URBAN AND REGIONAL PROGRAMMING
WITH FUZZY INFORMATION

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To the memory of Sai Wing Tam

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I. INTRODUCTION

Efficient and effective policy design and management have played an important role in the shaping and monitoring of urban and regional development in the developed countries. Though they have not been equally successful in all developed countries, they have proved to be crucial mechanisms of urban and regional planning. Developing countries, on the other hand, have been plagued by urban and regional problems, such as insufficient housing, inadequate public and private facilities, high unemployment rates, inferior environmental conditions, and high disparity of regional growth, in the past decades. The feeble economic base of the countries, compounded by the relatively inadequate planning practice is possibly a major factor contributing to such urban and regional plights. To achieve a healthier state of urban and regional growth, improvement in the efficiency and effectiveness of policy design and management is thus imperative.

Successful policy design and management, to a large extent, depend on a sound data base so that reliable estimation and prediction can be obtained. Unfortunately, most of the developing countries lack a sound data base. Information is in general weak in nature. Weak, here, connotes limited, and/or incomplete, and/or imprecise. For example, population-related planning has long suffered from a weak data base (see for instance Coale and Demeny, 1966; United Nations, 1967, 1970; Carrier and Hobcraft, 1971; Doeve, 1981; and Kwon, 1981). The prevailing weak information has, in turn, prevented successful applications of planning methods conventionally designed for the developed countries where data are relatively more readily available, more complete, or more precise.

Thus, a sound data base is mandatory for enhancing the effectiveness and efficiency of urban and regional policy design and management in developing countries. Nevertheless, existing conditions in most of these countries make the task of obtaining precise information economically, politically, or socially infeasible. While a sound data base is still beyond reach, methods which enable logical or formal analysis designed to account for a weak information base should be employed in the planning process.

Over the years, various methods have been developed to deal with the representation, analysis, and inference with weak information. Among others, fuzzy sets theory (Zadeh, 1965) is one of the methodologies which has plausible application value in the field of planning. Instead of treating precise information as obsolete, and fuzzy cognitive and decision-making processes as absurd, the theory regards them as prevailing phenomena which can be formally analyzed. Through the theory, human subjectivity and inexact information can be formally represented and analyzed. Since information on which policy design and management in developing countries are based is imprecise, fuzzy sets theory seems to be able to serve as an appropriate framework for the analysis of the decision-making processes.

The purpose of this paper is to introduce the basic ideas of this theory in general, and fuzzy linear programming in particular, to researchers or practitioners in urban and regional planning, especially to those who are involved with planning in the developing countries. Through this presentation, it is hoped that more research may be stimulated and the appropriateness of the theory in planning may be further evaluated. The core of the paper deals with allocation of resources for urban and regional development in an inexact environment. To facilitate this discussion, some basic concepts of fuzzy sets theory are summarized in the following section. Concepts and techniques of fuzzy linear programming are then discussed through a framework of urban land use allocation in section III. A simple application of the technique to a regional planning problem is then presented in section IV. To conclude the paper, section V provides an examination of the plausible applications of fuzzy sets theory to urban and regional planning in general, and to that of the developing countries in particular.

II. SOME BASIC CONCEPTS OF FUZZY SETS THEORY

This section summarizes some basic definitions and operations in fuzzy sets theory which are relevant to the discussion in section III. It is not intended to serve as a complete presentation of the theory. A more thorough examination of the theoretical foundation may be found in Zadeh (1965), Goguen (1967), and Kaufmann (1975).

A basic idea of the theory is the concept of a fuzzy subset. In conventional set theory, we have a clear-cut boundary between membership and non-membership of an element to a set. Under fuzzy sets theory, a gradual transition from membership to non-membership is considered as more realistic.

Definition 1 (Fuzzy subset). Let U be a universe of discourse, let x be an element of U. Then a fuzzy subset A in U is a set of ordered pairs

$$\{ (x, \mu_A(x)) \}, \text{ for all } x \in U, \tag{1}$$

where, $\mu_A : U \to M$ is a membership function which takes its values in a totally ordered set M, the membership set, and $\mu_A(x)$ indicates the grade of membership of x in A. The membership set M can be the closed interval [0, 1], (Zadeh, 1965), or a more general structure, e.g. a lattice (Goguen, 1967).

In this paper, the membership set is restricted to the closed interval [0, 1], with 0 and 1 representing the lowest and highest grades of membership respectively.

Example 1. In urban planning, decision-makers often encounter some types of budget constraints. Available budget may be exactly specified. However, it may only be approximately known sometimes. For instance, instead of "The budget is α," the statement "The budget is *about* α" may be the only information a planner can obtain. It is a statement which involves an inexact term "*about* α." The information conveyed is thus imprecise. To enable formal analysis, the term, "*about* α," can be represented as a fuzzy subset defined by the following membership function

$$\mu_{about\ \alpha}(x) = e^{-k(x-\alpha)^2}, \; k > 1. \tag{2}$$

(see Fig. 1)

Insert Fig. 1 about here

Example 2. In some urban design plans, an objective may be to maximize net revenue. Nevertheless, the maximization process may be subjected to a fuzzy condition. For example, the statement "net revenue should be *much greater than* β" may serve as a goal under the maximization scheme. Its defining membership function may be specified as

$$\mu_{much\ greater\ than\ \beta}(x) = 1 - e^{-k(x-\beta)}, \; k > 1. \tag{3}$$

(see Fig. 2)

Insert Fig. 2 about here

Example 3. Let $\{x_1, x_2, x_3, x_4\}$ be a set of objectives. With respect to the term *important*, the following fuzzy subset may be derived:

$$\{0.1/x_1,\ 1/x_2,\ 0.5/x_3,\ 0.8/x_4\}, \tag{4}$$

where, $\mu_{important}(x_i)$ is a subjectively assigned value with 1 representing the full membership of being *important*, and 0 representing the full non-membership of being *important*. For instance, the degree of importance of x_3 is 0.5. Here, the membership function μ does not take on a specific form.

