>R_{L}$ and $R_{s}=R_{H}$ with a probability $\rho$ in each period, $s$. In each period, each individual has an income $\omega>0$.

The utility of a member of group $J_{s}$ is given as

$$
\alpha_{s} G_{s}+c^{J_{s}}=\alpha_{s} G_{s}+\left(1-t^{J_{s}}\right) \omega
$$

where $c^{J_{s}}$ is his consumption, $\alpha_{s} \in\left\{\alpha_{L}, \alpha_{H}\right\}$ where $\alpha_{H}>2>\alpha_{L}$, and $\alpha_{s}=\alpha_{H}$ with probability $\phi$ in each period $s$. Following Besley and Persson (2010), we interpret $G_{s}$ as defense activities against external threats and $\phi$ as the risk of external conflict.

The first-period fiscal capacity is given as $\tau_{1}>0$ and it does not depreciate between the periods. The second period fiscal capacity, $\tau_{2}$, is chosen by the first-period incumbent, $I_{1}$, with the restriction that $\tau_{2} \geq \tau_{1}$. Investment in fiscal capacity has a cost, $F\left(\tau_{2}-\tau_{1}\right)$, where $F($.$) is an increasing and convex function$ with $F(0)=F_{\tau}(0)=0$. So, in the second period, the government budget constraint is given as

$$
\begin{equation*}
\sum_{J_{2} \in\left\{I_{2}, O_{2}\right\}} \frac{t^{J_{2}} \omega^{J_{2}}}{2}-g_{2}+R_{2}=0, \tag{4.1}
\end{equation*}
$$

and in the first period it is given as

$$
\begin{equation*}
\sum_{J_{1} \in\left\{I_{1}, O_{1}\right\}} \frac{t^{J_{1}} \omega^{J_{1}}}{2}-g_{1}-F\left(\tau_{2}-\tau_{1}\right)+R_{1}=0 . \tag{4.2}
\end{equation*}
$$

The exact timing of the events in each period, $s$, is as follows.
Stage 1. The initial conditions, $\tau_{s}$ and $G_{s-1}$, and the last period's incumbent, $I_{s-1}$, are given.

Stage 2. The value of the public good, $\alpha_{s}$, and the income from natural resources, $R_{s}$, are realized.

Stage 3. The last period's incumbent, $I_{s-1}$, remains in power with probability $1-\gamma$.

Stage 4. The current period's incumbent chooses the tax rates and investment in the public good, $\left\{\left\{t^{J_{s}}\right\}_{J_{s} \in\left\{I_{s}, O_{s}\right\}}, g_{s}\right\}$. In the first period, the incumbent $I_{1}$ also
determines the fiscal capacity in the second period, $\tau_{2}$.
Stage 5. The payoffs for period $s$ are realized.
We focus on subgame-perfect equilibria of the game.
The Second Period. In the second period, once the tax rates are chosen, investment in the public good, $g_{2}$, is determined by the government budget constraint as given in Equation 4.1. So, the incumbent, $I_{2}$, chooses the tax rates, $t^{J_{2}}$, for each $J_{2} \in\left\{I_{2}, O_{2}\right\}$ to maximize his utility,

$$
\omega\left(1-t^{I_{2}}\right)+\alpha_{2}\left(\frac{t^{I_{2}} \omega+t^{O_{2}} \omega}{2}+R_{2}+(1-d) G_{1}\right)
$$

subject to the constraints $t^{J_{2}} \leq \tau_{2}$ for each $J_{2} \in\left\{I_{2}, O_{2}\right\}$. Because of the linear structure of the utility functions, the incumbent spends all tax revenue either for transfers to his group or investment in the public good. Therefore, if $\alpha_{2}=\alpha_{H}>2$, both groups are taxed at the highest rate, $t^{I_{2}}=t^{O_{2}}=\tau_{2}$, and all government revenue is invested in the public good. If $\alpha_{2}=\alpha_{L}<2$, the opposition group is taxed at the highest rate, $t^{O_{2}}=\tau_{2}$, and all government revenue is transferred to the incumbent group, $t^{I_{2}}=-\frac{\tau_{2} \omega+2 R_{2}}{\omega}$. We can collect these results in the following proposition.

Proposition 4.1. We have $t^{O_{2}}=\tau_{2}$. Moreover, we have $t^{I_{2}}=\tau_{2}$ and $g_{2}=\tau \omega+R_{2}$ if $\alpha_{2}=\alpha_{H}$, and $t^{I_{2}}=-\tau_{2}-\frac{2 R_{2}}{\omega}$ and $g_{2}=0$ if $\alpha_{2}=\alpha_{L}$.

The First Period. In the first period, once the tax rates and second-period fiscal capacity are chosen, investment in the public good, $g_{1}$, is determined by the government budget constraint as given in Equation 4.2. So, considering the equilibrium policies given in Proposition 4.1, in the first period, the incumbent, $I_{1}$,
chooses a tax rate, $t^{J_{1}}$, for each $J_{1} \in\left\{I_{1}, O_{1}\right\}$ and a fiscal capacity, $\tau_{2}$, for the next period to maximize his expected utility:

$$
\begin{aligned}
& \omega\left(1-t^{I_{1}}\right)+\alpha_{1} G_{1} \\
& +\phi\left[\omega\left(1-\tau_{2}\right)+\alpha_{H}\left(\tau_{2} \omega+(1-d) G_{1}+E\left(R_{2}\right)\right)\right] \\
& +(1-\phi)(1-\gamma)\left[\omega\left(1+\tau_{2}\right)+2 E\left(R_{2}\right)+\alpha_{L}(1-d) G_{1}\right] \\
& +(1-\phi) \gamma\left[\omega\left(1-\tau_{2}\right)+\alpha_{L}(1-d) G_{1}\right]
\end{aligned}
$$

subject to the constraints

$$
\begin{gathered}
G_{1}=\frac{t^{I_{1}} \omega+t^{O_{1}} \omega}{2}-F\left(\tau_{2}-\tau_{1}\right)+R_{1}+(1-d) G_{0} \\
\\
\frac{t^{I_{1}} \omega+t^{O_{1}} \omega}{2}-F\left(\tau_{2}-\tau_{1}\right)+R_{1} \geq 0
\end{gathered}
$$

$t^{J_{1}} \leq \tau_{1}$ for each $J_{1} \in\left\{I_{1}, O_{1}\right\}$, and $\tau_{2} \geq \tau_{1}$ where $E\left(R_{2}\right)$ is the expected value of natural-resource income in the second period. While deciding on the allocation of tax revenues between transfers to his group and investment in the public good, the incumbent compares their marginal benefits. Since, the public good is durable, its marginal benefit is given as $\alpha_{1}+\left(\phi \alpha_{H}+(1-\phi) \alpha_{L}\right)(1-d)$, which we denote by $\beta_{1}$. So, if $\beta_{1}>2$, both groups are taxed at the highest rate, $t^{I_{1}}=t^{O_{1}}=\tau_{1}$, and all government revenue not invested in fiscal capacity is invested in the public good. If $\beta_{1}<2$, the opposition group is taxed at the highest rate, $t^{O_{1}}=\tau_{1}$, and all government revenue not invested in fiscal capacity is transferred to the incumbent group. ${ }^{1}$ We can collect these results in the following proposition.

Proposition 4.2. We have $t^{O_{1}}=\tau_{1}$. Moreover, we have $t^{I_{1}}=\tau_{1}$ and $g_{1}=$

[^16]$\tau_{1} \omega-F\left(\tau_{2}-\tau_{1}\right)+R_{1}$ if $\beta_{1}>2$, and $t^{I_{1}}=-\tau_{1}-\frac{2\left(-F\left(\tau_{2}-\tau_{1}\right)+R_{1}\right)}{\omega}$ and $g_{1}=0$ if $\beta_{1}<2$.

Given the equilibrium policies in Proposition 4.2, we can show that the firstorder condition for the optimal level of investment in fiscal capacity is

$$
\begin{equation*}
\omega\left[E\left(\lambda_{2}\right)-1\right] \leq \lambda_{1} F_{\tau}\left(\tau_{2}-\tau_{1}\right), \tag{4.3}
\end{equation*}
$$

where $E\left(\lambda_{2}\right)=\phi \alpha_{H}+(1-\phi)(1-\gamma) 2$ is the expected value of public funds in the second period and $\lambda_{1}=\max \left\{\alpha_{1}+\left(\phi \alpha_{H}+(1-\phi) \alpha_{L}\right)(1-d), 2\right\}$ is the value of public funds in the first period. The proof of the following proposition is given in the Appendix D.1.

Proposition 4.3. Assume that $E\left(\lambda_{2}\right)>1$. Then, investment in fiscal capacity is positive and increases with one of the following.
(i) An increase in income, $\omega$.
(ii) An increase in political stability, $1-\gamma$.
(iii) An increase in the depreciation rate of the public good, $d$, if $\beta_{1}>2$.
(iv) An increase in the risk of external conflict, $\phi$, if $\beta_{1}<2$ or $\left.\frac{\alpha_{H}-2(1-\gamma)}{\left(\alpha_{H}-\alpha_{L}\right)(1-d)}\right\rangle$ $\frac{E\left(\lambda_{2}\right)-1}{\beta_{1}}$.
(v) A decrease in the risk of external conflict, $\phi$, if $\beta_{1}>2$ and $\frac{\alpha_{H}-2(1-\gamma)}{\left(\alpha_{H}-\alpha_{L}\right)(1-d)}<$ $\frac{E\left(\lambda_{2}\right)-1}{\beta_{1}}$.
(vi) A decrease in the cost of investment in fiscal capacity (for given $\tau$ ).

If $E\left(\lambda_{1}\right)>1$, the left-hand side of Equation 4.3 is positive and, since $F_{\tau}(0)=0$, we have a positive investment in fiscal capacity as stated in Proposition 4.3. An increase in income increases tax revenue, so investment in fiscal capacity increases
as stated in Part (i). When political stability increases, the expected value of second-period public funds increases for the incumbent, engendering additional investment in fiscal capacity as stated in Part (ii). If $\beta_{1}>2$, the incumbent chooses between investment in the public good and fiscal capacity for the allocation of tax revenues. Since an increase in the depreciation rate of the public good decreases the benefit of investing in the public good, it increases investment in fiscal capacity as stated in Part (iii). Notice that the depreciation rate of the public good does not affect the existence of investment in fiscal capacity, as it does not appear in $E\left(\lambda_{2}\right)$.

We now look at the effects of a change in the risk of external conflict, $\phi$, on the investment in fiscal capacity as given in Parts (iv) and (v). If $\beta_{1}<2$, the incumbent allocates the tax revenue between transfers to his group and investment in fiscal capacity. Since an increase in the risk of external conflict increases the expected value of second-period public funds, it increases investment in fiscal capacity. If $\beta_{1}>2$, the incumbent allocates the tax revenue between investment in the public good and fiscal capacity. An increase in the risk of external conflict increases both the value of investment in the public good and the expected value of second-period public funds. Its effect on the former is $\left(\alpha_{H}-\alpha_{L}\right)(1-d)$ and on the latter, $\alpha_{H}-2(1-\gamma)$. Thus, if $\frac{\alpha_{H}-2(1-\gamma)}{\left(\alpha_{H}-\alpha_{L}\right)(1-d)}$ is large, an increase in the risk of external conflict increases investment in fiscal capacity as stated in Part (iv); otherwise, it decreases investment in fiscal capacity as stated in Part (v). Finally, when the marginal cost of investment in fiscal capacity decreases, it is clear that investment in fiscal capacity increases as stated in Part (vi).

We can understand the effects of risk of external conflict on the investment
in fiscal capacity better by examining the ratio $\frac{\alpha_{H}-2(1-\gamma)}{\left(\alpha_{H}-\alpha_{L}\right)(1-d)}$ more closely. If (a) $d$ is close to one or (b) $\alpha_{H}$ is close to $\alpha_{L}$, an increase in the risk of external conflict increases investment in fiscal capacity. The intuition behind condition (a) is clear. For condition (b) to hold, both $\alpha_{H}$ and $\alpha_{L}$ should be close to two. So, under this condition, being in a war does not change the value of defense activities much and alternative uses of public funds have similar values for the incumbent in the second period. Then why does an increase in the risk of external conflict increase fiscal-capacity investment? To answer this question, note that the benefit of investment in the public good is

$$
\begin{equation*}
\alpha_{1}+\left(\phi \alpha_{H}+(1-\phi) \alpha_{L}\right)(1-d) \tag{4.4}
\end{equation*}
$$

and it does not change much with $\phi$ if $\alpha_{H}$ is close to $\alpha_{L}$. The expected value of second-period public funds is

$$
\begin{equation*}
\gamma \phi \alpha_{H}+(1-\gamma)\left(\phi \alpha_{H}+(1-\phi) 2\right), \tag{4.5}
\end{equation*}
$$

and it also does not change much with $\phi$ if the incumbent stays in power in the second period and $\alpha_{H}$ is close to two. However, if the incumbent loses power, an increase in $\phi$ increases the expected value of second-period public funds. This effect is the source of increase in fiscal-capacity investment under condition (b). Thus, the increase in fiscal-capacity investment is greater if the incumbent is more likely to lose power in the second period.

Finally, if (c) $\alpha_{H}$ is close to $2(1-\gamma)$, an increase in the risk of external conflict decreases investment in fiscal capacity. For condition (c) to hold, $\gamma$ should be close
to zero and $\alpha_{H}$ should be close to two. So, under this condition, political stability is high and, even if there is a war, the incumbent is indifferent between transferring tax revenue to his group or spending it on defense activities. It is clear that an increase in $\phi$ does not affect the expected value of second-period public funds much if $\alpha_{H}$ is close to $2(1-\gamma)$. However, it increases the benefit of investment in the public good. This is the source of the decrease in fiscal-capacity investment under condition (c). Thus, the decrease in fiscal-capacity investment is greater if $\alpha_{L}$ is lower.

### 4.4 Conclusion

In this paper, we analyzed a two-period model in which two groups in society decide on government policy - taxes and provision of a public good - in each period and investment in the state's fiscal capacity in the first period. Following Besley and Persson (2010), we interpreted the public good as defense activities against external threats, but, relaxing their assumption, we let it accumulate between periods. First, we showed that an increase in the depreciation rate of the public good increases investment in fiscal capacity. Second, we showed that, under certain conditions, an increase in the risk of external conflict decreases it, contrary to the common result in the literature. Additionally, we presented some new insights into the effects of the risk of external conflict on fiscal-capacity investment. Our plan for future research is to extend our model to an infinite horizon.

## Appendix A

## Appendix for Chapter 1

## A. 1 Equilibrium Definition

For any $\mathbf{y}=\left(y_{1}, y_{2}\right) \in \mathbb{R}^{2}$ let $\mathbf{y}_{t}=y_{t}$ for any $t \in\{1,2\}$.

Definition A.1. A strategy profile $\left(b_{i}, a_{i}, c_{i}\right)_{i \in\{1,2\}}$ is an equilibrium if and only if (E1) For any $i \in\{1,2\}, \beta_{i} \in\{0,1\}, \mathbf{b} \in B_{\sigma}$ and $\sigma \in\{s, d\}$, we have

$$
\begin{gather*}
c_{i}\left(\mathbf{b}, \beta_{i}\right) \in \arg \max _{\mathbf{q}^{\prime}=\left(l_{1}^{\prime}, l_{2}^{\prime}\right)} u_{i}\left(1-\tau, \alpha_{1} l_{1}^{\prime}, \alpha_{2} l_{2}^{\prime} ; \beta_{i}\right)  \tag{A.1}\\
\text { s.t. } \mathbf{q}^{\prime} \in Q_{\sigma}(\mathbf{b})
\end{gather*}
$$

where $\tau$ is the first component of $\mathbf{b}$.
(E2) For any $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$ and $\mathbf{b} \in B_{s} \cup B_{d}$, we have $a_{i}(\mathbf{b})=1$ if and only if

$$
\begin{equation*}
E u_{i}\left(1-\tau, \alpha_{1} c_{i^{\prime}, 1}\left(\mathbf{b}, \beta_{i^{\prime}}\right), \alpha_{2} c_{i^{\prime}, 2}\left(\mathbf{b}, \beta_{i^{\prime}}\right) ; \beta_{i}\right) \geq 1 \tag{A.2}
\end{equation*}
$$

where $\tau$ is the first component of $\mathbf{b}$ and

$$
\begin{aligned}
& E u_{i}\left(1-\tau, \alpha_{1} c_{i^{\prime}, 1}\left(\mathbf{b}, \beta_{i^{\prime}}\right), \alpha_{2} c_{i^{\prime}, 2}\left(\mathbf{b}, \beta_{i^{\prime}}\right) ; \beta_{i}\right)= \\
& \lambda_{i}\left[\lambda_{i^{\prime}} u_{i}\left(1-\tau, \alpha_{1} c_{i^{\prime}, 1}(\mathbf{b}, 1), \alpha_{2} c_{i^{\prime}, 2}(\mathbf{b}, 1) ; 1\right)\right. \\
& \left.+\left(1-\lambda_{i^{\prime}}\right) u_{i}\left(1-\tau, \alpha_{1} c_{i^{\prime}, 1}(\mathbf{b}, 0), \alpha_{2} c_{i^{\prime}, 2}(\mathbf{b}, 0) ; 1\right)\right] \\
& +\left(1-\lambda_{i}\right)\left[\lambda_{i^{\prime}} u_{i}\left(1-\tau, \alpha_{1} c_{i^{\prime}, 1}(\mathbf{b}, 1), \alpha_{2} c_{i^{\prime}, 2}(\mathbf{b}, 1) ; 0\right)\right. \\
& \left.+\left(1-\lambda_{i^{\prime}}\right) u_{i}\left(1-\tau, \alpha_{1} c_{i^{\prime}, 1}(\mathbf{b}, 0), \alpha_{2} c_{i^{\prime}, 2}(\mathbf{b}, 0) ; 0\right)\right]
\end{aligned}
$$

(E3) For any $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$, we have

$$
\begin{gather*}
b_{i} \in \arg \max _{\mathbf{b}^{\prime}} E u_{i}\left(1-\tau^{\prime}, \alpha_{1} c_{i, 1}\left(\mathbf{b}^{\prime}, \beta_{i}\right), \alpha_{2} c_{i, 2}\left(\mathbf{b}^{\prime}, \beta_{i}\right) ; \beta_{i}\right) \\
\text { s.t. } \mathbf{b}^{\prime} \in B_{s} \cup B_{d}, \\
E u_{i^{\prime}}\left(1-\tau^{\prime}, \alpha_{1} c_{i, 1}\left(\mathbf{b}^{\prime}, \beta_{i}\right), \alpha_{2} c_{i, 2}\left(\mathbf{b}^{\prime}, \beta_{i}\right) ; \beta_{i^{\prime}}\right) \geq 1 \tag{A.3}
\end{gather*}
$$

where $\tau^{\prime}$ is the first component of $\mathbf{b}^{\prime}$.

Condition (E1) states that for any preference parameter $\beta_{i}$ and budget $\mathbf{b}$, a legislator's equilibrium labor-hiring policy maximizes his payoff. Condition (E2) says that legislator $i$ accepts a budget proposal if and only if his expected payoff from the proposal is higher than his payoff from the status-quo policy. Condition (E3) requires that legislator $i$ 's equilibrium budget proposal maximizes his expected payoff subject to party $i^{\prime}$ accepting the proposal.

## A. 2 Proof of Lemmas, Propositions and Corollaries

## A.2.1 Proof of Lemma 1.1

We have $p\left(\frac{1}{2}, \lambda_{2}\right)=p\left(\lambda_{1}, \frac{1}{2}\right)=\frac{1}{2}, p_{1}\left(\lambda_{1}, \lambda_{2}\right)=1-2 \lambda_{2}$ and $p_{2}\left(\lambda_{1}, \lambda_{2}\right)=1-2 \lambda_{1}$ for all $\lambda_{1}, \lambda_{2} \in(0,1)$.

Take $\lambda \in\left(0, \frac{1}{2}\right)$. Take $\lambda_{1}, \lambda_{2} \in(0, \lambda)$. Since $\lambda_{2}<\frac{1}{2}$, we have $p_{1}\left(\lambda_{1}, \lambda_{2}\right)>0$. So, we have $p\left(\lambda_{1}, \lambda_{2}\right)<p\left(\frac{1}{2}, \lambda_{2}\right)=\frac{1}{2}$ since $\lambda_{1}<\frac{1}{2}$.

Take $\lambda_{1}^{\prime} \in(0, \lambda)$ and $\lambda_{2}^{\prime} \in(1-\lambda, 1)$. Since $\lambda_{2}^{\prime}>\frac{1}{2}$, we have $p_{1}\left(\lambda_{1}^{\prime}, \lambda_{2}^{\prime}\right)<0$. So, we have $p\left(\lambda_{1}^{\prime}, \lambda_{2}^{\prime}\right)>p\left(\frac{1}{2}, \lambda_{2}^{\prime}\right)=\frac{1}{2}$ since $\lambda_{1}^{\prime}<\frac{1}{2}$.

Take $\lambda_{1}^{\prime \prime} \in(1-\lambda, 1)$ and $\lambda_{2}^{\prime} \in(0, \lambda)$. Since $\lambda_{1}^{\prime \prime}>\frac{1}{2}$, we have $p_{2}\left(\lambda_{1}^{\prime \prime}, \lambda_{2}^{\prime \prime}\right)<0$. So, we have $p\left(\lambda_{1}^{\prime \prime}, \lambda_{2}^{\prime \prime}\right)>p\left(\lambda_{1}^{\prime \prime}, \frac{1}{2}\right)=\frac{1}{2}$ since $\lambda_{2}^{\prime \prime}<\frac{1}{2}$.

Take $\lambda_{1}^{\prime \prime \prime}, \lambda_{2}^{\prime \prime \prime} \in(1-\lambda, 1)$. Since $\lambda_{1}^{\prime \prime \prime}>\frac{1}{2}$, we have $p_{2}\left(\lambda_{1}^{\prime \prime \prime}, \lambda_{2}^{\prime \prime \prime}\right)<0$. So, we have $p\left(\lambda_{1}^{\prime \prime \prime}, \lambda_{2}^{\prime \prime \prime}\right)<p\left(\lambda_{1}^{\prime \prime \prime}, \frac{1}{2}\right)=\frac{1}{2}$ since $\lambda_{2}^{\prime \prime \prime}>\frac{1}{2}$.

So, $p\left(\lambda_{1}, \lambda_{2}\right), p\left(\lambda_{1}^{\prime \prime \prime}, \lambda_{2}^{\prime \prime \prime}\right)<p\left(\lambda_{1}^{\prime}, \lambda_{2}^{\prime}\right)$ and $p\left(\lambda_{1}, \lambda_{2}\right), p\left(\lambda_{1}^{\prime \prime \prime}, \lambda_{2}^{\prime \prime \prime}\right)<p\left(\lambda_{1}^{\prime \prime}, \lambda_{2}^{\prime \prime}\right)$.

## A.2.2 Proof of Proposition 1.1

In Part (a) of the proof, we prove Parts (i)-(ii) of Proposition 1.1, and in Part (b) of the proof, we prove existence and uniqueness of the equilibrium.

Part (a). Let $\Gamma$ be an economic environment that satisfies Assumptions 1.11.3. Assume that $h \in H_{m}$ where $m \in\{0, \ldots, n-1\}$. Let $\left(b_{i}, a_{i}, c_{i}\right)_{i \in\{1,2\}}$ be an equilibrium of $\Gamma$. Let legislator $i \in\{1,2\}$ be the proposer and legislator $i^{\prime} \in\{1,2\}$ be the responder with $i \neq i^{\prime}$. Take $\mathbf{b}_{s}=\frac{k}{2 n} L \in B_{s}$. Then it is clear that
$c_{i}\left(\mathbf{b}_{s}, 0\right)=\left(0, \frac{k}{n} L\right)$ and $c_{i}\left(\mathbf{b}_{s}, 1\right)=\left(\frac{k}{n} L, 0\right)$. Moreover, $a_{i^{\prime}}\left(\mathbf{b}_{s}\right)=1$ if and only if

$$
\begin{align*}
& E u_{i^{\prime}}\left(1-\tau, \alpha c_{i, 1}\left(\mathbf{b}_{s}, \beta_{i}\right), \alpha c_{i, 2}\left(\mathbf{b}_{s}, \beta_{i}\right) ; \beta_{i^{\prime}}\right)= \\
& \lambda_{i^{\prime}}\left[\lambda_{i}\left(1-\frac{k}{2 n} L+h\left(\alpha \frac{k}{n} L\right)\right)+\left(1-\lambda_{i}\right)\left(1-\frac{k}{2 n} L\right)\right] \\
& +\left(1-\lambda_{i^{\prime}}\right)\left[\lambda_{i}\left(1-\frac{k}{2 n} L\right)+\left(1-\lambda_{i}\right)\left(1-\frac{k}{2 n} L+h\left(\alpha \frac{k}{n} L\right)\right)\right]  \tag{A.4}\\
& =1-\frac{k}{2 n} L+\left[\lambda_{i^{\prime}} \lambda_{i}+\left(1-\lambda_{i^{\prime}}\right)\left(1-\lambda_{i}\right)\right] h\left(\alpha \frac{k}{n} L\right) \geq 1 .
\end{align*}
$$

Take $\mathbf{b}_{d}=\left(\frac{l_{1}+l_{2}}{2 n}, \frac{l_{1}}{n}, \frac{l_{2}}{n}\right) \in B_{d}$. Then $a_{i^{\prime}}\left(\mathbf{b}_{d}\right)=1$ if and only if

$$
\begin{align*}
& E u_{i^{\prime}}\left(1-\tau, \alpha c_{i, 1}\left(\mathbf{b}_{d}, \beta_{i}\right), \alpha c_{i, 2}\left(\mathbf{b}_{d}, \beta_{i}\right) ; \beta_{i^{\prime}},\right)= \\
& 1-\frac{l_{1}+l_{2}}{2 n}+\lambda_{i^{\prime}} h\left(\alpha \frac{l_{1}}{n}\right)+\left(1-\lambda_{i^{\prime}}\right) h\left(\alpha \frac{l_{2}}{n}\right) \geq 1 . \tag{A.5}
\end{align*}
$$

Moreover, we have

$$
\begin{align*}
E u_{i}\left(1-\tau, \alpha c_{i, 1}\left(\mathbf{b}_{s}, \beta_{i}\right), \alpha c_{i, 2}\left(\mathbf{b}_{s}, \beta_{i}\right) ; \beta_{i},\right)= & \lambda_{i}\left(1-\frac{k}{2 n} L+h\left(\alpha \frac{k}{n} L\right)\right) \\
& +\left(1-\lambda_{i}\right)\left(1-\frac{k}{2 n} L+h\left(\alpha \frac{k}{n} L\right)\right) \\
= & 1-\frac{k}{2 n} L+h\left(\alpha \frac{k}{n} L\right) \tag{A.6}
\end{align*}
$$

and
$E u_{i}\left(1-\tau, \alpha c_{i, 1}\left(\mathbf{b}_{d}, \beta_{i}\right), \alpha c_{i, 2}\left(\mathbf{b}_{d}, \beta_{i}\right) ; \beta_{i},\right)=1-\frac{l_{1}+l_{2}}{2 n}+\lambda_{i} h\left(\alpha \frac{l_{1}}{n}\right)+\left(1-\lambda_{i}\right) h\left(\alpha \frac{l_{2}}{n}\right)$.

Fact 1: Legislator $i$ can propose $\mathbf{b}_{d}^{\prime}=\left(\frac{2}{2 n} L, \frac{1}{n} L, \frac{1}{n} L\right) \in B_{d}$, which gives a payoff equal to $1-\frac{2}{2 n} L+h\left(\alpha \frac{1}{n} L\right)$ to both legislators. Since, by Assumption 1.3,
$1-\frac{2}{2 n} L+h\left(\alpha \frac{1}{n} L\right)>1$, this proposal is accepted by legislator $i^{\prime}$. Therefore, the equilibrium payoff of legislator $i$ is greater than 1 .

Fact 2: Consider the budget $\mathbf{b}_{s}^{\prime}=\frac{l_{1}+l_{2}}{2 n} \in B_{s}$. If $l_{j}>0$ for some $j$, by strict concavity of $h$, it is clear from Equations A. 6 and A. 7 that
$E u_{i}\left(1-\tau, \alpha c_{i, 1}\left(\mathbf{b}_{s}^{\prime}, \beta_{i}\right), \alpha c_{i, 2}\left(\mathbf{b}_{s}^{\prime}, \beta_{i}\right) ; \beta_{i}\right)>E u_{i}\left(1-\tau, \alpha c_{i, 1}\left(\mathbf{b}_{d}, \beta_{i}\right), \alpha c_{i, 2}\left(\mathbf{b}_{d}, \beta_{i}\right) ; \beta_{i}\right)$
for any $\lambda_{i} \in(0,1)$.
Let $B_{s}^{\prime} \subseteq B_{s}$ denote the set of feasible one-item budget proposals that give legislator $i$ an expected payoff greater than 1 . By Facts 1 and $2, B_{s}^{\prime}$ is nonempty. For any $b \in B_{s}^{\prime}$, there exists $\lambda_{b, \Gamma} \in(0,1)$ such that, from Equations A. 4 and A.6, $E u_{i^{\prime}}\left(1-\tau, \alpha c_{i, 1}\left(b, \beta_{i}\right), \alpha c_{i, 2}\left(b, \beta_{i}\right) ; \beta_{i^{\prime}},\right) \geq 1$ if $\lambda_{i^{\prime}}, \lambda_{i} \in\left[0, \lambda_{b, \Gamma}\right]$ or $\lambda_{i^{\prime}}, \lambda_{i} \in$ $\left[1-\lambda_{b, \Gamma}, 1\right]$. Let $\lambda_{\Gamma}^{\prime}=\min _{b \in B_{s}^{\prime}} \lambda_{b, \Gamma}$. By Fact 1, legislator $i$ never proposes $(0,0,0) \in$ $B_{d}$ at an equilibrium. Then, by Fact 2, legislator $i$ proposes a one-item budget if $\lambda_{i^{\prime}}, \lambda_{i} \in\left(0, \lambda_{\Gamma}^{\prime}\right]$ or $\lambda_{i^{\prime}}, \lambda_{i} \in\left[1-\lambda_{\Gamma}^{\prime}, 1\right)$.

For any $b \in B_{s}^{\prime}$, there exists $\lambda_{b, \Gamma}^{\prime} \in(0,1)$ such that, from Equation A.4,

$$
E u_{i^{\prime}}\left(1-\tau, \alpha c_{i, 1}\left(b, \beta_{i}\right), \alpha c_{i, 2}\left(b, \beta_{i}\right) ; \beta_{i^{\prime}},\right)<1 \text { if } \lambda_{i^{\prime}} \in\left[0, \lambda_{b, \Gamma}^{\prime}\right) \text { and } \lambda_{i} \in\left(1-\lambda_{b, \Gamma}^{\prime}, 1\right] \text {, }
$$ or vice versa. Let $\lambda_{\Gamma}^{\prime \prime}=\min _{b \in B_{s}^{\prime}} \lambda_{b, \Gamma}^{\prime}$. If $\lambda_{i^{\prime}} \in\left(0, \lambda_{\Gamma}^{\prime \prime}\right)$ and $\lambda_{i} \in\left(1-\lambda_{\Gamma}^{\prime \prime}, 1\right)$, or vice versa, then, by Fact 1, legislator $i$ proposes a two-item budget, $\left(\tau, l_{1}, l_{2}\right) \in B_{d}$. Thus, if $\lambda_{i^{\prime}}, \lambda_{i} \in\left(0, \min \left\{\lambda_{\Gamma}^{\prime}, \lambda_{\Gamma}^{\prime \prime}\right\}\right)$ or $\lambda_{i^{\prime}}, \lambda_{i} \in\left(\max \left\{1-\lambda_{\Gamma}^{\prime}, 1-, \lambda_{\Gamma}^{\prime \prime}\right\}, 1\right)$, legislator $i$ proposes a one-item budget. However, if $\lambda_{i} \in\left(0, \min \left\{\lambda_{\Gamma}^{\prime}, \lambda_{\Gamma}^{\prime \prime}\right\}\right)$ and $\lambda_{i^{\prime}} \in\left(\max \left\{1-\lambda_{\Gamma}^{\prime}, 1-, \lambda_{\Gamma}^{\prime \prime}\right\}, 1\right)$ or vice versa, then legislator $i$ proposes a two-item budget.

There exists $\lambda_{\Gamma}^{\prime \prime \prime} \in(0,1)$ such that $1-\frac{1}{2 n} L+\lambda_{i^{\prime}} h\left(\alpha \frac{1}{n} L\right)<1$ for all $\lambda_{i^{\prime}} \in\left[0, \lambda_{\Gamma}^{\prime \prime \prime}\right)$.

By Assumption 1.3, there exists $\lambda_{\Gamma}^{\prime \prime \prime \prime} \in(0,1)$ such that $1-\frac{m+2}{2 n} L+(1-$ d) $h\left(\alpha \frac{1}{n} L\right) \geq 1$ for all $\lambda \in\left[0, \lambda_{\Gamma}^{\prime \prime \prime \prime}\right]$.