Definition 2 (Inclusion). A fuzzy subset A is included in a fuzzy subset B, denoted as $A \subset B$, if and only if $\mu_A(x) \leq \mu_B(x)$, for all $x \in U$.

Definition 3 (Equality). Fuzzy subsets A and B are equal, denoted as

A = B, if and only if $\mu_A(x) = \mu_B(x)$, for all $x \in U$.

Definition 4 (Complementation). Fuzzy subset B is the complement of fuzzy subset A, if and only if $\mu_B(x) = 1 - \mu_A(x)$, for all $x \in U$.

Definition 5 (Intersection). The intersection of fuzzy subsets A and B, denoted as $A \cap B$, is defined by

$$\mu_{A \cap B}(x) = \min\, [\mu_A(x), \mu_B(x)]\ , \text{ for all } x \in U, \tag{5}$$

or employing the conjunctive symbol

$$\mu_{A \cap B}(x) = \mu_A(x) \wedge \mu_B(x), \text{ for all } x \in U\,. \tag{6}$$

Example 4. The intersection of the fuzzy subsets defined by equations (2) and (3) is depicted in Fig. 3.

Insert Fig. 3 about here

Remark Intersection corresponds to the connective "and." When defined by the min- operation, it is the largest fuzzy subset that is contained in both A and B. Viewing example 4 in the context of urban planning, intersection of the two fuzzy subsets can be interpreted as a policy design rule "The budget is *about* α and net revenue should be *much greater than* β." The min-operation is usually regarded as a hard "and" for it does not allow trade-off between fuzzy subsets.

Definition 6 (Union). The union of fuzzy subsets A and B, denoted as $A \cup B$, is defined by

$$\mu_{A \cup B}(x) = \max\, [\mu_A(x), \mu_B(x)]\ , \text{ for all } x \in U, \tag{7}$$

or employing the disjunctive symbol

$$\mu_{A \cup B}(x) = \mu_A(x) \vee \mu_B(x), \text{ for all } x \in U\,. \tag{8}$$

Example 5. The union of the fuzzy subsets defined by equations (2) and (3) is depicted in Fig. 3.

Remark. Union corresponds to the connective "or." When defined by the max- operation, it becomes the smallest fuzzy subset which contains both A and B. The max-operation is usually interpreted as a hard "or."

Definition 7 (Algebraic product). The algebraic product of fuzzy subsets A and B, denoted as $A \cdot B$, is defined by

$$\mu_{A \cdot B}(x) = \mu_A(x) \cdot \mu_B(x), \text{ for all } x \in U. \tag{9}$$

Remark In the intersection of fuzzy subsets, the algebraic product is usually interpreted as the connective "and" in a soft sense. It is employed when dependence of two fuzzy subsets is to be represented. The relationship between the two operations is $\mu_A(x) \wedge \mu_B(x) \geqq \mu_A(x) \cdot \mu_B(x)$, for all $x \in U$.

Definition 8 (Algebraic sum). The algebraic sum of fuzzy subsets A and B, denoted as $A \hat{+} B$, is defined by

$$\mu_{A \hat{+} B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x), \text{ for all } x \in U. \tag{10}$$

Remark. In the union of fuzzy subsets, the algebraic sum is usually interpreted as the connective "or" in the soft sense. An immediate result is $\mu_{A \hat{+} B}(x) \geqq \mu_A(x) \vee \mu_B(x)$, for all $x \in U$.

Researchers should realize that the min- and the algebraic product are not the only operations defining intersection of fuzzy subsets. Similarly, the max- and the algebraic sum are just two operations by which union can be defined (see, for example, Giles, 1976, for other definitions). The author's opinion is that in selecting or constructing a specific type of operation, we should pay attention to its mathematical justifications (for the justification of min- and max-, see Bellman and Giertz, 1973) and its relevance under specific context. Sometimes, an operation is mathematically feasible but does not possess any significant value in interpretation.

Definition 9 (Fuzzy relation). A n-ary fuzzy relation is a fuzzy subset in $U_1 \times U_2 \times \ldots \times U_n$ defined by

$$\mu_R (x_1, x_2, \ldots, x_n) \in [0, 1] , x_i \in U_i, \text{ for } i = 1, 2, \ldots, n. \quad (11)$$

Specifically, a binary fuzzy relation is a fuzzy subset in $U_1 \times U_2$.

Since a fuzzy relation is a fuzzy subset, all the operations discussed above can likewise be applied to the operations on fuzzy relations. Thus, further elaborations are not attempted here.

III. FUZZY LINEAR PROGRAMMING AND URBAN LAND USE PLANNING

Urban and regional planning in developed countries ordinarily involves the allocation of limited resources to competing activities in the most efficient way. Such a problem is especially important in developing countries where scarcity of resources is the rule rather than than the exception, and selection and implementation of the most appropriate programs are crucial for development.