There exists $\lambda_{\Gamma}^{\prime \prime \prime \prime \prime} \in(0,1)$ such that $(1-\lambda) h\left(\alpha \frac{1}{n} L\right)<\frac{1}{2 n} L$ for all $\lambda \in\left(\lambda_{\Gamma}^{\prime \prime \prime \prime}, 1\right)$.
There exists $\lambda_{\Gamma}^{\prime \prime \prime \prime \prime \prime} \in(0,1)$ such that

$$
\begin{equation*}
\lambda\left[h\left(\alpha \frac{m+1}{n} L\right)-h\left(\alpha \frac{m}{n} L\right)\right]>\frac{1}{2 n} L \tag{A.9}
\end{equation*}
$$

and

$$
\begin{equation*}
\lambda\left[h\left(\alpha \frac{m+2}{n} L\right)-h\left(\alpha \frac{m+1}{n} L\right)\right]<\frac{1}{2 n} L \tag{A.10}
\end{equation*}
$$

if $m \in\{0, \ldots, n-2\}$, and

$$
\begin{equation*}
\lambda\left[h(\alpha L)-h\left(\alpha \frac{n-1}{n} L\right)\right]>\frac{1}{2 n} L \tag{A.11}
\end{equation*}
$$

if $m=n-1$ for all $\lambda \in\left(\lambda_{\Gamma}^{\prime \prime \prime \prime \prime \prime}, 1\right)$.
Let $\lambda_{\Gamma}=\min \left\{\lambda_{\Gamma}^{\prime}, \lambda_{\Gamma}^{\prime \prime}, \lambda_{\Gamma}^{\prime \prime \prime}, \lambda_{\Gamma}^{\prime \prime \prime \prime}, 1-\lambda_{\Gamma}^{\prime \prime \prime \prime \prime}, 1-\lambda_{\Gamma}^{\prime \prime \prime \prime \prime \prime}\right\}$.
By construction, if $\lambda_{i}, \lambda_{i^{\prime}} \in\left(0, \lambda_{\Gamma}\right)$ or $\lambda_{i}, \lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$ for each $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$, then $b^{i}, b^{i^{\prime}} \in B_{s}$. Moreover, if $\lambda_{i} \in\left(0, \lambda_{\Gamma}\right)$ and $\lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$ for each $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$, then $b^{i}, b^{i^{\prime}} \in B_{d}$.

It is clear that $\lambda_{\Gamma}>0$. We will now show that $\lambda_{\Gamma} \leq \frac{1}{2}$. We will show this by showing that $\lambda_{b}^{\Gamma}=\min \left\{\lambda_{b, \Gamma}, \lambda_{b, \Gamma}^{\prime}\right\} \leq \frac{1}{2}$ for any $b \in B_{s}^{\prime}$. Suppose not; that is, suppose there exists $b \in B_{s}^{\prime}$ such that $\lambda_{b}^{\Gamma}>\frac{1}{2}$. Take $\lambda_{1}, \lambda_{2} \in\left(1-\lambda_{b}^{\Gamma}, \lambda_{b}^{\Gamma}\right)$. Since $\lambda_{1}, \lambda_{2} \in\left(0, \lambda_{b}^{\Gamma}\right)$, we have $E u_{1}\left(1-\tau, \alpha c_{2,1}\left(b, \beta_{2}\right), \alpha c_{2,2}\left(b, \beta_{2}\right) ; \beta_{1},\right) \geq 1$. On the other hand, since $\lambda_{1} \in\left(0, \lambda_{b}^{\Gamma}\right)$ and $\lambda_{2} \in\left(1-\lambda_{b}^{\Gamma}, 1\right)$, we have $E u_{1}(1-$ $\left.\tau, \alpha c_{2,1}\left(b, \beta_{2}\right), \alpha c_{2,2}\left(b, \beta_{2}\right) ; \beta_{1},\right)<1$. So, we have a contradiction.

Now we will now show that also $\lambda_{\Gamma} \neq \frac{1}{2}$. Assume that $\lambda_{\Gamma}=\frac{1}{2}$ and there exists $b, b^{\prime} \in B_{s}^{\prime}$ such that $b \neq b^{\prime}$. Assume that $b^{\prime}<b$. Since $\lambda_{\Gamma}=\frac{1}{2}$, we have $\lambda_{b, \Gamma}^{\prime}=$ $\lambda_{b^{\prime}, \Gamma}^{\prime}=\frac{1}{2}$. If $1-b+\left[\lambda_{i^{\prime}} \lambda_{i}+\left(1-\lambda_{i^{\prime}}\right)\left(1-\lambda_{i}\right)\right] h(\alpha 2 b)<1$ for each $\lambda_{i^{\prime}} \in\left(0, \lambda_{b, \Gamma}^{\prime}\right)$ and $\lambda_{i} \in\left(1-\lambda_{b, \Gamma}^{\prime}, 1\right)$ or vice versa, and $1-b^{\prime}+\left[\lambda_{i^{\prime}} \lambda_{i}+\left(1-\lambda_{i^{\prime}}\right)\left(1-\lambda_{i}\right)\right] h\left(\alpha 2 b^{\prime}\right)<1$ for each $\lambda_{i^{\prime}} \in\left(0, \lambda_{b^{\prime}, \Gamma}^{\prime}\right)$ and $\lambda_{i} \in\left(1-\lambda_{b^{\prime}, \Gamma}^{\prime}, 1\right)$ or vice versa, then we need $\lambda_{b^{\prime}, \Gamma}<\lambda_{b, \Gamma}$, because of strict concavity of $h$. So, we have a contradiction. Thus, we have $\lambda_{\Gamma}=\frac{1}{2}$ only if there exists a unique $b \in B_{s}^{\prime}$. However, we have $\frac{1}{2 n} L, \frac{1}{n} L \in B_{s}^{\prime}$. So, $\lambda_{\Gamma}<\frac{1}{2}$.

Part(b). We now need to show existence and uniqueness of the equilibrium. We achieve this by characterizing the equilibrium.

Part (b.i). Assume that $\lambda_{i^{\prime}}, \lambda_{i} \in\left(0, \lambda_{\Gamma}\right)$ or $\lambda_{i^{\prime}}, \lambda_{i} \in\left(1-\lambda_{\Gamma}, 1\right)$. Then we know that legislator $i$ proposes a one-item budget.

It is clear that $c_{i}\left(\mathbf{b}_{s}, 0\right)=\left(0, \frac{k}{n} L\right)$ and $c_{i}\left(\mathbf{b}_{s}, 1\right)=\left(\frac{k}{n} L, 0\right)$ for any $\mathbf{b}_{s}=\frac{k}{n} L \in B_{s}$.
If $m \in\{0, \ldots, n-2\}$, we have

$$
\begin{equation*}
1-\frac{m+1}{2 n} L+h\left(\alpha \frac{m+1}{n} L\right)>1-\frac{m}{2 n} L+h\left(\alpha \frac{m}{n} L\right), \tag{A.12}
\end{equation*}
$$

and

$$
\begin{equation*}
1-\frac{m+2}{2 n} L+h\left(\alpha \frac{m+2}{n} L\right)<1-\frac{m+1}{2 n} L+h\left(\alpha \frac{m+1}{n} L\right) . \tag{A.13}
\end{equation*}
$$

Thus, from the quasilinear form of the utility functions and strict concavity of $h$, we have $b_{i}=\frac{m+1}{2 n} L \in B_{s}$.

If $m=n-1$, then we have

$$
\begin{equation*}
1-\frac{1}{2} L+h(\alpha L)>1-\frac{n-1}{2 n} L+h\left(\alpha \frac{n-1}{n} L\right) . \tag{A.14}
\end{equation*}
$$

Thus, by quasilinear form of the utility functions and strict concavity of $h$, we have $b_{i}=\frac{1}{2} L \in B_{s}$.

Part (b.ii). Assume that $\lambda_{i^{\prime}} \in\left(0, \lambda_{\Gamma}\right)$ and $\lambda_{i} \in\left(1-\lambda_{\Gamma}, 1\right)$. Then we know that legislator $i$ proposes a two-item budget.

Since, $\lambda_{i^{\prime}} \in\left(0,1-\lambda_{\Gamma}^{\prime \prime \prime \prime \prime}\right)$, legislator $i$ 's budget proposal must allocate at least $\frac{1}{n} L$ amount of labor for provision of $g_{2}$.

Since $\lambda_{i} \in\left(\lambda_{\Gamma}^{\prime \prime \prime \prime \prime}, 1\right)$, we have $\left(1-\lambda_{i}\right) h\left(\alpha \frac{1}{n} L\right)<\frac{1}{2 n} L$. Therefore, from the quasilinear form of the payoff functions and strict concavity of $h$, legislator $i$ does not propose a budget which allocates more than $\frac{1}{n} L$ amount of labor for provision of $g_{2}$.

If $m \in\{0, \ldots, n-2\}$, as $\lambda_{i} \in\left(\lambda_{\Gamma}^{\prime \prime \prime \prime \prime \prime}, 1\right)$, we have

$$
\begin{align*}
& 1-\frac{m+2}{2 n} L+\lambda_{i} h\left(\alpha \frac{m+1}{n} L\right)+\left(1-\lambda_{i}\right) h\left(\alpha \frac{1}{n} L\right)>  \tag{A.15}\\
& 1-\frac{m+1}{2 n} L+\lambda_{i} h\left(\alpha \frac{m}{n} L\right)+\left(1-\lambda_{i}\right) h\left(\alpha \frac{1}{n} L\right)
\end{align*}
$$

and

$$
\begin{align*}
& 1-\frac{m+3}{2 n} L+\lambda_{i} h\left(\alpha \frac{m+2}{n} L\right)+\left(1-\lambda_{i}\right) h\left(\alpha \frac{1}{n} L\right)<  \tag{A.16}\\
& 1-\frac{m+2}{2 n} L+\lambda_{i} h\left(\alpha \frac{m+1}{n} L\right)+\left(1-\lambda_{i}\right) h\left(\alpha \frac{1}{n} L\right) .
\end{align*}
$$

Thus, due to the quasilinear form of the utility functions and strict concavity of $h$, it is optimal for legislator $i$ to offer $\left(\frac{m+2}{2 n} L, \frac{m+1}{n} L, \frac{1}{n} L\right) \in B_{d}$.

If $m=n-1$, as $\lambda_{i} \in\left(\lambda_{\Gamma}^{\prime \prime \prime \prime \prime \prime}, 1\right)$, we have

$$
\begin{align*}
& 1-\frac{1}{2} L+\lambda_{i} h\left(\alpha \frac{n-1}{n} L\right)+\left(1-\lambda_{i}\right) h\left(\alpha \frac{1}{n} L\right)>  \tag{A.17}\\
& 1-\frac{n-1}{2 n} L+\lambda_{i} h\left(\alpha \frac{n-2}{n} L\right)+\left(1-\lambda_{i}\right) h\left(\alpha \frac{1}{n} L\right) .
\end{align*}
$$

Thus, due to the quasilinear form of the utility functions and strict concavity of $h$, it is optimal for legislator $i$ to offer $\left(\frac{1}{2} L, \frac{n-1}{n} L, \frac{1}{n} L\right) \in B_{d}$.

Since $\lambda_{i^{\prime}} \in\left(0, \lambda_{\Gamma}^{\prime \prime \prime \prime}\right)$, legislator $i^{\prime}$ accepts these offers. Thus, we have $b_{i}=$ $\left(\frac{m+2}{2 n} L, \frac{m+1}{n} L, \frac{1}{n} L\right)$ if $m \in\{0, \ldots, n-2\}$, and $b_{i}=\left(\frac{1}{2} L, \frac{n-1}{n} L, \frac{1}{n} L\right)$ if $m=n-1$.

Assume that $\lambda_{i} \in\left(0, \lambda_{\Gamma}\right)$ and $\lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$. Then, by following similar steps, it is easy to show that $b_{i}=\left(\frac{m+2}{2 n} L, \frac{1}{n} L, \frac{m+1}{n} L\right) \in B_{d}$ if $m \in\{0, \ldots, n-2\}$, and $b_{i}=\left(\frac{1}{2} L, \frac{1}{n} L, \frac{n-1}{n} L\right) \in B_{d}$ if $m=n-1$.

## A.2.3 Proof of Proposition 1.2

Let $\Gamma$ be an economic environment that satisfies Assumptions 1.1-1.3. Assume that $h \in H_{m}$ where $m \in\{0, \ldots, n-1\}$. Assume further that $\lambda_{i}, \lambda_{i^{\prime}} \in\left(0, \lambda_{\Gamma}\right)$ or $\lambda_{i}, \lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$ for each $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$. Let $\left(b_{i}, a_{i}, c_{i}\right)_{i \in\{1,2\}}$ be the unique equilibrium of $\Gamma$. Then, as we showed in Part (b.i) of the proof of Proposition 1.1, we have $b_{i}=\frac{m+1}{2 n} L \in B_{s}$ for each $i \in\{1,2\}$. We also have $c_{i}\left(\mathbf{b}_{s}, 0\right)=\left(0, \frac{k}{n} L\right)$ and $c_{i}\left(\mathbf{b}_{s}, 1\right)=\left(\frac{k}{n} L, 0\right)$ for any $\mathbf{b}_{s}=\frac{k}{n} L \in B_{s}$ and $i \in\{1,2\}$. Thus, we have $c_{i}\left(b^{i}, 0\right)=\left(\frac{m+1}{2 n} L, 0\right)$ and $c_{i}\left(b^{i}, 1\right)=\left(0, \frac{m+1}{2 n} L\right)$ for each $i \in\{1,2\}$.

## A.2.4 Proof of Proposition 1.3

Let $\Gamma$ be an economic environment that satisfies Assumptions 1.1-1.3. Assume that $h \in H_{m}$, where $m \in\{0, \ldots, n-1\}$. Assume further that $\lambda_{i} \in\left(0, \lambda_{\Gamma}\right)$ and $\lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$ for each $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$. Let $\left(b_{i}, a_{i}, c_{i}\right)_{i \in\{1,2\}}$ be the unique equilibrium of $\Gamma$. Then, as we showed in Part (b.ii) of the proof of Proposition 1, we have $b_{i}=\left(\frac{m+2}{2 n} L, \frac{1}{n} L, \frac{m+1}{n} L\right)$ and $b_{i^{\prime}}=\left(\frac{m+2}{2 n} L, \frac{m+1}{n} L, \frac{1}{n} L\right)$ if $m \in\{0, \ldots, n-2\}$, and $b_{i}=\left(\frac{1}{2} L, \frac{1}{n} L, \frac{n-1}{n} L\right)$ and $b_{i^{\prime}}=\left(\frac{1}{2} L, \frac{n-1}{n} L, \frac{1}{n} L\right)$ if $m=n-1$.

## A.2.5 Proof of Corollary 1.1

The corollary is a direct result of Propositions 1.2 and 1.3.

## A.2.6 Proof of Proposition 1.4

Let legislator $i$ be the proposer and legislator $i^{\prime}$ be responder where $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$. Without loss of generality assume that $\lambda_{i} \in\left(0, \lambda_{\Gamma}\right)$ and $\lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$.

In the proof of Proposition 1.3, we showed that when the probability of polarization is high, the responder rejects any one-item budget proposal with a positive income tax rate. So, if legislator $i$ chooses to offer a one-item budget, he has to offer the budget $b_{s}=0$. Assume that legislator $i$ proposes a two-item budget, $\left(\tau, l_{1}, l_{2}\right) \in B_{d}$, such that $l_{2}>0$. Then, legislator $i^{\prime}$ rejects $l_{1}$ as $\lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}^{\prime \prime \prime}, 1\right)$. So, if legislator $i$ prefers to offer a two-item budget, $\left(\tau, l_{1}, l_{2}\right)$, he has to choose $l_{2}$ equal to 0 . Given this fact, since $\lambda_{i} \in\left(0, \lambda_{\Gamma}^{\prime \prime \prime}\right)$, legislator $i$ also chooses $l_{1}$ equal to 0 . So, if legislator $i$ prefers to offer a two-item budget, $\left(\tau, l_{1}, l_{2}\right) \in B_{d}$, then he chooses $\tau=0$. Since he is indifferent between this budget and the one-item budget, $b_{s}=0$.

He proposes the latter by our assumption.

## A.2.7 Proof of Proposition 1.5

Let legislator $i$ be the proposer and legislator $i^{\prime}$ be the responder where $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$. Existence of an equilibrium is easy to prove by backward induction. So, we skip that part.

Assume that $h \in H_{m}$, where $m \in\{0, \ldots, n-2\}$. Assume that $\lambda_{i} \in\left(0, \lambda_{\Gamma}\right)$ and $\lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$. We will show that under a flexible two-item budget the unique optimal proposal for legislator $i$ is the two-item budget $\left(\tau, l_{1}, l_{2}, k\right)=$ $\left(\frac{m+2}{2 n} L, \frac{m+2}{n} L, 0, \frac{m+1}{n} L\right)$. Assume that the optimal income tax rate is $\frac{m+2}{2 n} L$. Since $\lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}^{\prime \prime \prime}, 1\right)$, legislator $i$ needs to offer a two-item budget in which he promises at least $\frac{1}{n} L$ amount of labor for provision of public good 1 . Therefore, the question is how he should allocate the remaining $\frac{m+1}{n} L$ of labor. If he allocates $l_{2} \leq \frac{m+1}{n} L$ for provision of public good 2 , then he can choose $k$ at most equal to $\frac{m+1}{n} L-l_{2}$. However, it is in the interest of legislator $i$ to have $k$ as large as possible. Therefore, he chooses $l_{2}=0, l_{1}=\frac{m+2}{n} L$ and $k=\frac{m+1}{n} L$. Given the budget proposal $\left(\tau, l_{1}, l_{2}, k\right)=\left(\frac{m+2}{2 n} L, \frac{m+2}{n} L, 0, \frac{m+1}{n} L\right)$, the payoff of legislator $i^{\prime}$ is

$$
\begin{align*}
1-\frac{m+2}{2 n} L & +\lambda_{i^{\prime}}\left[\lambda_{i} h\left(\alpha \frac{m+2}{n} L\right)+\left(1-\lambda_{i}\right) h\left(\alpha \frac{1}{n} L\right)\right]  \tag{A.18}\\
& +\left(1-\lambda_{i^{\prime}}\right)\left[\lambda_{i} h(0)+\left(1-\lambda_{i}\right) h\left(\alpha \frac{m+1}{n} L\right)\right]
\end{align*}
$$

and the payoff of legislator $i$ is

$$
\begin{equation*}
1-\frac{m+2}{2 n} L+\lambda_{i} h\left(\alpha \frac{m+2}{n} L\right)+\left(1-\lambda_{i}\right) h\left(\alpha \frac{m+1}{n} L\right) . \tag{A.19}
\end{equation*}
$$

Considering Equation A. 19 , since $h \in H_{m}$ and $\lambda_{i} \in\left(0,1-\lambda_{\Gamma}^{\prime \prime \prime \prime \prime \prime}\right)$, the optimal income tax rate is $\frac{m+2}{2 n} L$. Moreover, legislator $i^{\prime}$ accepts the proposal since $\lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}^{\prime \prime \prime \prime}, 1\right)$ If $\lambda_{i^{\prime}} \in\left(0, \lambda_{\Gamma}\right)$ and $\lambda_{i} \in\left(1-\lambda_{\Gamma}, 1\right)$, then following similar steps it can be shown that the unique optimal proposal for legislator $i$ is the two-item budget $\left(\tau, l_{1}, l_{2}, k\right)=\left(\frac{m+2}{2 n} L, 0, \frac{m+2}{2 n} L, \frac{m+1}{2 n} L\right)$.