Mathematical programming models, especially linear programming techniques, have been applied to economic planning in developing countries (see for example Gotsch, 1968; Bowles, 1969; MacEwan, 1971; and Wengel, 1980). In addition to a sound conceptual framework, the success of various programming models largely depends on the availability of data with exactitude. Unfortunately, the weak data base in developing countries often makes the task formidable or impossible. Though probabilistic and stochastic programming have been developed to handle planning under uncertainty, they are not designed to analyze uncertainty due to vagueness in meaning of data.[1] To better handle the problem of a weak data base, resource allocation in developing countries needs a flexible programming method so that inexact data can be tolerated and a higher degree of flexibility in programming can be accomplished.

In recent years, fuzzy mathematical programming (Bellman and Zadeh, 1970) has become an offspring of fuzzy sets research. It deals with optimization in a fuzzy decision-making environment in which objectives, constraints, or coefficients are vague. Generally, the weak data base of developing countries would likely force planners to formulate vague objectives and constraints. Thus, mathematical programming with fuzzy information seems to be appropriate for

[1] Note that probability and stochastic processes deal with uncertainty related to the randomness of occurrence, while fuzzy sets deals with that related to the vagueness of meaning.

urban and regional planning in these countries.

In this section, the basic concepts of fuzzy linear programming are discussed. To make the presentation more relevant to urban and regional planning, instead of introducing the method in a general context (Zimmermann, 1976; Negoita and Sularia, 1976), without loss of generality, I have chosen to examine it through a land use allocation problem.

Let the following be a linear programming model (modified from Schlager, 1965; Reif, 1973; and Leung, 1976) for allocating land to activities in a land use plan design:

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{12}$$

$$\text{s.t.} \sum_{i=1}^{m} \sum_{j=1}^{n} P_{ij} X_{ij} \geqq p \tag{13}$$

$$\sum_{j=1}^{n} x_{ij} \leqq f_i, \text{ for } i = 1, 2, \ldots, m \tag{14}$$

$$\sum_{i=1}^{m} d_j x_{ij} = e_j, \text{ for } j = 1, 2, \ldots, n \tag{15}$$

$$x_{ij} \geqq 0, \quad \text{for } i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n \tag{16}$$

where,

m = number of zones of equal area which form an exhaustive subdivision of the land area under study,

n = number of land use categories such as residential, industrial, agricultural, considered by the designer,

x_{ij} = number of acres of zone i to be allocated to land use category j,

c_{ij} = cost of developing an acre of zone i for allocation to land use category j,

P_{ij} = net revenue of developing an acre of zone i for allocation to land use category j,

p = expected total net return from land use allocation,

f_i = limit on the amount of land from zone i which can be allocated to land uses,

d_j = service ratio coefficient which provides for supporting service land requirements which are necessary for development of land use category j,

e_j = total demand of land use category j

The problem is to determine the optimal allocation of land to activities so that the total development cost is minimized and prescribed design standards are satisfied. The constraint in equation 13 requires total net revenue to be greater than a specific value p. Constraints in equation 14 set an exact limit on the total amount of land in each zone which can be allocated to varying land uses. Constraints in equation 15 ensure that the total allocation exactly equals the total demand in each land use category.

If our information becomes imprecise, the exactness of constraints in equations 13, 14, and 15 may decrease accordingly. That is, it may become impossible for planners to prescribe exact limits on the availability of land, f_i's, or it may become unrealistic to force an exact expectation on the total net revenue, p, from investment, or to set precise demands, e_j's, for each land use category. Thus, decision-makers may have to specify fuzzy versions of the exact constraints.

With regard to the constraint in equation 13, a fuzzy constraint

"Net return should be *greater than* p or *not much smaller than* p" (17)

may be more realistic. This constraint implies that the total net return should preferably be greater than p. In case such a requirement cannot be satisfied, due to uncertainty, it can only be smaller than p to a *small* magnitude. Since "*greater than* p" and "*not much smaller than* p" are linguistic criteria which can be treated as fuzzy subsets, they can be approximated by the functions depicted in Fig. 4a and 4b respectively. The fuzzy constraint in equation 17 is then the union of these two fuzzy subsets which imposes a fuzzy interval t on the base variable, in monetary unit (See Fig. 4c).

Insert Fig.'s 4a, 4b, and 4c about here

Thus, instead of forcing the total net return to be greater than a specific value p, a permissable level of violation, t, of p is incorporated in the fuzzy constraint. The exact constraint in equation 13 is now transformed into an inexact constraint denoted as

$$\sum_{i=1}^{m} \sum_{j=1}^{n} P_{ij}x_{ij} \underset{\sim}{\geq} p;\ p - t, \tag{18}$$

where $\underset{\sim}{\geq}$ stands for the fuzzy version of $\geq$, and p and p - t are the two extreme points of the fuzzy interval.

Accordingly, the planner's degree of satisfaction about the value $\sum_{i=1}^{m} \sum_{j=1}^{n} P_{ij}x_{ij}$ with respect to the statement in equation 17 may be approximated by the following membership function:

$$\mu\left(\sum_{i=1}^{m} \sum_{j=1}^{n} p_{ij}x_{ij}\right) = \begin{cases} 0 & \text{if } \sum_{i=1}^{m} \sum_{j=1}^{n} P_{ij}x_{ij} < p - t \\ 1 - \dfrac{P - \sum_{i=1}^{m} \sum_{j=1}^{n} P_{ij}x_{ij}}{t} & \text{if } p - t \leq \sum_{i=1}^{m} \sum_{j=1}^{n} P_{ij}x_{ij} < P \\ 1 & \text{if } \sum_{i=1}^{m} \sum_{j=1}^{n} P_{ij}x_{ij} \geq P \end{cases} \tag{19}$$

That is, when the total net revenue is greater than p, the planner is completely satisfied with the grade of membership equal to 1. His degree of satisfaction then decreases monotonically to zero towards the value p - t.