Assume that $h \in H_{n-1}$ and $\lambda_{i} \in\left(0, \lambda_{\Gamma}\right)$ and $\lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$. Then following similar steps it can be shown that the unique optimal proposal for legislator $i$ is the two-item budget $\left(\tau, l_{1}, l_{2}, k\right)=\left(\frac{1}{2} L, L, 0, \frac{n-1}{n} L\right)$. If $\lambda_{i^{\prime}} \in\left(0, \lambda_{\Gamma}\right)$ and $\lambda_{i} \in\left(1-\lambda_{\Gamma}, 1\right)$, then following similar steps it can be shown that the unique optimal proposal for legislator $i$ is the two-item budget $\left(\tau, l_{1}, l_{2}, k\right)=\left(\frac{1}{2} L, 0, L, \frac{n-1}{n} L\right)$. This proves uniqueness of the equilibrium.

## A.2.8 Proof of Proposition 1.6

Let legislator $i$ be the proposer and legislator $i^{\prime}$ be the responder where $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$. Assume that $h \in H_{m}$ where $m \in\{0, \ldots, n-2\}$. Assume that $\lambda_{i} \in\left(0, \lambda_{\Gamma}\right)$ and $\lambda_{i^{\prime}} \in\left(1-\lambda_{\Gamma}, 1\right)$. By Proposition 1.3, under a strict two-item budget, the payoff of legislator $i^{\prime}$ is

$$
\begin{equation*}
1-\frac{m+2}{2 n} L+\lambda_{i^{\prime}} h\left(\alpha \frac{1}{n} L\right)+\left(1-\lambda_{i^{\prime}}\right) h\left(\alpha \frac{m+1}{n} L\right), \tag{A.20}
\end{equation*}
$$

and the payoff of legislator $i$ is

$$
\begin{equation*}
1-\frac{m+2}{2 n} L+\lambda_{i} h\left(\alpha \frac{1}{n} L\right)+\left(1-\lambda_{i}\right) h\left(\alpha \frac{m+1}{n} L\right) . \tag{A.21}
\end{equation*}
$$

If we compare Equations A. 21 and A.19, it is clear that a flexible two-item
budget increases the payoff of legislator $i$. If we subtract Equation A. 20 from A.18, then we have

$$
\begin{equation*}
\lambda_{i^{\prime}} \lambda_{i}\left[h\left(\alpha \frac{m+2}{n} L\right)-h\left(\alpha \frac{1}{n} L\right)\right]+\left(1-\lambda_{i^{\prime}}\right) \lambda_{i}\left[-h\left(\alpha \frac{m+1}{n} L\right)\right] . \tag{A.22}
\end{equation*}
$$

Equation A. 22 is greater than 0 if and only if

$$
\begin{equation*}
\lambda_{i^{\prime}}>\frac{h\left(\alpha \frac{m+1}{n} L\right)}{h\left(\alpha \frac{m+2}{n} L\right)-h\left(\alpha \frac{1}{n} L\right)+h\left(\alpha \frac{m+1}{n} L\right)} . \tag{A.23}
\end{equation*}
$$

Assume that $\lambda_{i^{\prime}} \in\left(0, \lambda_{\Gamma}\right)$ and $\lambda_{i} \in\left(1-\lambda_{\Gamma}, 1\right)$. Then we can show that flexible two-item budget increases the payoff of legislator $i^{\prime}$ if and only if

$$
\begin{equation*}
\frac{h\left(\alpha \frac{m+2}{n} L\right)-h\left(\alpha \frac{1}{n} L\right)}{h\left(\alpha \frac{m+2}{n} L\right)-h\left(\alpha \frac{1}{n} L\right)+h\left(\alpha \frac{m+1}{n} L\right)}>\lambda_{i^{\prime}} . \tag{A.24}
\end{equation*}
$$

So, we choose $\tilde{\lambda}_{\Gamma}=\min \left\{\lambda_{\Gamma}, \frac{h\left(\alpha \frac{m+2}{n} L\right)-h\left(\alpha \frac{1}{n} L\right)}{h\left(\alpha \frac{m+2}{n} L\right)-h\left(\alpha \frac{1}{n} L\right)+h\left(\alpha \frac{m+1}{n} L\right)}\right\}$.
Assume that $h \in H_{n-1}$. Then, following similar steps, we can show that flexible two-item budget increases the payoff of legislator $i$ compared to strict two-item budget. Moreover, we can choose $\tilde{\lambda}_{\Gamma}=\min \left\{\lambda_{\Gamma}, \frac{h(\alpha L)-h\left(\alpha \frac{1}{n}\right)}{h(\alpha L)-h\left(\alpha \frac{1}{n}\right)+h\left(\alpha \frac{n-1}{n} L\right)}\right\}$.

## A. 3 Lack of Ethnic Tension and the Number of

## Line-Item Appropriations



Figure A.1: Lack of ethnic tension and the number of line-item appropriations for countries included in the OECD budget surveys.

## Appendix B

## Appendix for Chapter 2

## B. 1 Equilibrium Definition

Definition B.1. A strategy profile $\left(v^{i}, \rho^{i},\left(b_{\sigma}^{i}, a_{\sigma}^{i}, c_{\sigma}^{i}\right)_{\sigma \in\{s, d\}}\right)_{i \in\{1,2\}}$ is an equilibrium if and only if
(E1) For any $i \in\{1,2\}, \sigma \in\{s, d\}$ and $\mathbf{b}_{\sigma} \in B_{\sigma}$, we have

$$
\begin{gather*}
c_{\sigma}^{i}\left(\mathbf{b}_{\sigma}\right) \in \arg \max _{\mathbf{q}^{\prime}=\left(l_{1}^{\prime}, l_{2}^{\prime}\right)} u^{i}\left(1-\tau, \alpha_{1} l_{1}^{\prime}, \alpha_{2} l_{2}^{\prime}\right)  \tag{B.1}\\
\text { s.t. } \mathbf{q}^{\prime} \in Q_{\sigma}\left(\mathbf{b}_{\sigma}\right)
\end{gather*}
$$

where $\tau$ is the first component of $\mathbf{b}_{\sigma}$.
(E2) For any $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}, \sigma \in\{s, d\}$ and $\mathbf{b}_{\sigma} \in B_{\sigma}$, we have $a_{\sigma}^{i}\left(\mathbf{b}_{\sigma}\right)=1$ if and only if

$$
\begin{equation*}
u^{i}\left(1-\tau, \alpha_{1} c_{\sigma 1}^{i^{\prime}}\left(\mathbf{b}_{\sigma}\right), \alpha_{2} c_{\sigma 2}^{i^{\prime}}\left(\mathbf{b}_{\sigma}\right)\right) \geq 1 \tag{B.2}
\end{equation*}
$$

where $\tau$ is the first component of $\mathbf{b}_{\sigma}$.
(E3) For any $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$ and $\sigma \in\{s, d\}$ we have

$$
\begin{align*}
& b_{\sigma}^{i} \in \arg \max _{\mathbf{b}_{\sigma}^{\prime}} u^{i}\left(1-\tau^{\prime}, \alpha_{1} c_{\sigma 1}^{i}\left(\mathbf{b}_{\sigma}^{\prime}\right), \alpha_{2} c_{\sigma 2}^{i}\left(\mathbf{b}_{\sigma}^{\prime}\right)\right) \\
& \text { s.t. } \mathbf{b}_{\sigma}^{\prime} \in B_{\sigma} \\
&  \tag{B.3}\\
& \quad u^{i^{\prime}}\left(1-\tau^{\prime}, \alpha_{1} c_{\sigma 1}^{i}\left(\mathbf{b}_{\sigma}^{\prime}\right), \alpha_{2} c_{\sigma 2}^{i}\left(\mathbf{b}_{\sigma}^{\prime}\right)\right) \geq 1
\end{align*}
$$

where $\tau^{\prime}$ is the first component of $\mathbf{b}_{\sigma}^{\prime}$.
(E4) For any $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$ and $\sigma \in\{s, d\}$, we have $\rho^{i}(\sigma)=1$ if and only if

$$
\begin{equation*}
V^{i}\left(\sigma ;\left(b_{\sigma}^{k}, a_{\sigma}^{k}, c_{\sigma}^{k}\right)_{k \in\{1,2\}}\right) \geq V^{i}\left(\sigma^{s} ;\left(b_{\sigma^{s}}^{k}, a_{\sigma^{s}}^{k}, c_{\sigma^{s}}^{k}\right)_{k \in\{1,2\}}\right) \tag{B.4}
\end{equation*}
$$

where

$$
\begin{aligned}
V^{i}\left(\sigma^{\prime} ;\left(b_{\sigma^{\prime}}^{k}, a_{\sigma^{\prime}}^{k}, c_{\sigma^{\prime}}^{k}\right)_{k \in\{1,2\}}\right)= & \lambda_{i} u^{i}\left(1-\tau_{\sigma^{\prime}}^{i}, \alpha_{1} c_{\sigma^{\prime} 1}^{i}\left(b_{\sigma^{\prime}}^{i}\right), \alpha_{2} c_{\sigma^{\prime} 2}^{i}\left(b_{\sigma^{\prime}}^{i}\right)\right) \\
& +\left(1-\lambda_{i}\right) u^{i}\left(1-\tau_{\sigma^{\prime}}^{i^{\prime}}, \alpha_{1} c_{\sigma^{\prime} 1}^{i^{\prime}}\left(b_{\sigma^{\prime}}^{i^{\prime}}\right), \alpha_{2} c_{\sigma^{\prime} 2}^{i^{\prime}}\left(b_{\sigma^{\prime}}^{i^{\prime}}\right)\right)
\end{aligned}
$$

for any $\sigma^{\prime} \in\{s, d\}$ where $\tau_{\sigma^{\prime}}^{k}$ is the first component of $\mathbf{b}_{\sigma^{\prime}}^{k}$ for any $k \in\{1,2\}$.
(E5) For any $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$ we have

$$
\begin{align*}
& v^{i} \in \arg \max _{\sigma^{\prime} \in\{s, d\}} V^{i}\left(\sigma^{\prime} ;\left(b_{\sigma^{\prime}}^{k}, a_{\sigma^{\prime}}^{k}, c_{\sigma^{\prime}}^{k}\right)_{k \in\{1,2\}}\right)  \tag{B.5}\\
& \quad \text { s.t. } V^{i^{\prime}}\left(\sigma^{\prime} ;\left(b_{\sigma^{\prime}}^{k}, a_{\sigma^{\prime}}^{k}, c_{\sigma^{\prime}}^{k}\right)_{k \in\{1,2\}}\right) \geq V^{i^{\prime}}\left(\sigma^{s} ;\left(b_{\sigma^{s}}^{k}, a_{\sigma^{s}}^{k}, c_{\sigma^{s}}^{k}\right)_{k \in\{1,2\}}\right)
\end{align*}
$$

Condition (E1) states that for any budgeting rule $\sigma$ and budget $\mathbf{b}_{\sigma}$, a legislator's equilibrium labor-hiring policy maximizes his payoff. Condition (E2) says that for any budgeting rule $\sigma$, legislator $i$ accepts a budget proposal if and only if his payoff from the proposal is higher than his payoff from the status-quo policy.

Condition (E3) requires that for any budgeting rule $\sigma$, legislator $i$ 's equilibrium budget proposal maximizes his payoff subject to party $i^{\prime}$ accepting the proposal. Condition (E4) says that legislator $i$ accepts a budgeting-rule proposal if and only if his expected payoff from the proposal is higher than his expected payoff from the status-quo budgeting rule. Condition (E5) requires that legislator $i$ 's equilibrium budgeting-rule proposal maximizes his expected payoff subject to party $i^{\prime}$ accepting the proposal.

## B. 2 Proof of Propositions

## B.2.1 Proof of Proposition 2.1

For any economic environment $\Gamma$, by Assumption 2.3 and backward induction it is easy to show that there exists a unique equilibrium $\left(v^{i}, \rho^{i},\left(b_{d}^{i}, a_{d}^{i}, c_{d}^{i}\right)_{\sigma \in\{s, d\}}\right)_{i \in\{1,2\}}$.

Part (i). Without loss of generality assume that legislator $i=2$ proposes a budget and legislator $i^{\prime}=1$ responds by accepting or rejecting it. Under Assumptions 2.1-2.2, it is clear that $b_{d}^{2} \notin\left\{(0,0,0),\left(\frac{1}{4} L, \frac{1}{2} L, 0\right),\left(\frac{1}{2} L, L, 0\right)\right\}$. Moreover, if $u^{2}\left(\frac{1}{4} L, 0, \alpha \frac{1}{2} L\right)>u^{2}\left(1-\frac{1}{2} L, \alpha \frac{1}{2} L, \alpha \frac{1}{2} L\right)$ then $u^{1}\left(\frac{1}{4} L, 0, \alpha \frac{1}{2} L\right)<1$. Therefore, $b_{d}^{2} \neq\left(\frac{1}{4} L, 0, \frac{1}{2} L\right)$.

If (1) $b_{d}^{2}=\left(\frac{1}{2} L, 0, L\right)$, or (2) $b_{d}^{2}=\left(\frac{1}{2} L, \frac{1}{2} L, \frac{1}{2} L\right)$ and $u^{2}\left(1-\frac{1}{2} L, \alpha \frac{1}{2} L, \alpha \frac{1}{2} L\right)>$ $u^{2}\left(1-\frac{1}{2} L, 0, \alpha L\right)$, then $b_{s}^{2}=\tau_{d}^{2}$ and $c_{s}^{2}\left(b_{s}^{2}\right)=c_{d}^{2}\left(b_{d}^{2}\right)$ where $\tau_{d}^{2}$ is the first component of $b_{d}^{2}$. Therefore, $c_{d}^{2}\left(b_{d}^{2}\right) \neq c_{s}^{2}\left(b_{s}^{2}\right)$ if and only if (3) $b_{d}^{2}=\left(\frac{1}{2} L, \frac{1}{2} L, \frac{1}{2} L\right)$ and $u^{2}(1-$ $\left.\frac{1}{2} L, \alpha \frac{1}{2} L, \alpha \frac{1}{2} L\right)<u^{2}\left(1-\frac{1}{2} L, 0, \alpha L\right)$. Condition (3) holds if and only if $\beta>$ $\max \left\{\frac{h\left(\alpha \frac{1}{2} L\right)}{h(\alpha L)}, 1-\frac{L}{2 h(\alpha L)}\right\}$. Let $\underline{\beta}=\max \left\{\frac{h\left(\alpha \frac{1}{2} L\right)}{h(\alpha L)}, 1-\frac{L}{2 h(\alpha L)}\right\}$.

Part (ii). Assume that $\beta>\underline{\beta}$. Then it is clear that $b_{s}^{2} \neq \frac{1}{2} L$. If $\beta<1-\frac{L}{4 h\left(\alpha \frac{1}{2} L\right)}$,
we have $u^{1}\left(1-\frac{1}{4} L, 0, \alpha \frac{1}{2} L\right)>1$, which implies that $u^{2}\left(1-\frac{1}{4} L, 0, \alpha \frac{1}{2} L\right)>1$. So, we have $b_{s}^{2}=\frac{1}{4} L$ and $c_{s}^{2}\left(b_{s}^{2}\right)=\left(0, \frac{1}{2} L\right)$. Let $\bar{\beta}=1-\frac{L}{4 h\left(\alpha \frac{1}{2} L\right)}$.

Part (iii). If $\beta>1-\frac{L}{4 h\left(\alpha \frac{1}{2} L\right)}$, we have $u^{1}\left(1-\frac{1}{4} L, 0, \alpha \frac{1}{2} L\right)<1$. So, we have $b_{s}^{2}=0$ and $c_{s}^{2}\left(b_{s}^{2}\right)=(0,0)$.