By the same token, the imprecise information about the total amount of land available for development in each zone may force planners to replace each exact constraint in equation 14 by a fuzzy constraint

"Total area for development in zone i should be
smaller than f_i or *not much greater than* f_i." (20)

Symbolically, it may be stated as:

$$\sum_{j=1}^{n} x_{ij} \lesssim f_i;\ f_i + d_i,\ \text{for } i = 1, 2, \ldots, m. \tag{21}$$

Here, we are setting tolerance levels d_i's on the availabilities of land f_i's.

The planner's degree of satisfaction about the value $\sum_{j=1}^{n} x_{ij}$ may be approximated by

$$\mu_i\left(\sum_{j=1}^{n} x_{ij}\right) = \begin{cases} 1 & \text{if } \sum_{j=1}^{n} x_{ij} \leq f_i \\ 1 - \dfrac{\sum_{j=1}^{n} x_{ij} - f_i}{d_i} & \text{if } f_i < \sum_{j=1}^{n} x_{ij} \leq f_i + d_i \\ 0 & \text{if } \sum_{j=1}^{n} x_{ij} > f_i + d_i \end{cases} \tag{22}$$

(see Fig. 5).

Along the same line of reasoning, uncertainty of the future demands makes the formulation of exact constraints in equation 15 unrealistic. Planners may have to replace each of them by the following fuzzy constraint

> "Total allocation to land use category j should be *equal to* e_j or *not much greater than* e_j and *not much smaller than* e_j." (23)

Symbolically, the constraint is expressed as

$$\sum_{i=1}^{m} d_j x_{ij} \cong e_j;\ e_j - \underline{r},\ e_j + \overline{r},\ \text{for } j = 1, 2, \ldots, n. \tag{24}$$

In this formulation, permissible levels of violation on the left, $\underline{r}$, and on the right, $\overline{r}$, of e_j are specified.

The planner's degree of satisfaction about the value $\sum_{i=1}^{m} d_j x_{ij}$ may then be approximated by the membership function

$$\mu_j\left(\sum_{i=1}^{m} d_j x_{ij}\right)=\begin{cases} 0 & \text{if } \sum_{i=1}^{m} d_j x_{ij} < e_j - \underline{r} \\ 1 - \dfrac{e_j - \sum_{i=1}^{m} d_j x_{ij}}{\underline{r}} & \text{if } e_j - \underline{r} \le \sum_{i=1}^{m} d_j x_{ij} < e_j \\ 1 & \text{if } \sum_{i=1}^{m} d_j x_{ij} = e_j \\ 1 - \dfrac{\sum_{i=1}^{m} d_j x_{ij} - e_j}{\overline{r}} & \text{if } e_j < \sum_{i=1}^{m} d_j x_{ij} \le e_j + \overline{r} \\ 0 & \text{if } \sum_{i=1}^{m} d_j x_{ij} > e_j + \overline{r} \end{cases} \quad (25)$$

(see Fig. 6)

Now, let us suppose that instead of minimizing the total development cost in equation 12, planners would prefer to employ a target value, c, in the minimization process. The objective of the problem then becomes vague and may be stated as:

"Total development cost should be *smaller than* c or *not much greater than* c." (26)

The fuzzy objective may then be expressed as

$$\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \lesssim c,\ c + \ell. \quad (27)$$

The planner's degree of satisfaction about the value $\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}$ may be approximated by

$$\mu\left(\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}\right) = \begin{cases} 1 & \text{if } \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \le c \\ 1 - \dfrac{\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} - c}{\ell} & \text{if } c < \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \le c + \ell \\ 0 & \text{if } \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} > c + \ell \end{cases} \quad (28)$$

Since both the fuzzy objective and constraints are defined as fuzzy subsets, they are symmetric in the optimization process. The feasible solutions are those which satisfy both the fuzzy objective and constraints. Thus, the decision space may then be constructed as the intersection of the fuzzy subsets defining the fuzzy objective and constraints (Bellman and Zadeh, 1970). In our formulation, the decision space, D, is the intersection of the fuzzy subsets in equations 19, 22, 25, and 28. Its membership function is

$$\mu_D(x_{ij}) = \min\{\mu(\sum_{i=1}^{m}\sum_{j=1}^{n} P_{ij}x_{ij});\ \mu_i(\sum_{j=1}^{n} x_{ij}),\ \text{all } i;$$

$$\mu_j(\sum_{i=1}^{m} d_j x_{ij}),\ \text{all } j;\ \mu(\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij})\}. \qquad (29)$$