## B.2.2 Proof of Proposition 2.2

Without loss of generality assume that legislator $i=2$ proposes a budget and legislator $i^{\prime}=1$ responds by accepting or rejecting it.

Part (i). For any $\left(\tau, l_{1}, l_{2}\right) \in P, u^{1}\left(\tau, l_{1}, l_{2}\right)-u^{2}\left(\tau, l_{1}, l_{2}\right)=(1-2 \beta)\left(h\left(\alpha l_{2}\right)-\right.$ $\left.h\left(\alpha l_{1}\right)\right)$. Thus,

$$
\max _{\left(\tau, l_{1}, l_{2}\right) \in P}\left|u^{1}\left(\tau, l_{1}, l_{2}\right)-u^{2}\left(\tau, l_{1}, l_{2}\right)\right|=(1-2 \beta) h(\alpha L) .
$$

For any $\beta$, we have

$$
\max _{\tau^{\prime} \in\left\{0, \frac{1}{2 L} L, \ldots, \frac{1}{2} L\right\}} u^{2}\left(\tau^{\prime}, \alpha c_{s 1}^{2}\left(\tau^{\prime}\right), \alpha c_{s 2}^{2}\left(\tau^{\prime}\right) ; \beta\right) \geq u^{2}\left(\frac{1}{n} L, \alpha \frac{1}{n} L, \alpha \frac{1}{n} L ; \beta\right)>1 .
$$

Let $d=u^{2}\left(\frac{1}{n} L, \alpha \frac{1}{n} L, \alpha \frac{1}{n} L ; \beta\right)-1$. There exists $\underline{\boldsymbol{\beta}} \in\left(\frac{1}{2}, 1\right)$ such that (1$2 \beta) h(\alpha L)<d$ for any $\beta<\underline{\boldsymbol{\beta}}$. Take $\beta<\underline{\boldsymbol{\beta}}$. Let

$$
b_{s} \in \arg \max _{\tau^{\prime} \in\left\{0, \frac{1}{2 n} L, \ldots, \frac{1}{2} L\right\}} u^{2}\left(\tau^{\prime}, \alpha c_{s 1}^{2}\left(\tau^{\prime}\right), \alpha c_{s 2}^{2}\left(\tau^{\prime}\right) ; \beta\right)
$$

We have $u^{1}\left(b_{s}, \alpha c_{s 1}^{2}\left(b_{s}\right), \alpha c_{s 2}^{2}\left(b_{s}\right) ; \beta\right) \geq 1$. Thus for each $\beta<\underline{\boldsymbol{\beta}}$, we have $b_{d}^{2}=$ $\left(\tau_{d}^{2}, l_{d 1}^{2}, l_{d 2}^{2}\right)=\left(b_{s}^{2}, c_{s 1}^{2}\left(b_{s}^{2}\right), c_{s 2}^{2}\left(b_{s}^{2}\right)\right)$
and $b_{s}^{2} \in \arg \max _{\tau^{\prime} \in\left\{0, \frac{1}{2 n} L, \ldots, \frac{1}{2} L\right\}} u^{2}\left(\tau^{\prime}, \alpha c_{s 1}^{2}\left(\tau^{\prime}\right), \alpha c_{s 2}^{2}\left(\tau^{\prime}\right) ; \beta\right)$. So, if $\beta<\underline{\boldsymbol{\beta}}$ then
$c_{s}^{2}\left(b_{s}^{2}\right)=c_{d}^{2}\left(b_{d}^{2}\right)$.
Part (ii). For any $\left(\tau, l_{1}, l_{2}\right) \in Q$, if
$1-\tau+\beta h\left(\alpha l_{2}\right)+(1-\beta) h\left(\alpha l_{1}\right)<1-\tau+\beta h\left(\alpha\left(l_{2}+\frac{L}{n}\right)\right)+(1-\beta) h\left(\alpha\left(l_{1}-\frac{L}{n}\right)\right)$
then legislator 2 prefers to spend all tax revenue for $l_{2}$. If

$$
1-\frac{L}{2}+(1-\beta) h\left(\alpha \frac{L}{n}\right)+\beta h\left(\alpha\left(L-\frac{L}{n}\right)\right)<1-\frac{L}{2}+\beta h(\alpha L)
$$

that is

$$
\frac{h\left(\alpha \frac{L}{n}\right)}{h(\alpha L)-h\left(\alpha\left(L-\frac{L}{n}\right)\right)+h\left(\alpha \frac{L}{n}\right)}<\beta,
$$

Inequality B. 6 holds for all $\left(\tau, l_{1}, l_{2}\right) \in Q$. If

$$
1-\frac{L}{2 n}+(1-\beta) h\left(\alpha \frac{L}{n}\right)<1,
$$

that is

$$
1-\frac{1}{2 n h\left(\alpha \frac{L}{n}\right)}<\beta
$$

legislator 1 rejects any budget proposal that leads to a policy in which only public good 2 is provided.

Let

$$
\overline{\boldsymbol{\beta}}=\max \left\{\frac{h\left(\alpha \frac{L}{n}\right)}{h(\alpha L)-h\left(\alpha\left(L-\frac{L}{n}\right)\right)+h\left(\alpha \frac{L}{n}\right)}, 1-\frac{1}{2 n h\left(\alpha \frac{L}{n}\right)}\right\} .
$$

If $\beta>\overline{\boldsymbol{\beta}}$, it is clear that we have $b_{s}^{2}=0$ and $c_{s}^{2}\left(b_{s}^{2}\right)=(0,0)$. On the other hand, under two-bill budgeting, legislator 2 can propose the budget $\mathbf{b}_{d}=\left(\tau, l_{1}, l_{2}\right)=$
$\left(\frac{1}{n} L, \frac{1}{n} L, \frac{1}{n} L\right)$. Legislator 1 accepts the proposal and they can both get a payoff greater than one. Hence we have $b_{d}^{2} \in \mathbb{R}_{++}^{3}$ and $c_{d}^{2}\left(b_{d}^{2}\right) \in \mathbb{R}_{++}^{2}$.

## B.2.3 Proof of Proposition 2.3

Let $i \in\{1,2\}$ be the legislator selected by nature to propose a budgeting rule, and $i^{\prime} \in\{1,2\}$ with $i^{\prime} \neq i$.

Part (i). Assume that $\beta<\underline{\boldsymbol{\beta}}$. Because both budgeting rules result in the same government policy, $c_{s}=0$, and $c_{d}>0$ we have $v_{d}^{i}=s$.

Assume that $\beta>\overline{\boldsymbol{\beta}}$. Two-bill budgeting is better for legislator $i$ if and only if

$$
\begin{equation*}
\lambda_{i} \bar{u}_{d}+\left(1-\lambda_{i}\right) \underline{u}_{d}-c_{d} \geq 1, \tag{B.7}
\end{equation*}
$$

and for legislator $i^{\prime}$ if and only if

$$
\begin{equation*}
\lambda_{i} \underline{u}_{d}+\left(1-\lambda_{i}\right) \bar{u}_{d}-c_{d} \geq 1 . \tag{B.8}
\end{equation*}
$$

It is clear that if $\bar{u}_{d}<1+c_{d}$, one-bill budgeting is better for both legislators. Therefore, under any satus-quo budgeting rule, $\sigma^{s} \in\{s, d\}$, we have $v^{i}=s$.

Part (ii). Assume that $\beta>\overline{\boldsymbol{\beta}}, \underline{u}_{d}<1+c_{d}<\bar{u}_{d}$ and $\sigma^{s}=s$. Then for legislator $i$ to propose a two-bill budgeting,

$$
\begin{equation*}
\lambda_{i} \bar{u}_{d}+\left(1-\lambda_{i}\right) \underline{u}_{d}-c_{d} \geq 1 \tag{B.9}
\end{equation*}
$$

should hold, and for legislator $i^{\prime}$ to accept a two-bill budgeting proposal,

$$
\begin{equation*}
\lambda_{i} \underline{u}_{d}+\left(1-\lambda_{i}\right) \bar{u}_{d}-c_{d} \geq 1 \tag{B.10}
\end{equation*}
$$

should hold.

Thus $v^{i}=d$ if and only if Inequalities B.9-B. 10 hold. Rearranging Inequalities B.9-B.10, we have $v^{i}=d$ if and only if $\bar{u}_{d}-\left(1+c_{d}\right) \geq \lambda_{i}\left(\bar{u}_{d}-\underline{u}_{d}\right) \geq 1+c_{d}-\underline{u}_{d}$. Part (iii). Assume that $\beta>\overline{\boldsymbol{\beta}}, \underline{u}_{d}<1+c_{d}<\bar{u}_{d}$ and $\sigma^{s}=d$. Then for legislator $i$ to propose one-bill budgeting,

$$
\begin{equation*}
\lambda_{i} \bar{u}_{d}+\left(1-\lambda_{i}\right) \underline{u}_{d}-c_{d}<1 \tag{B.11}
\end{equation*}
$$

should hold, and for legislator $i^{\prime}$ to accept the one-bill budgeting proposal,

$$
\begin{equation*}
\lambda_{i} \underline{u}_{d}+\left(1-\lambda_{i}\right) \bar{u}_{d}-c_{d}<1 \tag{B.12}
\end{equation*}
$$

should hold.
Thus $v^{i}=s$ if and only if Inequalities B.11-B. 12 hold. Rearranging Inequalities B.11-B.12, we have $v^{i}=s$ if and only if $\bar{u}_{d}-\left(1+c_{d}\right)<\lambda_{i}\left(\bar{u}_{d}-\underline{u}_{d}\right)<1+c_{d}-\underline{u}_{d}$. This implies that $v^{i}=d$ if and only if $\bar{u}_{d}-\left(1+c_{d}\right) \geq \lambda_{i}\left(\bar{u}_{d}-\underline{u}_{d}\right)$ or $\lambda_{i}\left(\bar{u}_{d}-\underline{u}_{d}\right) \geq 1+c_{d}-\underline{u}_{d}$.

Part (iv). By Inequalities B.7-B.8, it is clear that if $\underline{u}_{d}>1+c_{d}$, then two-bill budgeting is better for both legislators. So, under any status-quo budgeting rule, $\sigma^{s} \in\{s, d\}$, we have $v^{i}=d$.

## B.2.4 Proof of Proposition 2.4

For any economic environment $\Gamma$, by backward induction it is easy to show that there exists an equilibrium $\left(\tilde{b}_{s}^{i}, \tilde{a}_{s}^{i}, \tilde{c}_{s}^{i}\right)_{i \in\{1,2\}}$ when group-specific transfers are allowed under one-bill budgeting.

Part (i). Without loss of generality assume that legislator $i=2$ is the proposer and legislator $i^{\prime}=1$ is the responder. By Assumption 2.2 and continuity there exists $\tilde{\beta}_{C}$ such that $\beta h\left(\alpha \frac{1}{n} L\right) \geq \frac{1}{n} L$ for all $\beta \geq \tilde{\beta}_{C}$. Let $\tilde{\boldsymbol{\beta}}=\max \left\{\tilde{\beta}_{C}, \overline{\boldsymbol{\beta}}\right\}$.

Assume that $\beta>\tilde{\boldsymbol{\beta}}$. By Proposition 2.2, we have $\left(\tau_{s}^{2}, c_{s 2}^{2}\left(b_{s}^{2}\right), c_{s 1}^{2}\left(b_{s}^{2}\right)\right)=\mathbf{0}$. We also have $\tilde{c}_{s 1}^{2}\left(\tilde{b}_{s}^{2}\right)=0$, and $\tilde{a}_{s}^{1}\left(\tilde{\mathbf{b}}_{s}^{2}\right)=0$ for any $\tilde{\mathbf{b}}_{s}^{2}=\left(\tilde{\tau}_{s}^{2}, s_{1}, s_{2}\right)$ such that $\tilde{\tau}_{s}^{2}>0$ and $s_{1}=0$, since $\beta>\overline{\boldsymbol{\beta}}$.

So, when group-specific transfers are allowed we can look for an equilibrium such that $\tilde{\tau}_{s}^{2}=0$, or $\tilde{\tau}_{s}^{2}>0, s_{1}>0$ and $s_{2}=0$. Then legislator 2's problem is

$$
\begin{align*}
& \max _{\left(\tau^{\prime}, s_{1}^{\prime}, l_{2}^{\prime}\right)} u^{2}\left(1-\tau^{\prime}, 0, \alpha l_{2}^{\prime}\right) \\
& \text { s.t. }\left(\tau^{\prime}, s_{1}^{\prime}, l_{2}^{\prime}\right) \in\left\{\left(\tau, s_{1}, l_{2}\right) \in(0,1] \times(0,1] \times\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}:\right.  \tag{B.13}\\
& \left.l_{2}+s_{1}=2 \tau^{\prime}\right\} \cup\{(0,0,0)\}, \\
& \\
& u^{1}\left(1-\tau^{\prime}+s_{1}^{\prime}, 0, \alpha l_{2}^{\prime}\right) \geq 1 .
\end{align*}
$$

Consider the point $\left(\tau^{\prime}, s_{1}^{\prime}, l_{2}^{\prime}\right)=\left(\frac{1}{n} L, \frac{1}{n} L, \frac{1}{n} L\right)$. Clearly it is in the constraint set of the legislator 2's problem. Moreover, we have $u^{2}\left(1-\frac{1}{n} L, 0, \alpha \frac{1}{n} L\right)>1$ since $\beta>\tilde{\beta}_{C}$. So, we have $\left(\tilde{\tau}_{s}^{2}, \tilde{c}_{s 2}^{2}\left(\tilde{b}_{s}^{2}\right), \tilde{c}_{s 1}^{2}\left(\tilde{b}_{s}^{2}\right)\right) \in \mathbb{R}_{++} \times \mathbb{R}_{++} \times\{0\}$.

Part (ii). Following from the arguments in the proof of Part (i), we have $u_{2 s}^{2}<\tilde{u}_{2 s}^{2}$ and $u_{2 s}^{1} \leq \tilde{u}_{2 s}^{1}$.

## B. 3 Larger Number of Public Goods

## B.3.1 The Economic Environment

The society is separated into two groups of citizens each with unit mass population and indexed by $i \in\{1,2\}$. Each citizen has an endowment of one unit of labor denoted by $l$. There is a single consumption good denoted by $z$, and two groups of public goods indexed by $t \in\{A, B\}$. In each group of public goods $t$, there are two public goods denoted by $g_{t}$ and indexed by $j \in\{1,2\}$. The consumption good is produced with the technology $f(l)=l$ and the public goods with the technology $f_{t j}(l)=\alpha_{t j} l$ where $\alpha_{t j}>0$ for each $t$ and $j$.

A citizen's utility function in group 1 is
$u_{1}\left(z, g_{A 1}, g_{A 2}, g_{B 1}, g_{B 2}\right)=z+\beta h\left(g_{A 1}\right)+(1-\beta) h\left(g_{A 2}\right)+\beta h\left(g_{B 1}\right)+(1-\beta) h\left(g_{B 2}\right)$
and in group 2 is
$u_{2}\left(z, g_{A 1}, g_{A 2}, g_{B 1}, g_{B 2}\right)=z+(1-\beta) h\left(g_{A 1}\right)+\beta h\left(g_{A 2}\right)+(1-\beta) h\left(g_{B 1}\right)+\beta h\left(g_{B 2}\right)$,
where $h: \mathbb{R}_{+} \rightarrow \mathbb{R}_{+}$is an increasing and strictly concave function such that $h(0)=0$. We assume that $\frac{1}{2} \leq \beta \leq 1$.

There is a competitive labor market. Thus, the assumption on the production technology of the consumption good implies a wage rate equal to 1 .

## B.3.2 Government Policies

A government policy is described by a quintuple ( $\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}$ ), where $\tau$ is the income tax rate and $l_{t j}$ is the amount of labor allocated for the production of public good $j$ in group $t$. The set of feasible government policies is given by

$$
\begin{aligned}
\hat{P}= & \left\{\left(\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right) \in[0,1] \times\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}^{4}: l_{A 1}+l_{A 2}+l_{B 1}+l_{B 2}=2 \tau\right. \\
& \text { and } \left.l_{t 1}+l_{t 2} \leq L \text { for each } t \in\{A, B\}\right\}
\end{aligned}
$$

where $L$ is the maximum amount of labor that can be allocated for the production of public goods in group $t$. Feasibility requires that $L \leq 1$, since the total endowment of labor in the economy is two. We assume that for the production of public good $j$ in group $t$ labor can only be hired at discrete levels; that is, $l_{t j} \in\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}$ where $n \in \mathbb{Z}_{+}$. Let

$$
\hat{Q}=\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}^{2}
$$

denote the set of labor-hiring policies for the provision of public goods in group $t$.
The economic environment with government policies can be summarized by

$$
\hat{\Gamma}=\left(\left(\alpha_{t j}\right)_{t \in\{A, B\}, j \in\{1,2\}}, \beta, h, n, L\right) .
$$

## B.3.3 The Political Process

Government policy decisions are made by a legislature consisting of representatives of both groups. The decision process is composed of two stages. In the first stage, nature selects one of the legislators to propose a budgeting rule, $\sigma \in\{s, d\}$,
where $s$ and $d$ represents one- and two-bill budgeting, respectively. The other legislator responds by either accepting or rejecting the proposal. If he accepts, in the second stage, the proposed budgeting rule is implemented. Otherwise, a status-quo budgeting rule, $\sigma^{s} \in\{s, d\}$, prevails. Implementing one-bill budgeting has a cost $c_{s}=0$ and two-bill budgeting has a cost $c_{d}>0$. In the second stage, nature selects one of the legislators to propose a budget. The probability that nature selects legislator $i$ is $\lambda_{i}$. We interpret $\lambda_{i}$ as the political power of legislator i. The proposer has to follow the budgeting rule determined in the first stage. Composition of a budget and the remainder of the process under each budgeting rule is described in the following subsections.