The optimization problem is then

$$\max_{x_{ij} \geq 0} \mu_D(x_{ij}). \qquad (30)$$

By making a simple arithmetic substitution and by dropping the 1's (Zimmermann, 1976) in equations 19, 22, 25, and 28, the optimization problem in equation 29 can be simplified to

$$\max_{x_{ij} \geq 0} \min\{\frac{1}{t}\sum_{i=1}^{m}\sum_{j=1}^{n} P_{ij}x_{ij} - \frac{P}{t};\ \frac{f_i}{d_i} - \frac{1}{d_i}\sum_{j=1}^{n} x_{ij},\ \text{all } i;$$

$$\frac{e_j}{\underline{r}} - \frac{1}{\underline{r}}\sum_{i=1}^{m} d_j x_{ij},\ \frac{1}{\overline{r}}\sum_{i=1}^{m} d_j x_{ij} - \frac{e_j}{\overline{r}},\ \text{all } j; \qquad (31)$$

$$\frac{c}{\ell} - \frac{1}{\ell}\sum_{i=1}^{m}\sum_{j=1}^{m} c_{ij}x_{ij}\}$$

Equivalently, it can be rewritten as a conventional linear program

$$\max \ \lambda \qquad (32)$$

$$\text{s.t.}\ \frac{1}{t}\sum_{i=1}^{m}\sum_{j=1}^{n} P_{ij}x_{ij} - \frac{P}{t} \geq \lambda \qquad (33)$$

$$\frac{f_i}{d_i} - \frac{1}{d_i}\sum_{j=1}^{n} x_{ij} \geq \lambda,\ \text{for } j = 1, 2, \ldots, m \qquad (34)$$

$$\frac{e_j}{r} - \frac{1}{r} \sum_{i=1}^{m} d_j x_{ij} \geq \lambda \text{ , for } j = 1, 2, \ldots, n \tag{35}$$

$$\frac{1}{r} \sum_{i=1}^{m} d_j x_{ij} - \frac{e_j}{r} \geq \lambda, \text{ for } j = 1, 2, \ldots, n \tag{36}$$

$$\frac{c}{\ell} - \frac{1}{\ell} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \geq \lambda \tag{37}$$

$$x_{ij} \geq 0, \text{ for } i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n \tag{38}$$

Therefore, with the linearity assumption, planning programs with fuzzy objective and constraints can be formulated as a conventional linear program which can be readily solved by existing algorithms.

With the availability of exact information on some planning aspects, planners may be able to specify some constraints with exactitude. For example, in addition to the fuzzy constraints previously discussed, planners may want to impose the following density constraints:

$$x_{ij} \leq g_{jk} x_{ik}, \qquad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, k, \ldots, n \tag{39}$$

$$x_{ij} \leq h_{jk} x_{hk}, \qquad i = 1, 2, \ldots, h, \ldots, m; \quad j = 1, 2, \ldots, k, \ldots, n \tag{40}$$

where,

g_{jk} = ratio of land use category j allowed relative to land use category k, with land use categories j and k in the same zone, and

k_{jk} = ratio of land use categories j allowed relative to land use category k, with land use categories j and k in different zones.

Such exact constraints can be added to the linear program, equations 32 to 38, and solve for x_{ij}'s simultaneously.

Though the basic concepts of fuzzy linear programming is introduced via the context of urban land use allocation, the fundamental principles of the technique can be applied to planning problems in various contexts. As an illustration, fuzzy linear programming is applied to a simple regional resource allocation problem in the following section.

IV. AN APPLICATION OF FUZZY LINEAR PROGRAMMING TO A REGIONAL RESOURCE ALLOCATION PROBLEM

In this section, fuzzy linear programming is applied to a simplified regional resource allocation problem (based on the example in Hillier and Lieberman, 1980, pp. 27-29) which involves the communal farming communities, the system of kibbutzim, in Israel.

It is common for groups of kibbutzim to join together as a confederation, to share common technical services and to coordinate their production. Within the confederation, the overall planning is carried out by a coordinating technical office. The function of the office is to plan agricultural production of the confederation for the coming year.

The present example concerns a confederation of three kibbutzim. The agricultural output of each kibbutz is limited by the amount of available irrigable land, which can be exactly prescribed, and by the quantity of water, which can only be vaguely specified, allocated for irrigation by the Water Commissioner (a national government official).
(Table 1).

Insert Table 1 about here

The crops under consideration are sugar beets, cotton, and sorghum. These crops differ in their expected net return and their consumption of water. Fuzzy maximum quota for the total amount of land that can be devoted to each of the crops are also imposed by the Ministry of Agriculture. These exact and inexact data are tabulated in Table 2.

Insert Table 2 About Here

The three kibbutzim of the Confederation have agreed that every kibbutz will plant the same proportion of its available irrigable land. Nevertheless, any combination of the crops may be grown at any of the kibbutzim.

For the coming year, the coordinating technical office has to determine the amount of land to be devoted to each crop in the respective kibbutzim so that the above prescribed restrictions are satisfied. In place of taking the maximization of total net return to the confederation as an objective, it is decided that a target value, 260,000 dollars, should be employed and the total net return is required to exceed the target value or be not much below it. Thus, the objective can be stated as

"Total net return should be greater than 260,000 dollars or not much smaller than 260,000 dollars,"

with the associated fuzzy interval [250,000 , 26,000] .