One-bill Budgeting. - Under one-bill budgeting, a budget is composed of an income tax rate $\tau$ and total amount of labor $l_{t}$ that will be hired for the provision of public goods in group $t$. Let

$$
\begin{aligned}
\hat{B}_{s}= & \left\{\hat{\mathbf{b}}_{s}=\left(\tau, l_{A}, l_{B}\right) \in\left\{0, \frac{1}{2 n} L, \frac{2}{2 n} L, \ldots, \frac{1}{2} L\right\} \times\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}^{2}:\right. \\
& \left.l_{A}+l_{B}=2 \tau\right\}
\end{aligned}
$$

denote the set of feasible budget proposals under one-bill budgeting.
Under this rule, the proposer offers a budget $\hat{\mathbf{b}}_{s} \in \hat{B}_{s}$. The other legislator responds by either accepting or rejecting it. If he accepts, then the proposer chooses
a labor-hiring policy, $\mathbf{q}=\left(l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right)$. Let

$$
\begin{aligned}
\hat{Q}_{s}\left(\hat{\mathbf{b}}_{s}\right)= & \left\{\left(l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right) \in\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}^{4}: l_{t 1}+l_{t 2}=l_{t}\right. \\
& \text { for each } t \in\{A, B\}\}
\end{aligned}
$$

denote the set of feasible labor-hiring policies given a budget $\hat{\mathbf{b}}_{s}=\left(\tau, l_{A}, l_{B}\right) \in$ $\hat{B}_{s}$. If the responder rejects the proposal, then the status-quo policy, $\hat{\mathbf{q}}^{s}=$ $\left(\tau^{s}, l_{A 1}^{s}, l_{A 2}^{s}, l_{B 1}^{s}, l_{B 2}^{s}\right)=(0,0,0,0,0)$, is implemented.

Two-bill Budgeting. - Under two-bill budgeting, a budget is composed of an income tax rate $\tau$ and amounts of labor $\left(l_{t 1}, l_{t 2}\right)$ that will be hired for the production of public goods $\left(g_{t 1}, g_{t 2}\right)$ in group $t$. Let

$$
\begin{gathered}
\hat{B}_{d}=\left\{\hat{\mathbf{b}}_{d}=\left(\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right) \in\left\{0, \frac{1}{2 n} L, \frac{2}{2 n} L, \ldots, \frac{1}{2} L\right\} \times\left\{0, \frac{1}{n} L, \frac{2}{n} L, \ldots, L\right\}^{4}\right. \\
\left.: l_{t 1}+l_{t 2} \leq L \text { for each } t \in\{A, B\} \text { and } l_{A 1}+l_{A 2}+l_{B 1}+l_{B 2}=2 \tau\right\}
\end{gathered}
$$

denote the set of feasible budget proposals under two-bill budgeting.
Under this rule, the proposer offers a budget $\hat{\mathbf{b}}_{d}=\left(\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right) \in \hat{B}_{d}$. The other legislator responds by either accepting or rejecting it. If he accepts, then the proposer implements the labor-hiring policy $\mathbf{q}=\left(l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right)$. For convenience of notation, let

$$
\hat{Q}_{d}\left(\hat{\mathbf{b}}_{d}\right)=\left\{\left(l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right)\right\}
$$

denote the set of feasible labor-hiring policies given a budget $\hat{\mathbf{b}}_{d}=\left(\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right)$
$\in \hat{B}_{d}$. If the responding legislator rejects the proposal, then the status-quo policy, $\hat{\mathbf{q}}^{s}$, is implemented.

## B.3.4 Political Equilbirium

A strategy for legislator $i,\left(\hat{v}^{i}, \hat{\rho}^{i},\left(\hat{b}_{\sigma}^{i}, \hat{a}_{\sigma}^{i}, \hat{c}_{\sigma}^{i}\right)_{\sigma \in\{s, d\}}\right)$, is composed of a budgetingrule proposal strategy, $\hat{v}^{i} \in\{s, d\}$, a budgeting-rule acceptance strategy, $\hat{\rho}^{i}$ : $\{s, d\} \rightarrow\{0,1\}$, and for each budgeting-rule $\sigma \in\{s, d\}$, a budget-proposal strategy, $\hat{b}_{\sigma}^{i} \in \hat{B}_{\sigma}$, a budget-acceptance strategy, $\hat{a}_{\sigma}^{i}: \hat{B}_{\sigma} \rightarrow\{0,1\}$, and a labor-hiring strategy, $\hat{c}_{\sigma}^{i}: \hat{B}_{\sigma} \rightarrow Q$, such that $\hat{c}_{\sigma}^{i}\left(\hat{\mathbf{b}}_{\sigma}\right) \in \hat{Q}_{\sigma}\left(\hat{\mathbf{b}}_{\sigma}\right)$ for each $\hat{\mathbf{b}}_{\sigma} \in \hat{B}_{\sigma}$. Legislator $i$ 's budgeting-rule acceptance strategy, $\hat{\rho}^{i}(\sigma)$, takes the value 1 if legislator $i$ accepts the budgeting-rule proposal, $\sigma$, offered by legislator $i^{\prime} \neq i$, and 0 otherwise. Legislator $i$ 's budget-acceptance strategy, $\hat{a}_{\sigma}^{i}\left(\hat{\mathbf{b}}_{\sigma}\right)$, takes the value 1 if legislator $i$ accepts the budget proposal, $\hat{\mathbf{b}}_{\sigma}$, offered by legislator $i^{\prime} \neq i$, and 0 otherwise.

We consider subgame-perfect equilibria. We restrict attention to equilibria in which (i) $\hat{\rho}^{i}(\sigma)=1$ when legislator $i$ is indifferent between $\sigma$ and $\sigma^{s}$, (ii) $\hat{a}_{\sigma}^{i}\left(\hat{\mathbf{b}}_{\sigma}\right)=1$ when legislator $i$ is indifferent between $\hat{c}_{\sigma}^{i^{\prime}}\left(\hat{\mathbf{b}}_{\sigma}\right)$ and $\hat{\mathbf{q}}^{s}$, (iii) $\hat{\rho}^{i}\left(\hat{v}^{i^{\prime}}\right)=1$ and (iv) $\hat{a}_{\sigma}^{i}\left(\hat{b}_{\sigma}^{i^{\prime}}\right)=1$ for all $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}$. That is, a responding legislator accepts any proposal that he is indifferent between accepting and rejecting, and equilibrium proposals are always accepted.

Assumption B.1. Production technologies of the two public goods are the same; specifically, $\alpha_{t j}=\alpha>0$ for each $j \in\{1,2\}$ and $t \in\{A, B\}$.

Assumption B.2. Public goods are valuable enough for citizens; specifically, $h\left(\alpha \frac{1}{n} L\right)>\frac{1}{n} L$.

Assumption B.3. Any two different government policies give different utilities; specifically, $u_{i}\left(1-\tau, \alpha l_{A 1}, \alpha l_{A 2}, \alpha l_{B 1}, \alpha l_{B 2}\right)-c_{\sigma} \neq u_{i}\left(1-\tau^{\prime}, \alpha l_{A 1}^{\prime}, \alpha l_{A 2}^{\prime}, \alpha l_{B 1}^{\prime}, \alpha l_{B 2}^{\prime}\right)-$ $c_{\sigma^{\prime}}$ for any $\left(\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right),\left(\tau^{\prime}, l_{A 1}^{\prime}, l_{A 2}^{\prime}, l_{B 1}^{\prime}, l_{B 2}^{\prime}\right) \in \hat{P}$ such that $\left(\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right) \neq\left(\tau^{\prime}, l_{A 1}^{\prime}, l_{A 2}^{\prime}, l_{B 1}^{\prime}, l_{B 2}^{\prime}\right), \sigma, \sigma^{\prime} \in\{s, d\}$ and $i \in\{1,2\}$.

For any $\mathbf{y}=\left(y_{1}, y_{2}, y_{3}, y_{4}\right) \in \mathbb{R}^{4}$, let $\mathbf{y}_{t}=y_{t}$ for any $t \in\{1,2,3,4\}$.

Proposition B.1. Let $\hat{\Gamma}$ be an economic environment that satisfies Assumptions B.1-B.3. The environment has a unique equilibrium $\left(\hat{v}^{i}, \hat{\rho}^{i},\left(\hat{b}_{\sigma}^{i}, \hat{a}_{\sigma}^{i}, \hat{c}_{\sigma}^{i}\right)_{\sigma \in\{s, d\}}\right)_{i \in\{1,2\}}$. Moreover, there exists $\boldsymbol{\beta}, \hat{\boldsymbol{\beta}} \in\left(\frac{1}{2}, 1\right)$ such that $\boldsymbol{\beta} \leq \hat{\boldsymbol{\beta}}$ and the following hold.
(i) If polarization is low, then the two budgeting rules lead to the same provision of all public goods; specifically, if $\beta<\boldsymbol{\beta}$, then $\hat{c}_{s}^{i}\left(\hat{b}_{s}^{i}\right)=\hat{c}_{d}^{i}\left(\hat{b}_{d}^{i}\right)$ for any $i \in\{1,2\}$.
(ii) If polarization is high, then under one-bill budgeting all public goods are provided at a lower level compared to two-bill budgeting; specifically, if $\beta>\hat{\boldsymbol{\beta}}$, then, for any $i, i^{\prime} \in\{1,2\}$ with $i \neq i^{\prime}, \hat{c}_{s}^{i}\left(\hat{b}_{s}^{i}\right)=(0,0,0,0), \hat{c}_{d t i}^{i}\left(\hat{b}_{d}^{i}\right)>0$ for any $t \in\{A, B\}$, and $\hat{c}_{d t i^{\prime}}^{i}\left(\hat{b}{ }_{d}^{i}\right) \geq 0$ for any $t \in\{A, B\}$ with strict inequality for some $t \in\{A, B\}$.

Proof. Existence of a unique equilibrium is easy to prove by backward induction. So, we skip this part. Without loss of generality, assume that legislator $i=2$ proposes a budget and legislator $i^{\prime}=1$ responds by either accepting or rejecting it.

Part (i). For any $\left(\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right) \in \hat{P}$, we have

$$
\begin{gathered}
u^{1}\left(\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right)-u^{2}\left(\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right) \\
=(1-2 \beta)\left[\left(h\left(\alpha l_{A 2}\right)-h\left(\alpha l_{A 1}\right)\right)+\left(h\left(\alpha l_{B 2}\right)-h\left(\alpha l_{B 1}\right)\right)\right] .
\end{gathered}
$$

Thus,

$$
\max _{\left(\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right) \in \hat{P}}\left|u^{1}\left(\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right)-u^{2}\left(\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right)\right|=(1-2 \beta) 2 h(\alpha L) .
$$

For any $\beta$, we have

$$
\begin{gathered}
\max _{\tau^{\prime} \in\left\{0, \frac{1}{2 n} L, \ldots, \frac{1}{2} L\right\}} u^{2}\left(\tau^{\prime}, \alpha c_{s 1}^{2}\left(\tau^{\prime}\right), \alpha c_{s 2}^{2}\left(\tau^{\prime}\right), \alpha c_{s 3}^{2}\left(\tau^{\prime}\right), \alpha c_{s 4}^{2}\left(\tau^{\prime}\right) ; \beta\right) \\
\quad \geq u^{2}\left(\frac{2}{n} L, \alpha \frac{1}{n} L, \alpha \frac{1}{n} L, \alpha \frac{1}{n} L, \alpha \frac{1}{n} L ; \beta\right)>1
\end{gathered}
$$

Let $d=\left(\frac{2}{n} L, \alpha \frac{1}{n} L, \alpha \frac{1}{n} L, \alpha \frac{1}{n} L, \alpha \frac{1}{n} L ; \beta\right)-1$. There exits $\underset{\sim}{\boldsymbol{\beta}} \in\left(\frac{1}{2}, 1\right)$ such that $(1-2 \beta) 2 h(\alpha L)<d$ for any $\beta<\boldsymbol{\beta}$. Take $\beta<\boldsymbol{\beta}$. Let

$$
\hat{b}_{s} \in \arg \max _{\tau^{\prime} \in\left\{0, \frac{1}{2 n} L, \ldots, \frac{1}{2} L\right\}} u^{2}\left(\tau^{\prime}, \alpha c_{s 1}^{2}\left(\tau^{\prime}\right), \alpha c_{s 2}^{2}\left(\tau^{\prime}\right), \alpha c_{s 3}^{2}\left(\tau^{\prime}\right), \alpha c_{s 4}^{2}\left(\tau^{\prime}\right) ; \beta\right) .
$$

We have $u^{1}\left(\hat{b}_{s}, \alpha c_{s 1}^{2}\left(\hat{b}_{s}\right), \alpha c_{s 2}^{2}\left(\hat{b}_{s}\right), \alpha c_{s 3}^{2}\left(\hat{b}_{s}\right), \alpha c_{s 4}^{2}\left(\hat{b}_{s}\right) ; \beta\right) \geq 1$.
Thus for each $\beta<\boldsymbol{\beta}$, we have

$$
\hat{b}_{d}^{2}=\left(\tau_{d}^{2}, l_{d A 1}^{2}, l_{d A 2}^{2}, l_{d B 1}^{2}, l_{d B 2}^{2}\right)=\left(\hat{b}_{s}^{2}, \hat{c}_{s A 1}^{2}\left(\hat{b}_{s}^{2}\right), \hat{c}_{s A 2}^{2}\left(\hat{b}_{s}^{2}\right), \hat{c}_{s B 1}^{2}\left(\hat{b}_{s}^{2}\right), \hat{c}_{s B 2}^{2}\left(\hat{b}_{s}^{2}\right)\right),
$$

where

$$
\hat{b}_{s}^{2} \in \arg \max _{b^{\prime} \in \hat{B}_{s}} u^{2}\left(\tau^{\prime}, \alpha \hat{c}_{s A 1}^{2}\left(b^{\prime}\right), \alpha \hat{c}_{s A 2}^{2}\left(b^{\prime}\right), \alpha \hat{c}_{s B 1}^{2}\left(b^{\prime}\right), \alpha \hat{c}_{s B 2}^{2}\left(b^{\prime}\right) ; \beta\right) .
$$

So, if $\beta<\underset{\sim}{\boldsymbol{\beta}}$, we have $\hat{c}_{s}^{2}\left(\hat{b}_{s}^{2}\right)=\hat{c}_{d}^{2}\left(\hat{b}_{d}^{2}\right)$.

Part (ii). For any $\left(\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right) \in \hat{Q}$, if

$$
\begin{equation*}
\beta h\left(\alpha l_{t 2}\right)+(1-\beta) h\left(\alpha l_{t 1}\right)<\beta h\left(\alpha\left(l_{t 2}+\frac{L}{n}\right)\right)+(1-\beta) h\left(\alpha\left(l_{t 1}-\frac{L}{n}\right)\right) \tag{B.14}
\end{equation*}
$$

for any $t \in\{A, B\}$, then legislator 2 prefers to spend no money for provision of $l_{A 1}$ or $l_{B 1}$. If

$$
(1-\beta) h\left(\alpha \frac{L}{n}\right)+\beta h\left(\alpha\left(L-\frac{L}{n}\right)\right)<\beta h(\alpha L),
$$

that is

$$
\frac{h\left(\alpha \frac{L}{n}\right)}{h(\alpha L)-h\left(\alpha\left(L-\frac{L}{n}\right)\right)+h\left(\alpha \frac{L}{n}\right)}<\beta,
$$

then Inequality B. 14 holds for all $\left(\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right) \in \hat{Q}$ and for any $t \in\{A, B\}$. If

$$
1-\frac{L}{2 n}+(1-\beta) h\left(\alpha \frac{L}{n}\right)<1,
$$

that is

$$
1-\frac{1}{2 n h\left(\alpha \frac{L}{n}\right)}<\beta,
$$

then legislator 1 rejects any budget proposal that leads to a policy in which only $g_{A 1}$ and $g_{B 1}$ are provided.