Based on the above information, the resource allocation problem of the Confederation can be formulated as a fuzzy linear program as follows:

Objective:

$$400(x_{11} + x_{12} + x_{13}) + 300(x_{21} + x_{22} + x_{23}) + 100(x_{31} + x_{32} + x_{33}) \geq 260{,}000;\ 250{,}000 \quad (41)$$

Constraints:

Water (fuzzy):

$$3\ x_{11} + 2\ x_{21} + x_{31} \leq 600;\ 660 \quad (42)$$

$$3\ x_{12} + 2\ x_{22} + x_{32} \leq 800;\ 840 \quad (43)$$

$$3\ x_{13} + 2\ x_{23} + x_{33} \leq 375;\ 450 \quad (44)$$

Crop (fuzzy):

$$x_{11} + x_{12} + x_{13} \leq 600;\ 650 \quad (45)$$

$$x_{12} + x_{22} + x_{32} \leq 500;\ 540 \quad (46)$$

$$x_{31} + x_{32} + x_{33} \leq 325;\ 350 \quad (47)$$

Land (exact):

$$x_{11} + x_{21} + x_{31} \leq 400 \quad (48)$$

$$x_{12} + x_{22} + x_{32} \leq 600 \quad (49)$$

$$x_{13} + x_{23} + x_{33} \leq 300 \quad (50)$$

Proportional utilization (exact):

$$\frac{1}{400}(x_{11} + x_{21} + x_{31}) = \frac{1}{600}(x_{12} + x_{22} + x_{32}) \quad (51)$$

$$\frac{1}{600}(x_{12} + x_{22} + x_{32}) = \frac{1}{300}(x_{13} + x_{23} + x_{33}) \quad (52)$$

$$\frac{1}{300}(x_{13} + x_{23} + x_{33}) = \frac{1}{400}(x_{11} + x_{21} + x_{31}) \quad (53)$$

Non-negativity:

$$x_{ij} \geq 0, \text{ for } i = 1, 2, 3 \text{ and } j = 1, 2, 3, \quad (54)$$

where x_{ij} = number of acres of land in kibbutz j to be allocated to plant crop i (1 = sugar beets, 2 = cotton, 3 = sorghum).

Following the arguments in section III, the above fuzzy linear program can be formulated as a regular linear program as follows:

$$\max \quad \lambda$$

$$\text{s.t.} -26 + .04(x_{11} + x_{12} + x_{13}) + .03(x_{21} + x_{22} + x_{23}) + .01(x_{31} + x_{32} + x_{33}) \geq \lambda \quad (5$$

$$10 - .05\ x_{11} - .03\ x_{21} - .02\ x_{31} \geq \lambda \tag{56}$$

$$20 - .08\ x_{12} - .05\ x_{22} - .03\ x_{32} \geq \lambda \tag{57}$$

$$5 - .04\ x_{13} - .03\ x_{23} - .01\ x_{33} \geq \lambda \tag{58}$$

$$12 - .02\ x_{11} - .02\ x_{12} - .02\ x_{13} \geq \lambda \tag{59}$$

$$12.5 - .03\ x_{21} - .03\ x_{22} - .03\ x_{23} \geq \lambda \tag{60}$$

$$13 - .04\ x_{31} - .04\ x_{32} - .04\ x_{33} \geq \lambda \tag{61}$$

$$x_{11} + x_{21} + x_{31} \leq 400 \tag{62}$$

$$x_{12} + x_{22} + x_{32} \leq 600 \tag{63}$$

$$x_{13} + x_{23} + x_{33} \leq 300 \tag{64}$$

$$3(x_{11} + x_{21} + x_{31}) - 2(x_{12} + x_{22} + x_{32}) = 0 \tag{65}$$

$$x_{12} + x_{22} + x_{32} - 2(x_{13} + x_{23} + x_{33}) = 0 \tag{66}$$

$$4(x_{13} + x_{23} + x_{33}) - 3(x_{11} + x_{21} + x_{31}) = 0 \tag{67}$$

$$x_{ij} \geq 0, \text{ for } i = 1, 2, 3 \text{ and } j = 1, 2, 3 \tag{68}$$

The optimal solution is

$$\begin{aligned}&(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33})\\ &= (140, 155, 112.5, 107.778, 316.667, 0, 0, 0, 73.333)\end{aligned} \tag{69}$$

V. CONCLUSION

Fuzzy sets theory has been introduced as an analytical framework by which urban and regional planning with inexact information can be handled. Specifically, fuzzy linear programming has been examined via an urban land use allocation framework and has been applied to a simplified regional resource allocation problem. While fuzzy linear programming appears to be appropriate in solving optimal allocation problems with fuzzy objectives and constraints, there are several aspects which require further attention.

1. In fuzzy linear programming, membership functions are approximated by linear functions. However, well-behaved monotonically increasing or decreasing functions can also be handled with no major difficulties.

2. In constructing the decision space, the min- operation is ordinarily employed as a rule of confluence. Nevertheless, operations such as the algebraic product (Zimmermann, 1978) and addition (Sommer and Pollatschek, 1980) can similarly be applied. At present, there are no definitive empirical verifications on how decision-makers combine fuzzy objectives and constraints. Thus, more empirical analyses are required before the most suitable operation can be determined.

3. In this paper, constraints become fuzzy when limitations on available resources become vague. Likewise, an objective becomes fuzzy when the target value of the objective function becomes ambiguous. However, they are not the only sources of fuzziness. When the coefficients of the objective function and that of the constraints are fuzzy numbers, the optimization problem again becomes a fuzzy mathematical programming problem with fuzzy objectives and constraints. Such a problem can also be effectively transformed into a regular linear program (Negoita and Sularia, 1976; Dubois and Prade, 1980).

4. Though only fuzzy linear programming with a single objective has been discussed, the framework can easily be extended to multiple objective optimization characterizing most planning problems. The symmetry of the objectives and constraints enables the formulation of a multiple objective fuzzy linear program as a single objective fuzzy linear program (Zimmermann, 1978).