Let

$$
\hat{\boldsymbol{\beta}}=\max \left\{\frac{h\left(\alpha \frac{L}{n}\right)}{h(\alpha L)-h\left(\alpha\left(L-\frac{L}{n}\right)\right)+h\left(\alpha \frac{L}{n}\right)}, 1-\frac{1}{2 n h\left(\alpha \frac{L}{n}\right)}\right\} .
$$

If $\beta>\hat{\boldsymbol{\beta}}$, it is clear that we have $\hat{b}_{s}^{2}=0$ and $\hat{c}_{s}^{2}\left(b_{s}^{2}\right)=(0,0,0,0)$. On the other
hand, under two-bill budgeting, legislator 2 can propose the budget

$$
\hat{\mathbf{b}}_{d}=\left(\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right)=\left(\frac{2}{n} L, \frac{1}{n} L, \frac{1}{n} L, \frac{1}{n} L, \frac{1}{n} L\right) .
$$

Legislator 1 accepts the proposal and they can both get a payoff greater than one. If $h\left(\frac{1}{n} L\right)$ is high enough then legislator 2 can choose to offer the budget

$$
\hat{\mathbf{b}}_{d}^{\prime}=\left(\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right)=\left(\frac{2}{n} L, 0, \frac{2}{n} L, \frac{1}{n} L, \frac{1}{n} L\right)
$$

or

$$
\hat{\mathbf{b}}_{d}^{\prime \prime}=\left(\tau, l_{A 1}, l_{A 2}, l_{B 1}, l_{B 2}\right)=\left(\frac{2}{n} L, \frac{1}{n} L, \frac{1}{n} L, 0, \frac{2}{n} L\right) .
$$

Hence, we have $\hat{b}_{d}^{2}=\left(\tau^{*}, l_{A 1}^{*}, l_{A 2}^{*}, l_{B 1}^{*}, l_{B 2}^{*}\right)$ such that $\tau^{*}, l_{A 2}^{*}, l_{B 2}^{*}>0$ and $l_{A 1}^{*}, l_{B 1}^{*} \geq 0$, with strict inequality for at least one, and $\hat{c}_{d}^{2}\left(\hat{b}_{d}^{2}\right)=\left(l_{A 1}^{*}, l_{A 2}^{*}, l_{B 1}^{*}, l_{B 2}^{*}\right)$.

Until now, we have taken the budgeting rules as given and analyzed their effects on public-good provision. We now focus on the determination of the budgeting rule in the game's first stage. Let $\hat{\Gamma}$ be an economic environment and $\left(\hat{v}^{i}, \hat{\rho}^{i},\left(\hat{b}_{\sigma}^{i}, \hat{a}_{\sigma}^{i}, \hat{c}_{\sigma}^{i}\right)_{\sigma \in\{s, d\}}\right)_{i \in\{1,2\}}$ be its unique equilibrium. Define

$$
\mathbf{u}_{i \sigma}^{i} \equiv u_{i}\left(1-\hat{\tau}_{\sigma}^{i}, \alpha \hat{c}_{\sigma A 1}^{i}\left(\hat{b}_{\sigma}^{i}\right), \alpha \hat{c}_{\sigma A 2}^{i}\left(\hat{b}_{\sigma}^{i}\right), \alpha \hat{c}_{\sigma B 1}^{i}\left(\hat{b}_{\sigma}^{i}\right), \alpha \hat{c}_{\sigma B 2}^{i}\left(\hat{b}_{\sigma}^{i}\right)\right)
$$

and

$$
\mathbf{u}_{i^{\prime} \sigma}^{i} \equiv u_{i^{\prime}}\left(1-\hat{\tau}_{\sigma}^{i}, \alpha \hat{c}_{\sigma A 1}^{i}\left(\hat{b}_{\sigma}^{i}\right), \alpha \hat{c}_{\sigma A 2}^{i}\left(\hat{b}_{\sigma}^{i}\right), \alpha \hat{c}_{\sigma B 1}^{i}\left(\hat{b}_{\sigma}^{i}\right), \alpha \hat{c}_{\sigma B 2}^{i}\left(\hat{b}_{\sigma}^{i}\right)\right)
$$

for any $\sigma \in\{s, d\}$, where $\hat{\tau}_{\sigma}^{i}$ is the first component of $\hat{b}{ }_{\sigma}^{i}$; that is, $\mathbf{u}_{i \sigma}^{i}$ and $\mathbf{u}_{i^{\prime} \sigma}^{i}$ are the equilibrium payoffs of legislators $i$ and $i^{\prime}$, respectively, when legislator $i$
is the budget proposer under budgeting rule $\sigma$. By Assumption B.1, we have $\mathbf{u}_{1 \sigma}^{1}=\mathbf{u}_{2 \sigma}^{2}=\hat{u}_{\sigma}$ and $\mathbf{u}_{1 \sigma}^{2}=\mathbf{u}_{2 \sigma}^{1}=u_{\sigma}$ for any $\sigma \in\{s, d\}$. It is clear that $\hat{u}_{\sigma} \geq u_{\sigma}$.

Proposition B.2. Let $\Gamma$ be an economic environment that satisfies Assumptions B.1-B.3 and $\left(\hat{v}^{i}, \hat{\rho}^{i},\left(\hat{b}_{\sigma}^{i}, \hat{a}_{\sigma}^{i}, \hat{c}_{\sigma}^{i}\right)_{\sigma \in\{s, d\}}\right)_{i \in\{1,2\}}$ be its unique equilibrium. Then the following hold.
(i) If polarization is low, or both polarization and cost of two-bill budgeting is high, then legislators propose one-bill budgeting; specifically, if $\beta<\boldsymbol{\beta}$, or $\beta>\hat{\boldsymbol{\beta}}$ and $\hat{u}_{d}<1+c_{d}$, then $\hat{v}^{i}=s$ for any $i \in\{1,2\}$.
(ii) If polarization is high, cost of two-bill budgeting is moderate, and the status-quo budgeting rule is one-bill budgeting, then legislators propose two-bill budgeting if and only if the political power of each legislators is high enough; specifically, if $\beta>\hat{\boldsymbol{\beta}}, u_{d}<1+c_{d}<\hat{u}_{d}$ and $\sigma^{s}=s$ then $\hat{v}^{i}=d$ if and only if $\hat{u}_{d}-\left(1+c_{d}\right) \geq \lambda_{i}\left(\hat{u}_{d}-u_{d}\right) \geq 1+c_{d}-u_{d}$ for any $i \in\{1,2\}$.
(iii) If polarization is high, cost of two-bill budgeting is moderate, and the status-quo budgeting rule is two-bill budgeting, then legislators propose two-bill budgeting if and only if the political power of at least one legislator is high enough; specifically, if $\beta>\hat{\boldsymbol{\beta}}, u_{d}<1+c_{d}<\hat{u}_{d}$ and $\sigma^{s}=d$ then $\hat{v}^{i}=d$ if and only if $\hat{u}_{d}-\left(1+c_{d}\right) \geq \lambda_{i}\left(\hat{u}_{d}-u_{d}\right)$ or $\lambda_{i}\left(\hat{u}_{d}-u_{d}\right) \geq 1+c_{d}-u_{d}$ for any $i \in\{1,2\}$.
(iv) If polarization is high and cost of two-bill budgeting is low, then legislators propose two-bill budgeting; specifically, if $\beta>\hat{\boldsymbol{\beta}}$ and $1+c_{d}<u_{d}$, then $\hat{v}^{i}=d$ for any $i \in\{1,2\}$.

The proof of Proposition B. 2 is similar to that of Proposition 2.3. So, we skip it.

## Appendix C

## Appendix for Chapter 3

## C. 1 Proof of Lemmas and Propositions

## C.1.1 Proof of Lemma 3.1

Let $T$ be given. If the home country maximizes its revenue by choosing $t \geq T$, its problem is given as

$$
\begin{equation*}
\max _{t^{\prime} \geq T} t^{\prime}\left(1-b-\frac{1}{d}\left(t^{\prime}-T\right)\right) . \tag{C.1}
\end{equation*}
$$

The solution to Problem C. 1 is given as

$$
t= \begin{cases}\frac{d(1-b)+T}{2} & \text { if } T \leq d(1-b)  \tag{C.2}\\ T & \text { if } T>d(1-b)\end{cases}
$$

If the home country maximizes its revenue by choosing $t \leq T$, its problem is given as

$$
\begin{equation*}
\max _{t^{\prime} \leq T} t^{\prime}\left(1-b+\frac{1}{D}\left(T-t^{\prime}\right)\right) \tag{C.3}
\end{equation*}
$$

The solution to Problem C. 3 is given as

$$
t= \begin{cases}T & \text { if } T<D(1-b)  \tag{C.4}\\ \frac{D(1-b)+T}{2} & \text { if } T \geq D(1-b)\end{cases}
$$

Assume that $D>d$. Take $T \leq d(1-b)$. If the home country maximizes it revenue by choosing $t \geq T$, its revenue is $\frac{(d(1-b)+T)^{2}}{4 d}$ by Equation C.2. If it maximizes its revenue by choosing $t \leq T$, its revenue is $T(1-b)$ by Equation C.4. Since the former revenue is greater than the latter, $t(T)=\frac{d(1-b)+T}{2}$.

Take $d(1-b)<T<D(1-b)$. Whether the home country maximizes its revenue by choosing $t \geq T$ or $t \leq T$, it is optimal to choose $t=T$ by Equations C. 2 and C.4. So, $t(T)=T$.

Take $T>D(1-b)$. If the home country maximizes its revenue by choosing $t \geq T$, its revenue is $T(1-b)$ by Equation C.2. If it maximizes its revenue by choosing $t \leq T$, its revenue is $\frac{(D(1-b)+T)^{2}}{4 D}$ by Equation C.4. Since the latter revenue is greater than the former, $t(T)=\frac{D(1-b)+T}{2}$.

Assume that $D<d$. Take $T<D(1-b)$. If the home country maximizes its revenue by choosing $t \geq T$, its revenue is $\frac{(d(1-b)+T)^{2}}{4 d}$ by Equation C.2. If it maximizes its revenue by choosing $t \leq T$, its revenue is $T(1-b)$ by Equation C.4. Since the former revenue is greater than the latter, $t(T)=\frac{d(1-b)+T}{2}$.

Take $D(1-b)<T<d(1-b)$. If the home country maximizes its revenue by choosing $t \geq T$, its revenue is $\frac{(d(1-b)+T)^{2}}{4 d}$ by Equation C.2. If it maximizes its revenue by choosing $t \leq T$, its revenue is $\frac{(D(1-b)+T)^{2}}{4 D}$ by Equation C.4. Since the former revenue is greater than or equal to the latter if and only if $T \leq(1-b) \sqrt{D d}$, $t(T)=\frac{d(1-b)+T}{2}$ if $T \leq(1-b) \sqrt{D d}$ and $t(T)=\frac{D(1-b)+T}{2}$ if $T \geq(1-b) \sqrt{D d}$.


Figure C.1: Best-response correspondences when $D \geq d$.

Take $T>d(1-b)$. If the home country maximizes its revenue by choosing $t \geq T$, its revenue is $T(1-b)$ by Equation C.2. If it maximizes its revenue by choosing $t \leq T$, its revenue is $\frac{(D(1-b)+T)^{2}}{4 D}$ by Equation C.4. Since the latter revenue is greater than the former, $t(T)=\frac{D(1-b)+T}{2}$.

## C.1.2 Proof of Proposition 3.1

Assume that $D \geq d$. Then the best-response correspondences are given as in Figure C.1. To have an equilibrium in pure strategies, $\left(T_{N}, t_{N}\right)$, the best-response correspondences of the two countries should intersect. As it is clear from the graph we can have this in two cases:
(i) First, $t(T)$ defined on $T \geq D(1-b)$ can intersect with $T(t)$ defined on $t \leq(1+b) \sqrt{D d}$. In this case, we have

$$
t_{N}=\frac{D(1-b)+T_{N}}{2} \text { and } T_{N}=\frac{D(1+b)+t_{N}}{2}
$$

by Lemma 3.1. Solving these equations, we have

$$
t_{N}=D\left(1-\frac{b}{3}\right) \text { and } T_{N}=D\left(1+\frac{b}{3}\right) .
$$



Figure C.2: Best-response correspondences when $D \leq d$.

We have $t_{N}=D\left(1-\frac{b}{3}\right) \leq(1+b) \sqrt{D d}$ if and only if $\frac{3-b}{3(1+b)} \leq \sqrt{\frac{d}{D}} . \quad T_{N}=$ $D\left(1+\frac{b}{3}\right) \geq D(1-b)$ holds for all $b \geq 0$.
(ii) Second, $t(T)$ defined on $T \leq d(1-b)$ can intersect with $T(t)$ defined on $t \geq(1+b) \sqrt{D d}$. In this case, we have

$$
t_{N}=\frac{d(1-b)+T_{N}}{2} \text { and } T_{N}=\frac{d(1+b)+t_{N}}{2}
$$

by Lemma 3.1. Solving these equations, we have

$$
t_{N}=d\left(1-\frac{b}{3}\right) \text { and } T_{N}=d\left(1+\frac{b}{3}\right) .
$$

We have $T_{N}=d\left(1+\frac{b}{3}\right) \leq d(1-b)$ if and only if $b=0$, and given $b=0$ we have $t_{N}=d\left(1-\frac{b}{3}\right) \geq(1+b) \sqrt{D d}$ if and only if $D=d$. However, this case is already covered in Case (i).

Assume that $D \leq d$. Then the best-response correspondences are given as in Figure C.2. To have an equilibrium in pure strategies, $\left(T_{N}, t_{N}\right)$, the best-response correspondences of the two countries should intersect. As it is clear from the graph we can have this in two cases:
(iii) First, $t(T)$ defined on $T \geq(1-b) \sqrt{D d}$ can intersect with $T(t)$ defined on $t \leq(1+b) D$. In this case, we have

$$
t_{N}=\frac{D(1-b)+T_{N}}{2} \text { and } T_{N}=\frac{D(1+b)+t_{N}}{2}
$$

by Lemma 3.1. Solving these equations, we have

$$
t_{N}=D\left(1-\frac{b}{3}\right) \text { and } T_{N}=D\left(1+\frac{b}{3}\right) .
$$

We have $t_{N}=D\left(1-\frac{b}{3}\right) \leq(1+b) D$ for all $b \geq 0, T_{N}=D\left(1+\frac{b}{3}\right) \geq(1-b) \sqrt{D d}$ if and only if $\frac{3+b}{3(1-b)} \geq \sqrt{\frac{d}{D}}$.
(iv) Second, $t(T)$ defined on $T \leq(1-b) \sqrt{D d}$ can intersect with $T(t)$ defined on $t \geq(1+b) d$. In this case, we have

$$
t_{N}=\frac{d(1-b)+T_{N}}{2} \text { and } T_{N}=\frac{d(1+b)+t_{N}}{2}
$$

by Lemma 3.1. Solving these equations, we have

$$
t_{N}=d\left(1-\frac{b}{3}\right) \text { and } T_{N}=d\left(1+\frac{b}{3}\right) .
$$

We have $t_{N}=d\left(1-\frac{b}{3}\right) \geq d(1+b)$ if and only if $b=0$ and given $b=0$ we have $T_{N}=d\left(1+\frac{b}{3}\right) \leq(1-b) \sqrt{D d}$ if and only if $D=d$. However, this case is already covered in Case (iii).

## C.1.3 Proof of Proposition 3.2

Assume that $\sqrt{\frac{d}{D}}<\frac{3-9 b}{3(1+b)}$. We will now show that a mixed strategy equilibrium exists as given in Part (i). Assume that $T_{N}=T_{1}=\frac{(1+b)(d+\sqrt{D d})}{2}$ with probability $\alpha$ and $T_{N}=T_{2}=\frac{(1+b)(D+\sqrt{D d})}{2}$ with probability $1-\alpha$. Consider the following problem:

$$
\max _{t^{\prime} \leq t\left(T_{1}\right)} \alpha t^{\prime}\left((1-b)+\frac{1}{D}\left(T_{1}-t^{\prime}\right)\right)+(1-\alpha) t^{\prime}\left((1-b)+\frac{1}{D}\left(T_{2}-t^{\prime}\right)\right) .
$$

It is easy to verify that the solution to this problem is $t_{1 s}=t\left(T_{1}\right)$. Now, consider the following problem:

$$
\max _{t^{\prime} \geq t\left(T_{2}\right)} \alpha t^{\prime}\left((1-b)-\frac{1}{d}\left(t^{\prime}-T_{1}\right)\right)+(1-\alpha) t^{\prime}\left((1-b)-\frac{1}{d}\left(t^{\prime}-T_{2}\right)\right) .
$$

It is also easy to verify that the solution to this problem is $t_{2 s}=t\left(T_{2}\right)$. Consider also the problem

$$
\max _{t^{\prime} \in\left[t\left(T_{1}\right), t\left(T_{2}\right)\right]} \alpha t^{\prime}\left((1-b)-\frac{1}{d}\left(t^{\prime}-T_{1}\right)\right)+(1-\alpha) t^{\prime}\left((1-b)+\frac{1}{D}\left(T_{2}-t^{\prime}\right)\right) .
$$

If we ignore the constraints the solution to this problem is

$$
\begin{equation*}
t_{s 3}(\alpha)=\frac{1-b+\alpha \frac{1}{d} T_{1}+(1-\alpha) \frac{1}{D} T_{2}}{2\left(\alpha \frac{1}{d}+(1-\alpha) \frac{1}{D}\right)} . \tag{C.5}
\end{equation*}
$$