5. To cope with the dynamic aspects of planning, the current framework can be extended to deal with fuzzy optimization over time (Bellman and Zadeh, 1970). In particular, fuzzy linear programming can easily be extended to solve multi-stage planning with time-dependent fuzzy objectives and constraint.

Though our discussion in this paper is restricted to urban and regional planning in the context of fuzzy mathematical programming, it has been demonstrated that fuzzy sets theory is also applicable in analyzing more general planning problems such as those involving hierarchical objectives (Leung, 1979) and multicriteria conflicts (Nijkamp, 1979; Leung, 1981; 1982, and 1983).

To recapitulate, new methodologies such as fuzzy sets theory appear to be appropriate for dealing with urban and regional planning in developing countries suffering from the acute problem of a weak data base. Though the theory is likely not the ultimate solution for the analysis of human subjectivity and inexact information, it is, however, more realistic than most of the conventional analytical techniques. Following the argument of the present paper, one may have the misconception that planning in developed countries is free from the problem of inexact information. On the contrary, experiences have demonstrated that human subjectivity and inexact information also prevail, albeit to a lesser extent, in the highly complex urban and regional systems and decision-making processes in developed countries. Thus, development of new methodologies such as fuzzy sets theory is pertinent for realistic planning.

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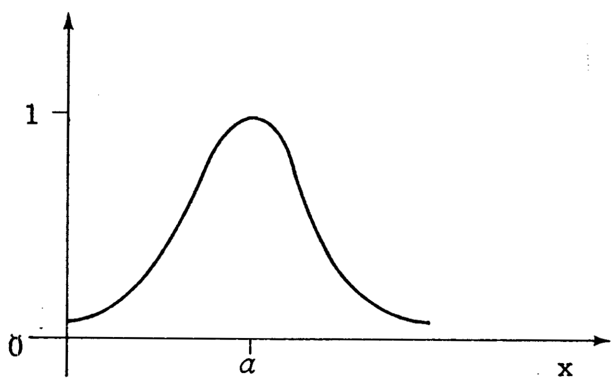

Fig. 1 Membership function of the fuzzy subset *"about* α*"*

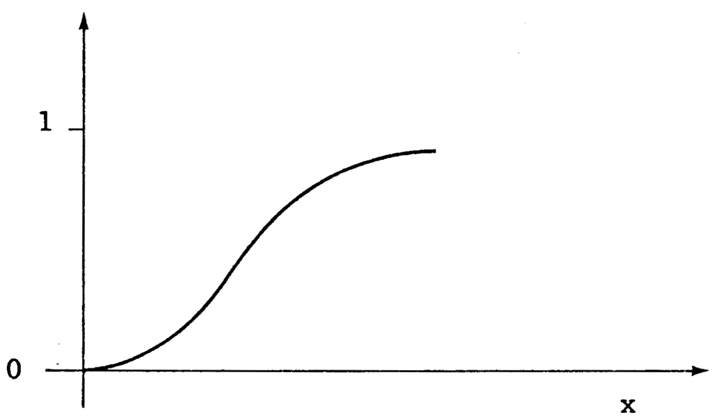

Fig. 2 Membership function of the fuzzy subset "*much greater than* β"

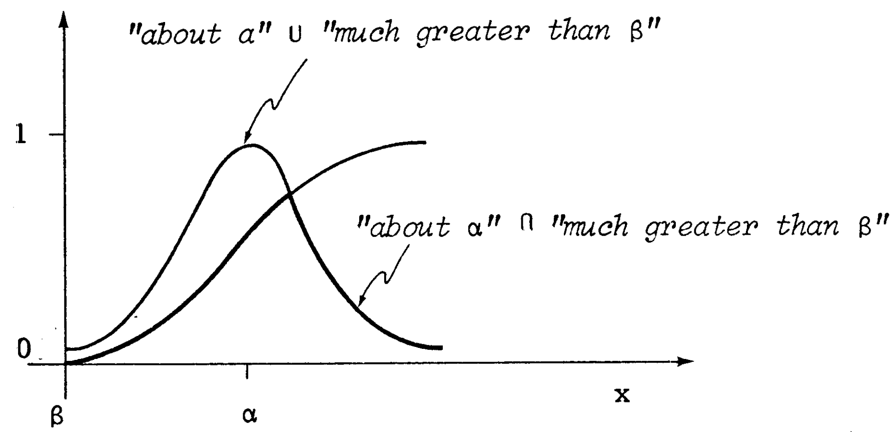

Fig. 3 Membership function of the intersection and union of fuzzy subsets *"about α"* and *"much greater than β"*

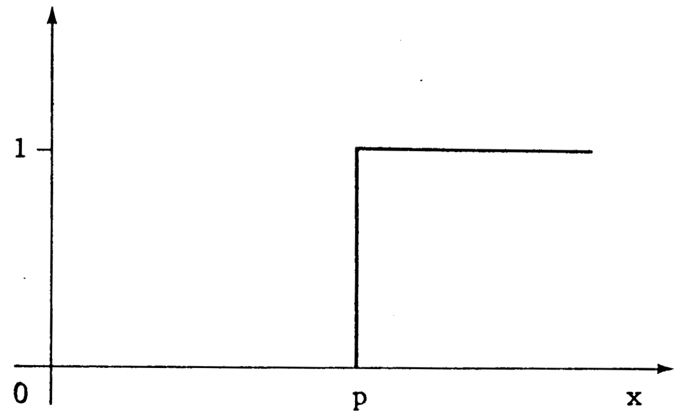

Fig. 4a Membership function of *"greater than* p"

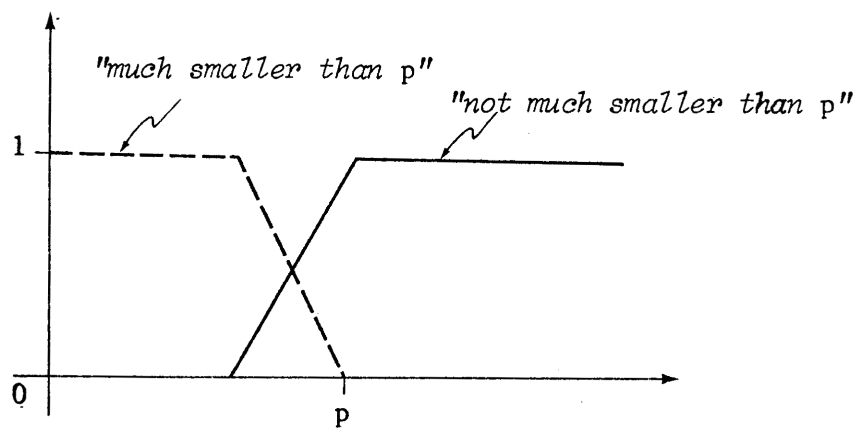

Fig. 4b Membership function of *"not much smaller than* p"

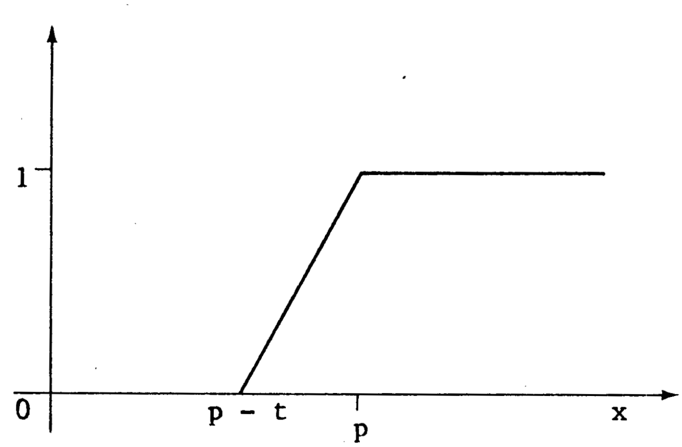

Fig. 4c Membership function of *"greater than* p" or *"not much smaller than* p"

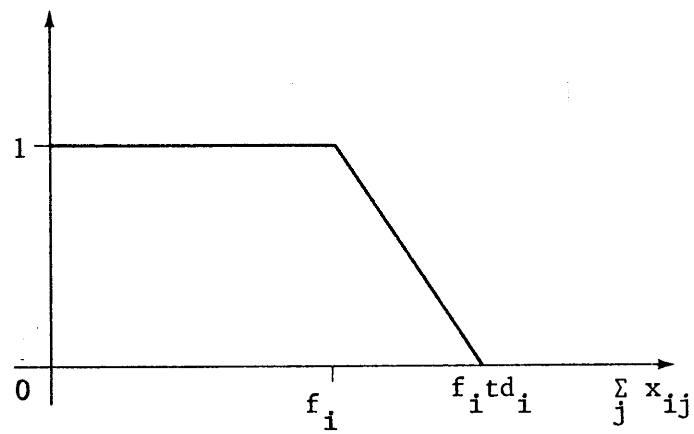

Fig. 5 Membership function of the degree of satisfaction about the value $\sum_j x_{ij}$

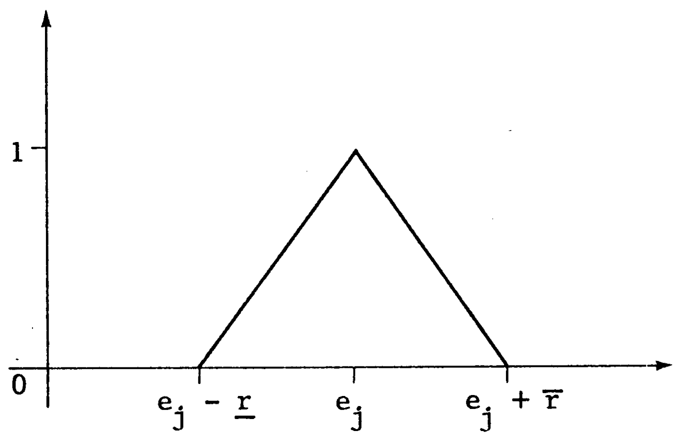

Fig. 6 Membership function of the degree of satisfaction about the value $\sum_i d_j x_{ij}$

Kibbutz	Irrigable land (acres)	water allocation (acre feet)	
		fuzzy specification	associated fuzzy interval
1	400	should be less than 600 or not much greater than 600	[600,660]
2	600	should be less than 800 or not much greater than 800	[800,840]
3	300	should be less than 375 or not much greater than 375	[375,450]

Table 1 Resources data for the Confederation of Kibbutzim

Crop	Net Return (dollars/acre)	(acre feet/acre)	maximum quota (acres)	
			fuzzy specification	associated fuzzy interval
Sugar beets	400	3	should be less than 600 or not much greater than 600	[600,650]
Cotton	300	2	should be less than 500 or not much greater than 500	[500,540]
Sorghum	100	1	should be less than 325 or not much greater than 325	[325,350]

Table 2 Crop data for the Confederation of Kibbutzim