One can show that $\frac{\partial t_{s 3}}{\partial \alpha}<0$. Since $t\left(T_{1}\right)<(1+b) \sqrt{D d}<t\left(T_{2}\right)$, there exists $\alpha_{N} \in(0,1)$ such that $t_{s 3}\left(\alpha_{N}\right)=(1+b) \sqrt{D d}$. So, we have a mixed strategy equilibrium where $T_{N}=T_{1}$ with probability $\alpha_{N}, T_{N}=T_{2}$ with probability $1-\alpha_{N}$
and $t_{N}=(1+b) \sqrt{D d}$. Equating $t_{s 3}\left(\alpha_{N}\right)=(1+b) \sqrt{D d}$ in Equation C.5, we can obtain

$$
\alpha_{N}=\frac{\frac{3-b}{2}-\frac{3}{2}(1+b) \sqrt{\frac{d}{D}}}{(1+b) \frac{3}{2} \frac{D-d}{\sqrt{D d}}}
$$

Assume that $\frac{3+9 b}{3(1-b)}<\sqrt{\frac{d}{D}}$. We will now show that a mixed strategy equilibrium exists as given in Part (ii). Assume that $t_{N}=t_{1}=\frac{(1-b)(D+\sqrt{D d})}{2}$ with probability $\beta$ and $t_{N}=t_{2}=\frac{(1-b)(d+\sqrt{D d})}{2}$ with probability $1-\beta$. Consider the following problem:

$$
\max _{T^{\prime} \leq T\left(t_{1}\right)} \beta T^{\prime}\left((1+b)+\frac{1}{d}\left(t_{1}-T^{\prime}\right)\right)+(1-\beta) T^{\prime}\left((1+b)+\frac{1}{d}\left(t_{2}-T^{\prime}\right)\right) .
$$

It is easy to verify that the solution to this problem is $T_{1 s}=T\left(t_{1}\right)$. Now, consider the following problem:

$$
\max _{T^{\prime} \geq T\left(t_{2}\right)} \beta T^{\prime}\left((1+b)-\frac{1}{D}\left(T^{\prime}-t_{1}\right)\right)+(1-\beta) T^{\prime}\left((1+b)-\frac{1}{D}\left(T^{\prime}-t_{2}\right)\right) .
$$

It is also easy to verify that the solution to this problem is $T_{2 s}=T\left(t_{2}\right)$. Consider also the problem

$$
\max _{T^{\prime} \in\left[T\left(t_{1}\right), T\left(t_{2}\right)\right]} \beta T^{\prime}\left((1+b)-\frac{1}{D}\left(T^{\prime}-t_{1}\right)\right)+(1-\beta) T^{\prime}\left((1+b)+\frac{1}{d}\left(t_{2}-T^{\prime}\right)\right) .
$$

If we ignore the constraints the solution to this problem is

$$
\begin{equation*}
T_{s 3}(\beta)=\frac{1+b+\beta \frac{1}{D} t_{1}+(1-\beta) \frac{1}{d} t_{2}}{2\left(\beta \frac{1}{D}+(1-\beta) \frac{1}{d}\right)} . \tag{C.6}
\end{equation*}
$$

One can show that $\frac{\partial T_{s 3}}{\partial \beta}<0$. Since $T\left(t_{1}\right)<(1-b) \sqrt{D d}<T\left(t_{2}\right)$, there exists $\beta_{N} \in(0,1)$ such that $T_{s 3}\left(\beta_{N}\right)=(1-b) \sqrt{D d}$. So, we have a mixed strategy
equilibrium where $t_{N}=t_{1}$ with probability $\beta_{N}, t_{N}=t_{2}$ with probability $1-\beta_{N}$ and $T_{N}=(1-b) \sqrt{D d}$. Equating $T_{s 3}\left(\beta_{N}\right)=(1-b) \sqrt{D d}$ in Equation C. 6 we can obtain

$$
\beta_{N}=\frac{\frac{3+b}{2}-\frac{3}{2}(1-b) \sqrt{\frac{D}{d}}}{(1-b) \frac{3}{2} \frac{d-D}{\sqrt{D d}}} .
$$

## C.1.4 Proof of Lemma 3.2

Let $T$ be given. If the home country maximizes its revenue by choosing $t \geq T$, its problem is given as

$$
\begin{equation*}
\max _{t^{\prime} \geq T} t^{\prime} h\left(1-\frac{1}{d}\left(t^{\prime}-T\right)\right) . \tag{C.7}
\end{equation*}
$$

The solution to Problem C. 1 is given as

$$
t= \begin{cases}\frac{d+T}{2} & \text { if } T \leq d  \tag{C.8}\\ T & \text { if } T>d\end{cases}
$$

If the home country maximizes its revenue by choosing $t \leq T$, its problem is given as

$$
\begin{equation*}
\max _{t^{\prime} \leq T} t^{\prime} h+t^{\prime} H \frac{1}{D}\left(T-t^{\prime}\right) \tag{C.9}
\end{equation*}
$$

The solution to Problem C. 3 is given as

$$
t= \begin{cases}T & \text { if } T<D \theta  \tag{C.10}\\ \frac{D \theta+T}{2} & \text { if } T \geq D \theta\end{cases}
$$

Assume that $D \theta>d$. Take $T<d$. If the home country maximizes its revenue by choosing $t \geq T$, its revenue is $h \frac{(d+T)^{2}}{4 d}$, by Equation C.8. If it maximizes its revenue by choosing $t \leq T$, its revenue is $T h$, by Equation C.10. Since the former revenue is higher than the latter, $t(T)=\frac{d+T}{2}$.

Take $d<T<D \theta$. Whether the home country maximizes its revenue by choosing $t \geq T$ or $t \leq T$, it is optimal to choose $t=T$, by Equations C. 8 and C.10. So, $t(T)=T$.

Take $T>D \theta$. If the home country maximizes its revenue by choosing $t \geq T$, its revenue is $T h$, by Equation C.8. If it maximizes its revenue by choosing $t \leq T$, its revenue is $h \frac{(D \theta+T)^{2}}{4 D \theta}$, by Equation C.10. Since the latter revenue is higher than the former, $t(T)=\frac{D \theta+T}{2}$.

Assume that $D \theta<d$. Take $T<D \theta$. If the home country maximizes its revenue by choosing $t \geq T$, its revenue is $h \frac{(d+T)^{2}}{4 d}$, by Equation C.8. If it maximizes its revenue by choosing $t \leq T$, its revenue is $T h$, by Equation C.10. Since the former revenue is higher than the latter, $t(T)=\frac{d+T}{2}$.

Take $D \theta<T<d$. If the home country maximizes its revenue by choosing $t \geq T$, its revenue is $h \frac{(d+T)^{2}}{4 d}$, by Equation C.8. If it maximizes its revenue by choosing $t \leq T$, its revenue is $h \frac{(D \theta+T)^{2}}{4 D \theta}$, by Equation C.10. Since the former revenue is greater than or equal to the latter if and only if $T \leq \sqrt{D \theta d}, t(T)=\frac{d(1-b)+T}{2}$ if $T \leq \sqrt{D \theta d}$ and $t(T)=\frac{D(1-b)+T}{2}$ if $T \geq \sqrt{D \theta d}$.

Take $t>d$. If the home country maximizes its revenue by choosing $t \geq T$, its revenue is $T h$, by Equation C.8. If it maximizes its revenue by choosing $t \leq T$, its revenue is $h \frac{(D \theta+T)^{2}}{4 D}$, by Equation C.10. Since the latter revenue is higher than the former, $t(T)=\frac{D \theta+T}{2}$.

## C.1.5 Proof of Proposition 3.3

The proof of Proposition 3.3 uses Lemma 3.2 and follows similar steps to those in the proof of Proposition 3.1.

## C.1.6 Proof of Proposition 3.4

The proof of Proposition 3.4 uses Lemma 3.2 and follows similar steps to those in the proof of Proposition 3.2.

## C. 2 Revenue Functions and Best-Response Cor-

 respondences of the Foreign CountryUnder Assumption 3.1, the revenue function of the foreign country is given as

$$
R(T, t)= \begin{cases}T\left(1+b+\frac{t-T}{d}\right) & \text { if } T \leq t  \tag{C.11}\\ T\left(1+b-\frac{T-t}{D}\right) & \text { if } T \geq t\end{cases}
$$

Under Assumption 3.1, the best-response correspondence of the foreign country, $T(t)$, takes two different shapes depending on the value of the transportation costs in the two countries. If $D \leq d$, we have

$$
T(t)= \begin{cases}\frac{D(1+b)+t}{2} & \text { if } t \leq D(1+b)  \tag{C.12}\\ t & \text { if } D(1+b)<t \leq d(1+b) \\ \frac{d(1+b)+t}{2} & \text { if } d(1+b)<t\end{cases}
$$

and if $D>d$, we have

$$
T(t)= \begin{cases}\frac{D(1+b)+t}{2} & \text { if } t \leq(1+b) \sqrt{D d}  \tag{C.13}\\ \frac{d(1+b)+t}{2} & \text { if }(1+b) \sqrt{D d} \leq t\end{cases}
$$

Under Assumption 3.2, the revenue function of the foreign country is given as

$$
R(T, t)=\left\{\begin{array}{l}
T H+T h \frac{t-T}{d} \text { if } T \leq t  \tag{C.14}\\
T H\left(1-\frac{T-t}{D}\right) \text { if } T \geq t
\end{array}\right.
$$

Under Assumption 3.2, the best-response correspondence of the foreign country, $T(t)$, takes two different shapes depending on the value of the transportation costs in the two countries. If $D \theta \leq d$, we have

$$
T(t)= \begin{cases}\frac{D+t}{2} & \text { if } t \leq D  \tag{C.15}\\ t & \text { if } D<t \leq d / \theta \\ \frac{d / \theta+t}{2} & \text { if } d / \theta<t\end{cases}
$$

and if $D \theta>d$, we have

$$
T(t)= \begin{cases}\frac{D+t}{2} & \text { if } t \leq \sqrt{d D / \theta}  \tag{C.16}\\ \frac{d / \theta+t}{2} & \text { if } \sqrt{d D / \theta} \leq t\end{cases}
$$

## Appendix D

## Appendix for Chapter 4

## D. 1 Proof of Proposition 4.3

Assume that $E\left(\lambda_{2}\right)>1$. So, we have a positive investment in fiscal capacity.
Part (i) follows from the fact that $\frac{\partial \omega\left[E\left(\lambda_{2}\right)-1\right]}{\partial \omega}>0$.
Part (ii) follows from the fact that $\frac{\partial E\left(\lambda_{2}\right)}{\partial \gamma}=-(1-\phi) 2<0$.
Part (iii) follows from the fact that if $\beta_{1}>2, \frac{\partial \lambda_{1}}{\partial d}=-\left(\phi \alpha_{H}+(1-\phi) \alpha_{L}\right)$.
The first result in Part (iv) follows from the fact that if $2>\beta_{1}, \lambda_{1}=2$ and $\frac{\partial E\left(\lambda_{2}\right)}{\partial \phi}=\alpha_{H}-2(1-\gamma)>0$.

Assume that $\beta_{1}>2$. Then we can rewrite Inequality 4.3 as

$$
\begin{equation*}
\omega \frac{\left[E\left(\lambda_{2}\right)-1\right]}{\beta_{1}} \leq F_{\tau}\left(\tau_{2}-\tau_{1}\right) . \tag{D.1}
\end{equation*}
$$

The derivative of left side of Inequality D. 1 is

$$
\begin{equation*}
\omega \frac{\left[\alpha_{h}-(1-\gamma) 2\right] \beta_{1}-\left[\left(\alpha_{H}-\alpha_{L}\right)(1-d)\right]\left[E\left(\lambda_{2}\right)-1\right]}{\beta_{1}^{2}} \tag{D.2}
\end{equation*}
$$

which is greater than zero if and only if

$$
\frac{\alpha_{H}-(1-\gamma) 2}{\left(\alpha_{H}-\alpha_{L}\right)(1-d)}>\frac{E\left(\lambda_{2}\right)-1}{\beta_{1}} .
$$

This proves Parts (iv) and (v).
Part (vi) refers to multiplicative downward shift of the cost function $F$ (.).

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## Curriculum Vitae

Sezer Yaşar obtained his B.Sc. and M.Sc. degrees in Economics from the Middle East Technical University, Turkey in 2007 and 2010, respectively. He started his doctoral studies in the Department of Economics at Johns Hopkins University in 2011. He will be an Assistant Professor in the Department of Economics at TED University, Turkey in September 2017.


[^0]:    ${ }^{1}$ See, for example, Wittman (1977, 1983), Besley and Coate (1997) and Osborne and Slivinski (1996).
    ${ }^{2}$ See, for example, Romer and Rosenthal $(1978,1979)$ and Baron and Ferejohn (1989).

[^1]:    ${ }^{3}$ See, for example, Barro (1973), Breton (1974), Brennan and Buchanan (1980), Ferejohn

[^2]:    ${ }^{5}$ See, for example, Tilly $(1985,1990)$, Levi (1988) and Brewer (1989).

[^3]:    ${ }^{1}$ The data on the number of line-item appropriations come from the 2007 and 2012 budget surveys of the OECD available in its International Budget Practices and Procedures Database, http://www.oecd.org/gov/budgeting/internationalbudgetpracticesandproceduresdatabase.htm. In 2007, there are no data for Estonia, Chile, and Russia. Slovenia has 8,500, Germany has 6,000, Spain has 13,000 and Turkey has 34,583 line items. In 2012, there are no data for Argentina, Costa Rica, Peru, and Venezuela. Luxembourg has 4,400, Germany has 6,000, Russia has 12,000, Spain has 15,749 , Japan has 23,000 , Turkey has 40,000 and Portugal has 46,000 line items.

[^4]:    ${ }^{2}$ The data on discretionary budget shares are taken from the Budget of the US Government Fiscal Year 2015, Historical Tables, Table 5.5., which is available at https://www.gpo.gov/fdsys/pkg/BUDGET-2015-TAB/pdf/BUDGET-2015-TAB.pdf. The data on the number of line-item appropriations are obtained from appropriation bills of the US government, which are available at https://www.congress.gov/resources/display/content/Appropriations+and+Budget. We considered each title in an appropriation act of a department as a line-item appropriation. In years in which continuation acts were enacted, we took the number of line-item appropriations to be equal to the previous year's number.

[^5]:    ${ }^{3}$ We make this assumption for tractability. It is a common assumption in models with legislative decision making and more than one public good (see, for example, Ferejohn et al., 1987; Lockwood, 2002, 2004).

[^6]:    ${ }^{4}$ Our main result, Proposition 1.1, continues to hold under a weaker assumption that the utility of the minimum provision of a public good is high enough for the citizens. Specifically, $h\left(\alpha \frac{1}{n} L\right)>\frac{1}{n} L$.

[^7]:    ${ }^{5}$ See Alesina et al. (1999) for some references in this literature.

[^8]:    ${ }^{6}$ A more detailed explanation of the lack of ethnic tension measure is given in ICRG Methodology of PRS Group.
    ${ }^{7}$ Lack of ethnic tension is the year's average. The figure covers all countries included in the 2007 and 2012 OECD budget surveys except for outlier countries with a high number of line-item appropriations. A scatter including the outlier countries is given in Appendix A.3.
    ${ }^{8}$ One can ask if using ethnic fractionalization to measure polarization confirms the prediction of our model. We check this using the ethnic fractionalization measure of Alesina et al. (2003) which takes the value 1 if ethnic fractionalization is maximum and 0 if ethnic fractionalization is minimum in a country. Excluding the outlier countries with a high number of line-item appropriations, we find that while the correlation between ethnic fractionalization and the number of line-item appropriations is 0.44 for countries included in the 2012 budget survey of the OECD, it is -0.23 for countries included in the 2007 budget survey of the OECD. So, the evidence with ethnic fractionalization data is mixed. However, ethnic tension may measure the conflict over the provision of public goods better than ethnic fractionalization. Scatters of ethnic fractionalization and the number of line-item appropriations are not presented in the paper, but they are available upon request by the author.

[^9]:    ${ }^{1}$ See, for example, Williamson (1967), for a review of bureaucratic theory on the connection between the "control-loss" phenomenon and the size of a government bureau, as formalized by Tullock (1965) and Downs (1966). Applying this approach to firms, Williamson provides a theory of optimal firm size.

[^10]:    ${ }^{2}$ We make this assumption for tractability. It is a common assumption in models with legislative decision making and more than one public good. See Ferejohn et al. (1987) and Lockwood (2002, 2004).

[^11]:    ${ }^{1}$ In our paper, we use the specification of Nielsen (2001), as it is easier to compare with the model of Kanbur and Keen (1993).

[^12]:    ${ }^{2} \mathrm{~A}$ sufficient condition for our results is that $v=V>\max \{\max \{D, d\}(1+$ b), $\max \{D, d\} \sqrt{h / H}\}$.

[^13]:    ${ }^{3}$ The revenue function and the best-response correspondence of the foreign country can be written analogously as given in Appendix C.2.

[^14]:    ${ }^{4}$ Combining the Nielsen and KK models, Liu and Madden (2013) also show that an equilibrium in pure strategies may not exist. They also claim that if there does not exist a pure-strategy equilibrium, a mixed-strategy equilibrium exits in which the larger country chooses a lower tax rate with a positive probability. However, they do not show that the large country's tax rate is a best response in the whole parameter set at this mixed-strategy equilibrium.

[^15]:    ${ }^{5}$ The foreign country's revenue function and best-response correspondence can be written analogously, as given in Appendix C.2.

[^16]:    ${ }^{1}$ To avoid nongeneric cases, we assume that $\beta_{1} \neq 2$.

